MORE ON SHAPE OPERATORS

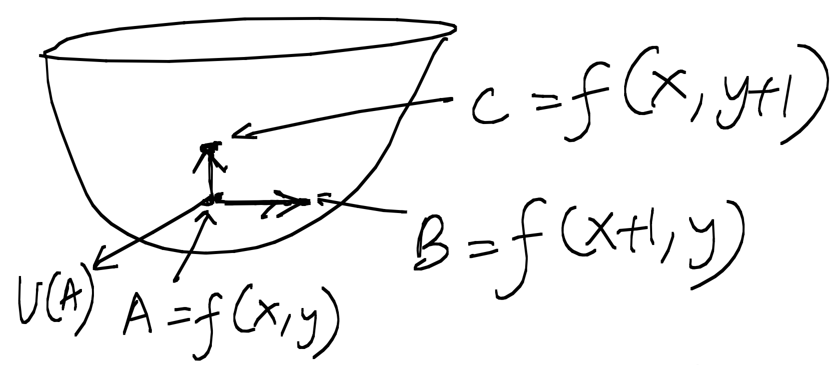
Note Title 4/17/2006

Shape operator for a generic surface

$M: \; z = f(x, y)$

Compute U : Cross product of two tangent vectors.

Consider three pts.



$A: (x, y, f(x, y))$

$B: (x+1, y, f(x+y) + f_x(x, y))$

$C: (x, y+1, f(x+y) + f_y(x+y))$

$$\text{dir of. } U(A) = (B-A) \times (C-A)$$

$$= (1, 0, f_x) \times (0, 1, f_y)$$

$$= (-f_x) U_1 + (-f_y) U_2 + U_3.$$

$$U(A) = \frac{(-f_x) U_1 + (-f_y) U_2 + U_3}{\sqrt{1 + f_x^2 + f_y^2}}$$

(at $(0,0)$ denominator is 1)

$u_1 = (1, 0, 0)$

$$\nabla_{u_1} U = u_1[-f_x] U_1 + u_1[-f_y] U_2 + u_1[1] U_3$$

$$= -f_{xx} U_1 - f_{xy} U_2$$

$$S(u_1) = -\nabla_{u_1} U = f_{xx}(0,0) U_1 + f_{xy}(0,0) U_2$$

Similarly

$$S(u_2) = f_{yx}(0,0) U_1 + f_{yy}(0,0) U_2$$