

Lecture 04

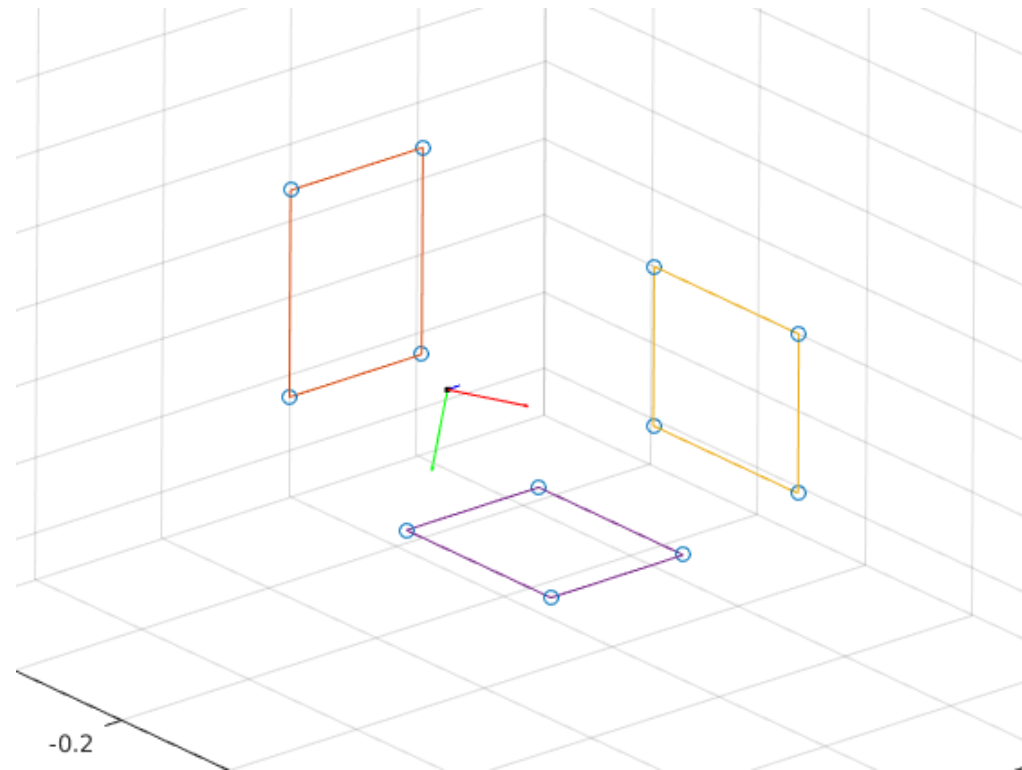
Image Filtering

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Lab Exercise 2 - Today afternoon

- Room ETH HG E 33.1 from 14:15 to 16:00
- Work description: your first camera motion estimator using DLT



Course Schedule update

For updates, slides, and additional material: <http://rpg.ifi.uzh.ch/teaching.html>

| Date | Time | Description of the lecture/exercise | Lecturer |
|------------|---------------|---|-------------------------------|
| 22.09.2016 | 10:15 - 12:00 | 01 – Introduction | Scaramuzza |
| 29.09.2016 | 10:15 - 12:00 | 02 - Image Formation 1: perspective projection and camera models | Scaramuzza |
| 06.10.2016 | 10:15 - 12:00 | 03 - Image Formation 2: camera calibration algorithms | Scaramuzza |
| | 14:15 – 16:00 | Lab Exercise 1: Augmented reality wireframe cube | Titus Cieslewski/Henri Rebecq |
| 13.10.2016 | 10:15 - 12:00 | 04 - Filtering & Edge detection | Scaramuzza |
| | 14:15 – 16:00 | Lab Exercise 2: PnP problem | Titus Cieslewski/Henri Rebecq |
| 20.10.2016 | 10:15 - 12:00 | 05 - Point Feature Detectors 1: Harris detector | Scaramuzza |
| | 14:15 – 16:00 | Lab Exercise 3: Harris detector + descriptor + matching | Titus Cieslewski/Henri Rebecq |
| 27.10.2016 | 10:15 - 12:00 | 06 - Point Feature Detectors 2: SIFT, BRIEF, BRISK | Scaramuzza |
| 3.11.2016 | 10:15 - 12:00 | 07 - Multiple-view geometry 1 | Scaramuzza |
| | 14:15 – 16:00 | Lab Exercise 4: Stereo vision: rectification, epipolar matching, disparity, triangulation | Titus Cieslewski/Henri Rebecq |
| 10.11.2016 | 10:15 - 12:00 | 08 - Multiple-view geometry 2 | Scaramuzza |
| | 14:15 – 16:00 | Exercise 5: Eight-point algorithm and RANSAC | Titus Cieslewski/Henri Rebecq |
| 17.11.2016 | 10:15 - 12:00 | 09 - Multiple-view geometry 3 | Scaramuzza |
| | 14:15 – 16:00 | Exercise 6: P3P algorithm and RANSAC | Titus Cieslewski/Henri Rebecq |
| 24.11.2016 | 10:15 - 12:00 | 10 - Dense 3D Reconstruction (Multi-view Stereo) | Scaramuzza |
| | 14:15 – 16:00 | Exercise 7: Intermediate VO Integration | Titus Cieslewski/Henri Rebecq |
| 01.12.2016 | 10:15 - 12:00 | 11 - Optical Flow and Tracking (Lucas-Kanade) | Scaramuzza |
| | 14:15 – 16:00 | Exercise 8: Lucas-Kanade tracker | Titus Cieslewski/Henri Rebecq |
| 08.12.2016 | 10:15 - 12:00 | 12 – Place recognition | Scaramuzza |
| | 14:15 – 16:00 | Exercise 9: Recognition with Bag of Words | Titus Cieslewski/Henri Rebecq |
| | 10:15 - 12:00 | 13 – Visual inertial fusion | Scaramuzza |
| 15.12.2016 | 14:15 – 16:00 | Exercise 10: Pose graph optimization and Bundle adjustment | Titus Cieslewski/Henri Rebecq |
| 22.12.2016 | 10:15 - 12:00 | 14 - Event based vision + lab visit and live demonstrations | Scaramuzza |
| | 14:15 – 16:00 | Exercise 11: final VO integration | Titus Cieslewski/Henri Rebecq |

Image filtering

- The word *filter* comes from frequency-domain processing, where “filtering” refers to the process of accepting or rejecting certain frequency components
- We distinguish between low-pass and high-pass filtering
 - A **low-pass filter** smooths an image (retains low-frequency components)
 - A **high-pass filter** retains the contours (also called edges) of an image (high frequency)



Low-pass filtered image



High-pass filtered image



Low-pass filtering

Low-pass filtering

Motivation: noise reduction

- **Salt and pepper noise:** random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise:** variations in intensity drawn from a Gaussian normal distribution



Original



Salt and pepper noise

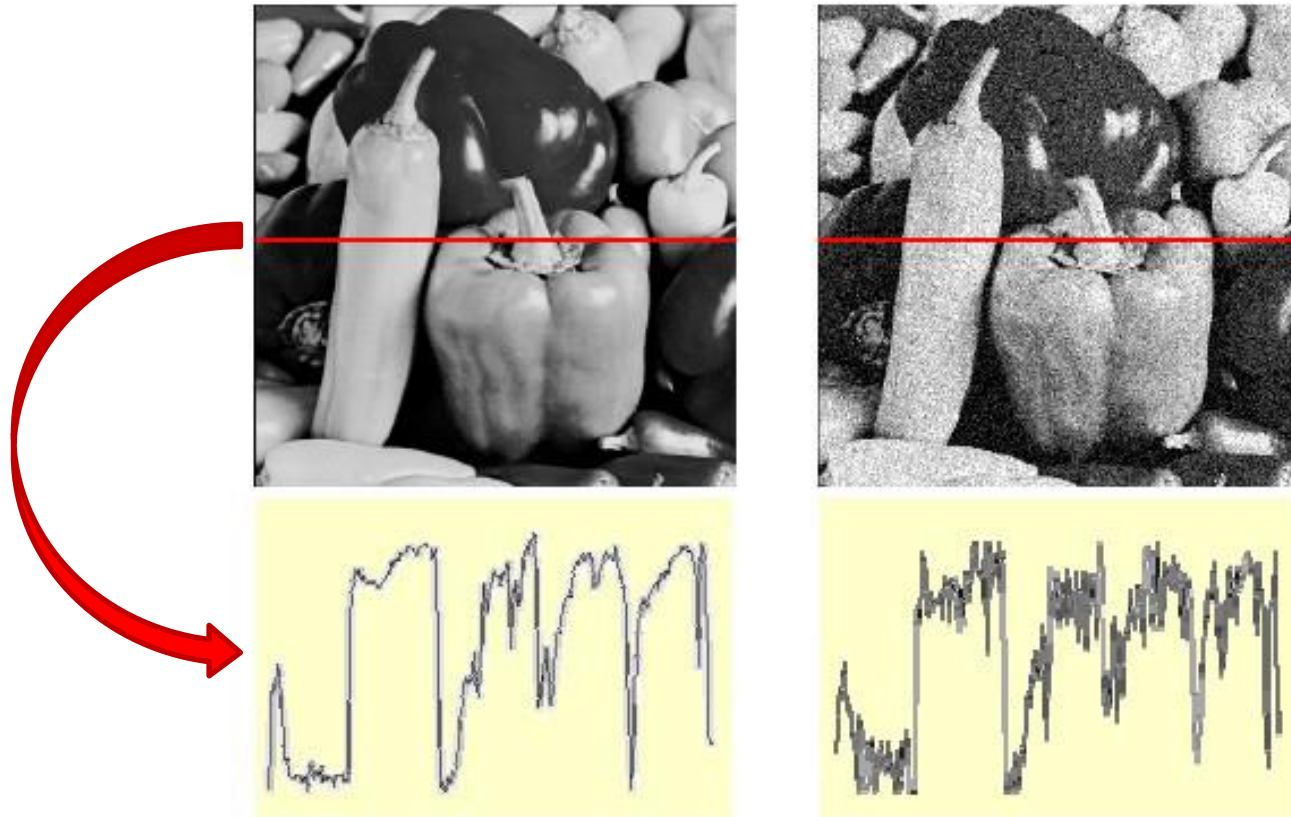


Impulse noise



Gaussian noise

Gaussian noise



$$f(x, y) = \overbrace{\hat{f}(x, y)}^{\text{Ideal Image}} + \overbrace{\eta(x, y)}^{\text{Noise process}}$$

Gaussian i.i.d. ("white") noise:
 $\eta(x, y) \sim \mathcal{N}(\mu, \sigma)$

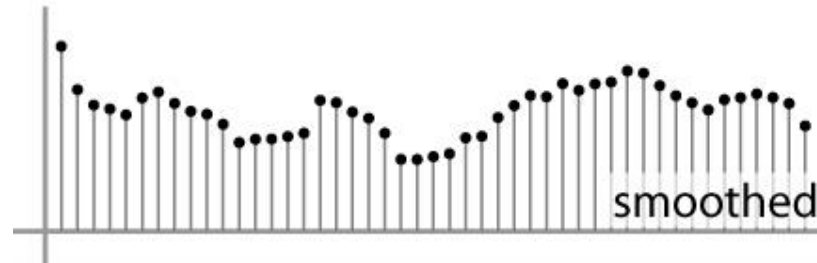
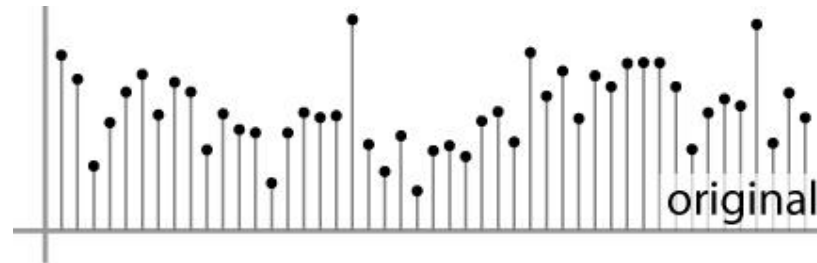
How could we reduce the noise to try to recover the "ideal image"?

Moving average

- Replaces each pixel with an average of all the values in its neighborhood
- Assumptions:
 - Expect pixels to be like their neighbors
 - Expect noise processes to be independent from pixel to pixel

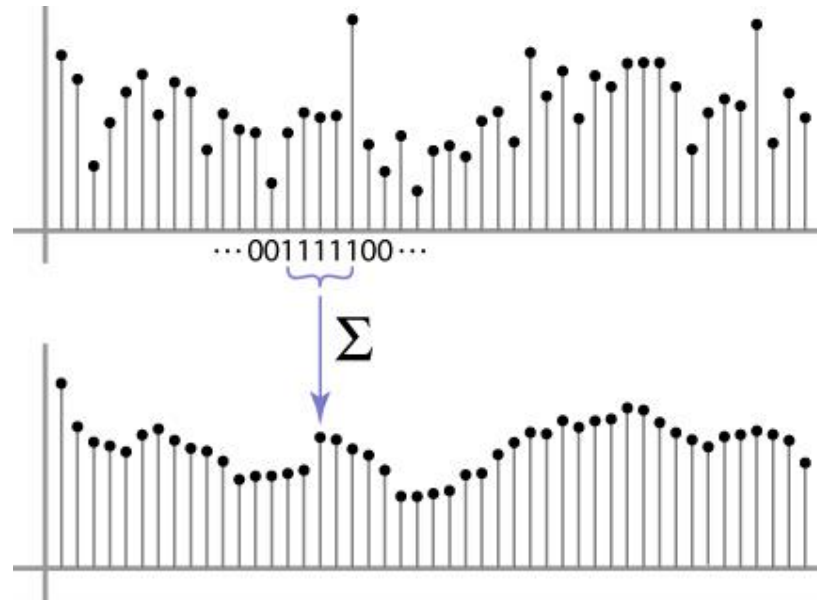
Moving average

- Replaces each pixel with an average of all the values in its neighborhood
- Moving average in 1D:



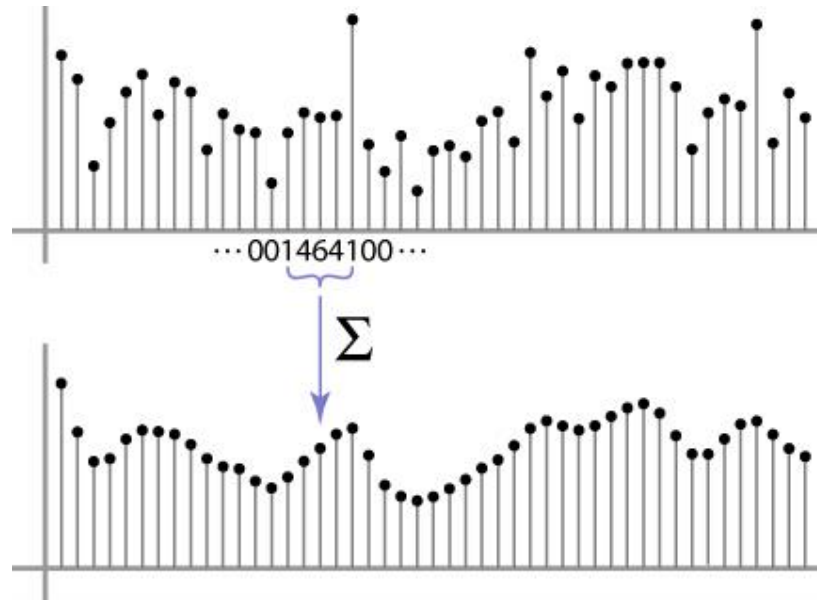
Weighted Moving Average

- Can add weights to our moving average
- *Weights* $[1, 1, 1, 1, 1] / 5$



Weighted Moving Average

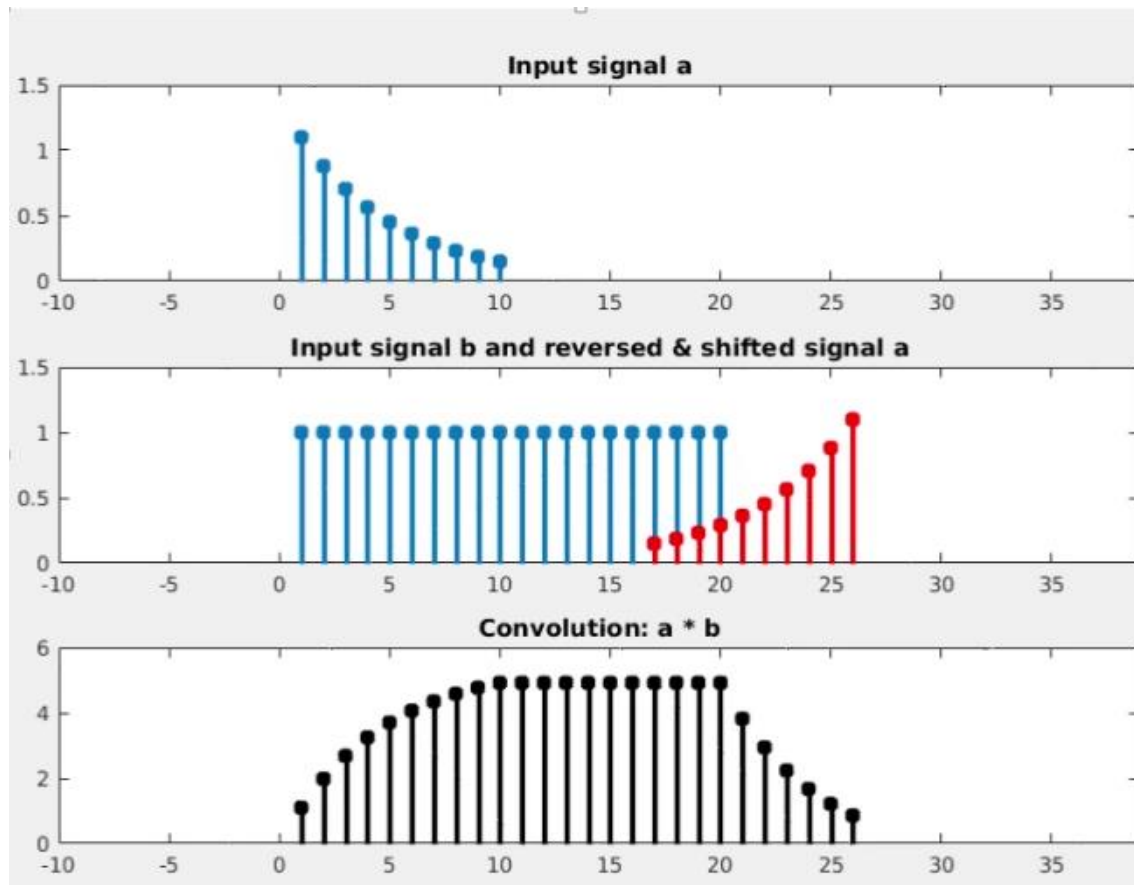
- Non-uniform weights [1, 4, 6, 4, 1] / 16



This operation is called *convolution*

Example of convolution of two sequences (or “signals”)

- One of the sequences is flipped (right to left) before sliding over the other
- Notation: $a \star b$
- Nice properties: linearity, associativity, commutativity, etc.

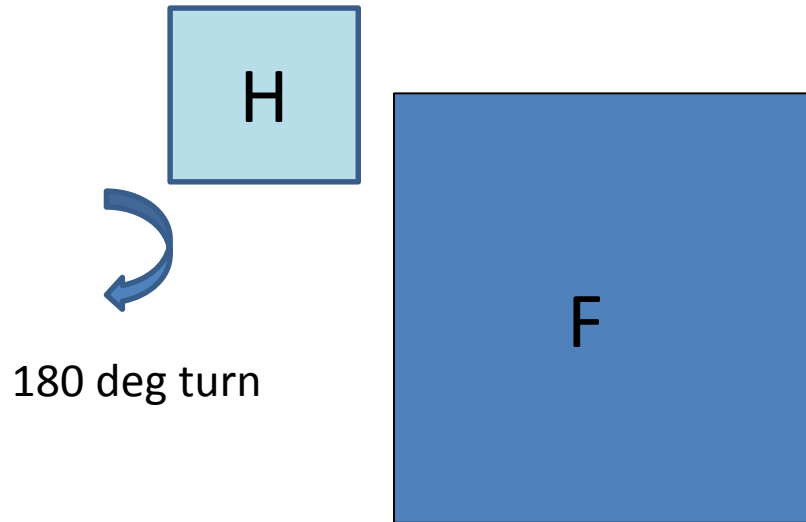


2D Filtering

- Convolution:
 - Flip the filter in both dimensions (bottom to top, right to left)
 - Then slide the filter over the image

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i - u, j - v]$$

$$G = H \star F$$



Filtering an image: replace each pixel with a linear combination of its neighbors.

The **filter** H is also called “**kernel**” or “**mask**”.

It allows to have different weights depending on neighboring pixel’s relative position.

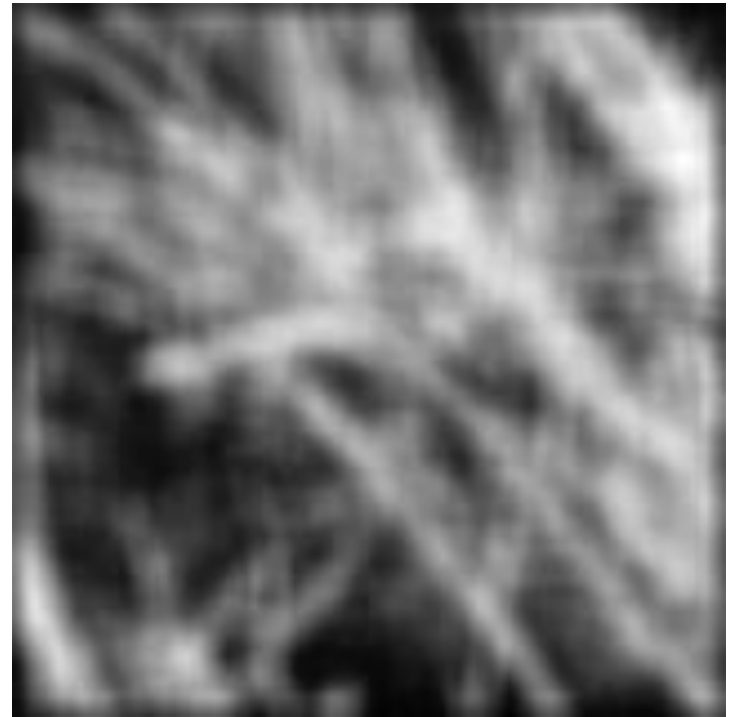
Smoothing by averaging



Box filter:
white = high value, black = low value



original



filtered

Gaussian filter

- What if we want the closest pixels to have higher influence on the output?

| | | | | | | | | | |
|---|---|----|----|----|----|----|----|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

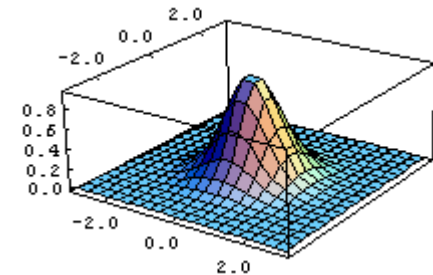
$F[x, y]$

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 1 | 2 | 1 |

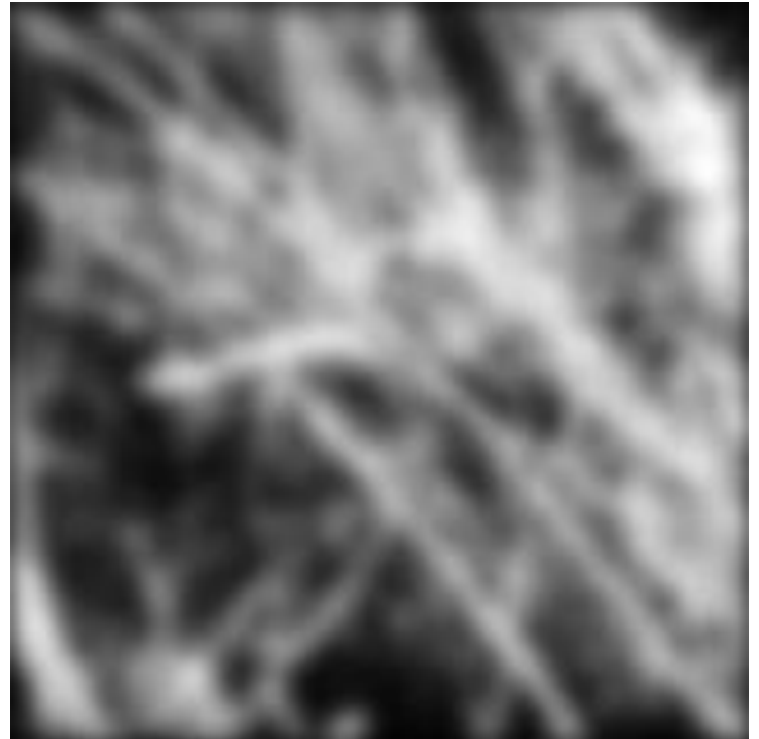
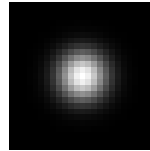
$H[u, v]$

This kernel is an approximation of a Gaussian function:

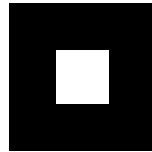
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



Smoothing with a Gaussian

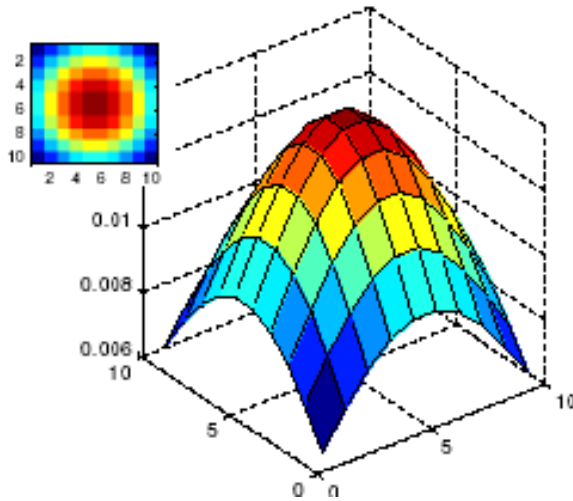


Compare the result with a box filter

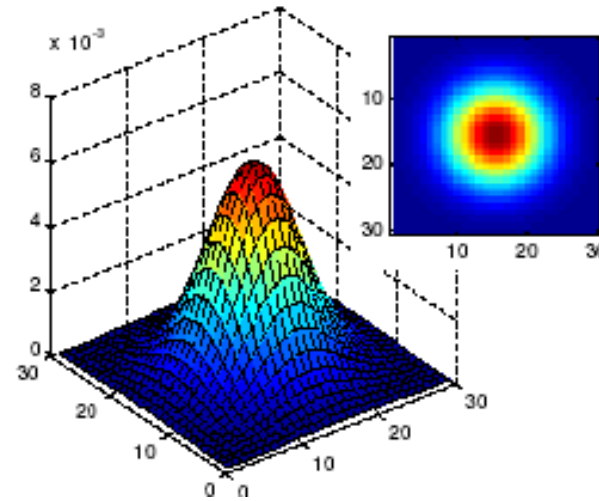


Gaussian filters

- What parameters matter?
- **Size** of kernel or mask
 - Note, Gaussian function has infinite support, but discrete filters use finite kernels



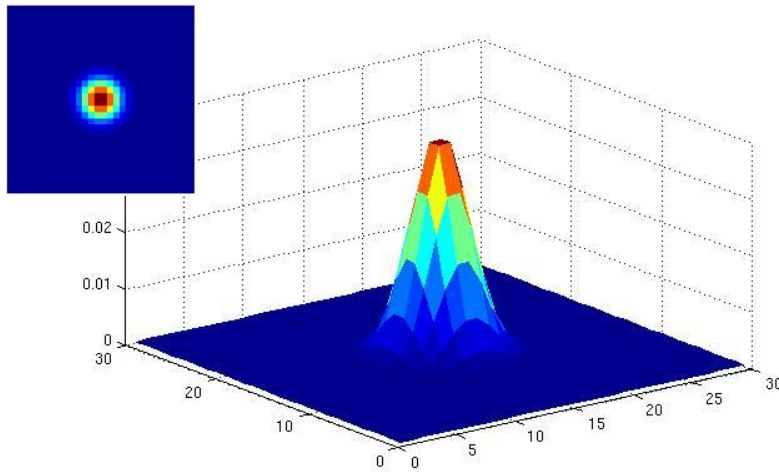
$\sigma = 5$ pixels
with 10 x 10 pixel kernel



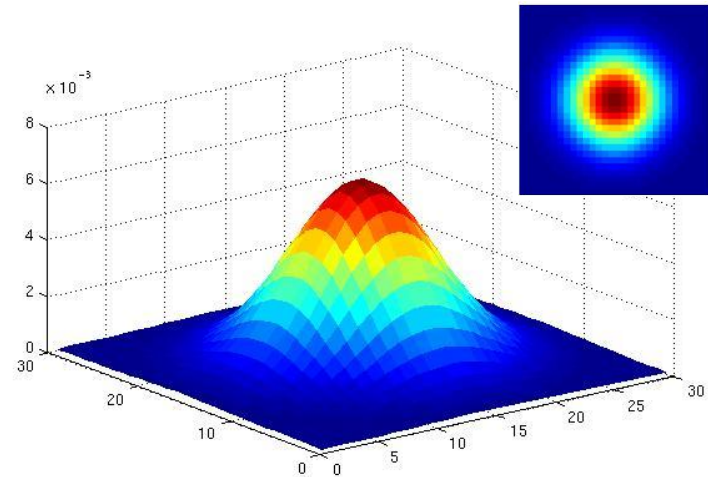
$\sigma = 5$ pixels
with 30 x 30 pixel kernel

Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing



$\sigma = 2$ pixels
with 30 x 30 pixel kernel

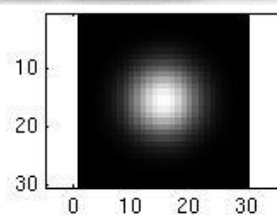
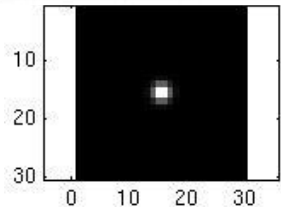


$\sigma = 5$ pixels
with 30 x 30 pixel kernel

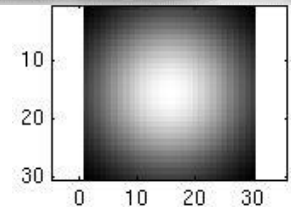
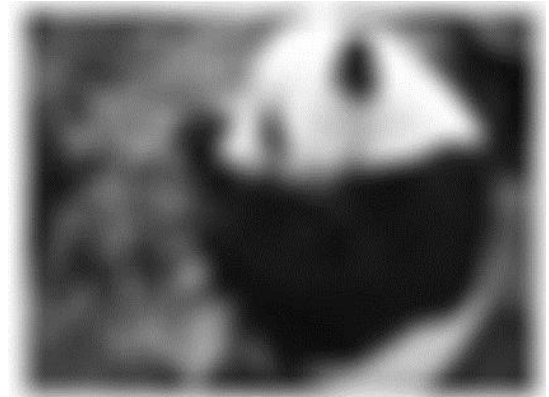
Recall: standard deviation = σ [pixels], variance = σ^2 [pixels²]

Smoothing with a Gaussian

Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.



...



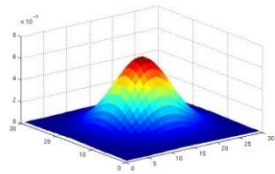
In practice there is just one parameter:

the kernel size is computed given the variance or vice versa.

Sample Matlab code

```
>> hsize = 10;  
>> sigma = 5;  
>> h = fspecial('gaussian' hsize, sigma);
```

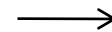
```
>> mesh(h);
```



```
>> imagesc(h);
```



```
>> im = imread('panda.jpg');  
>> outim = imfilter(im, h);  
>> imshow(outim);
```



outim

Boundary issues

- What about near the edge?
 - the filter window falls off the edge of the image
 - need to pad the image borders
 - methods:
 - zero padding (black)
 - wrap around
 - copy edge
 - reflect across edge



Summary on (linear) filters

- Smoothing

- Filter has positive values (also called coefficients)
- Sum to 1 \rightarrow preserve brightness of constant regions
- Amount of smoothing is proportional to mask size
- Remove “high-frequency” components; “low-pass” filter

Non-linear filtering

Effect of smoothing filters

5x5



Additive Gaussian noise

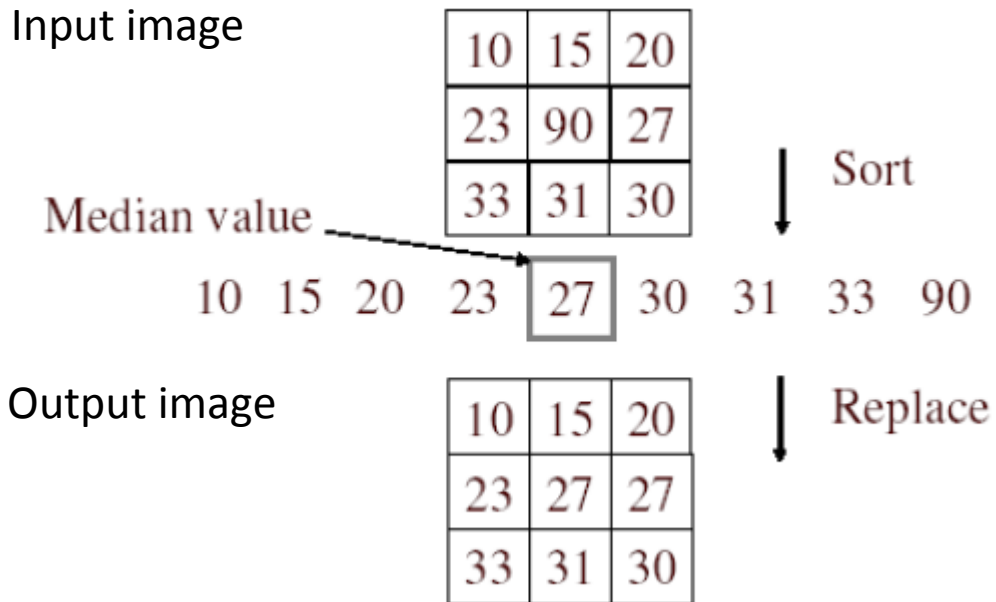


Salt and pepper noise

Linear smoothing filters do not alleviate salt and pepper noise!

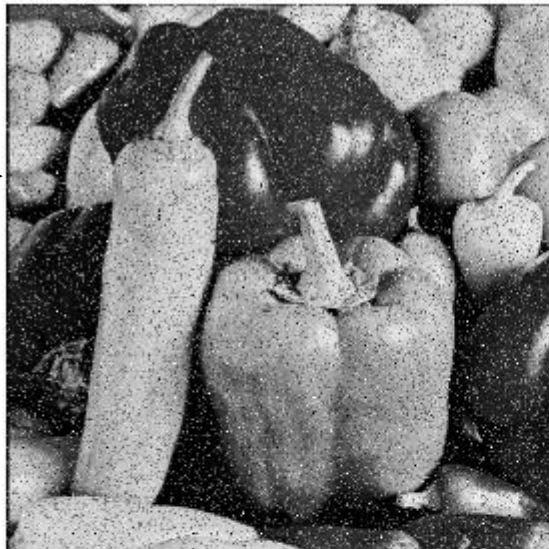
Median filter

- It is a non-linear filter
- Removes spikes: good for impulse, salt & pepper noise

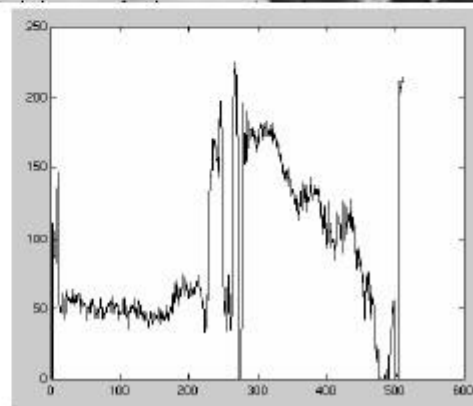
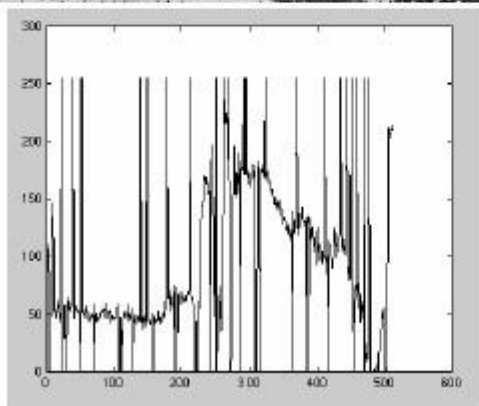


Median filter

Salt and pepper noise →



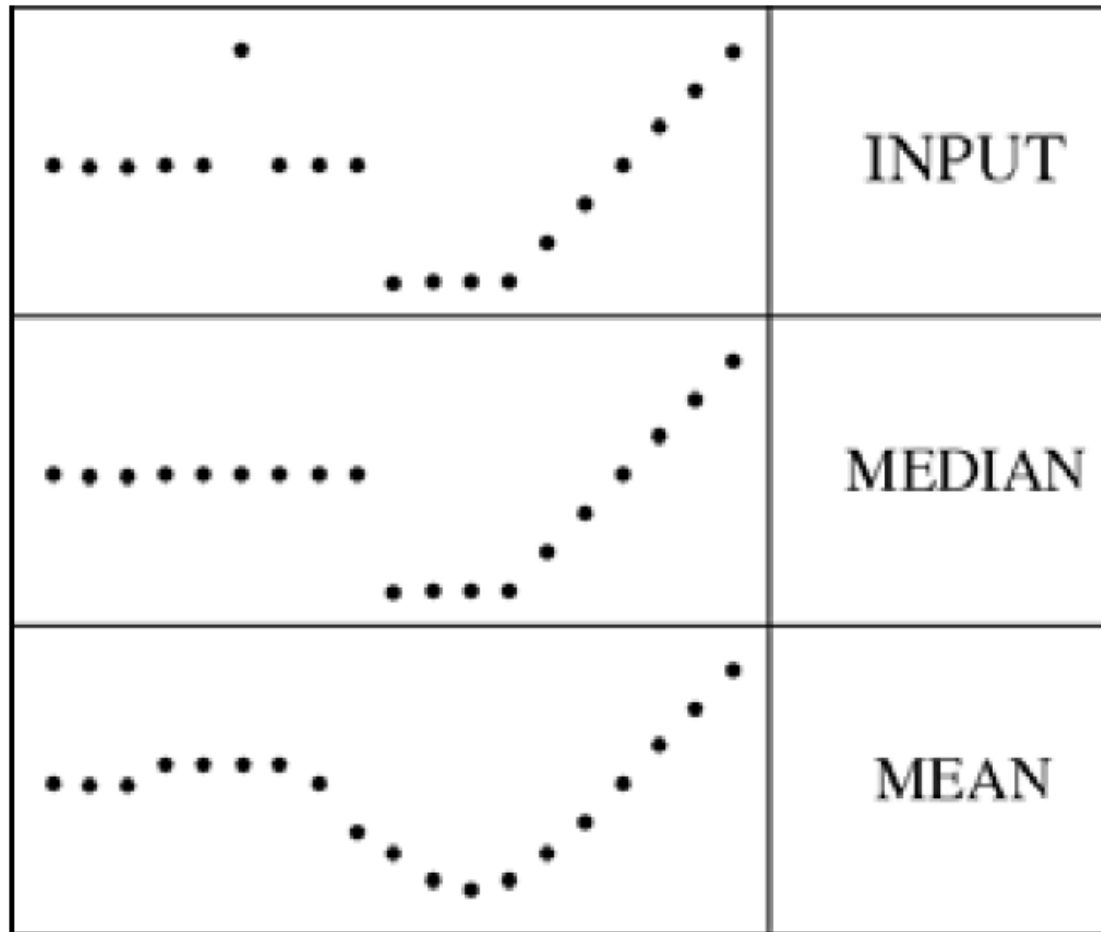
← Median filtered



Plots of a row of the image

Median filter

- Median filter preserves sharp transitions (i.e., edges),

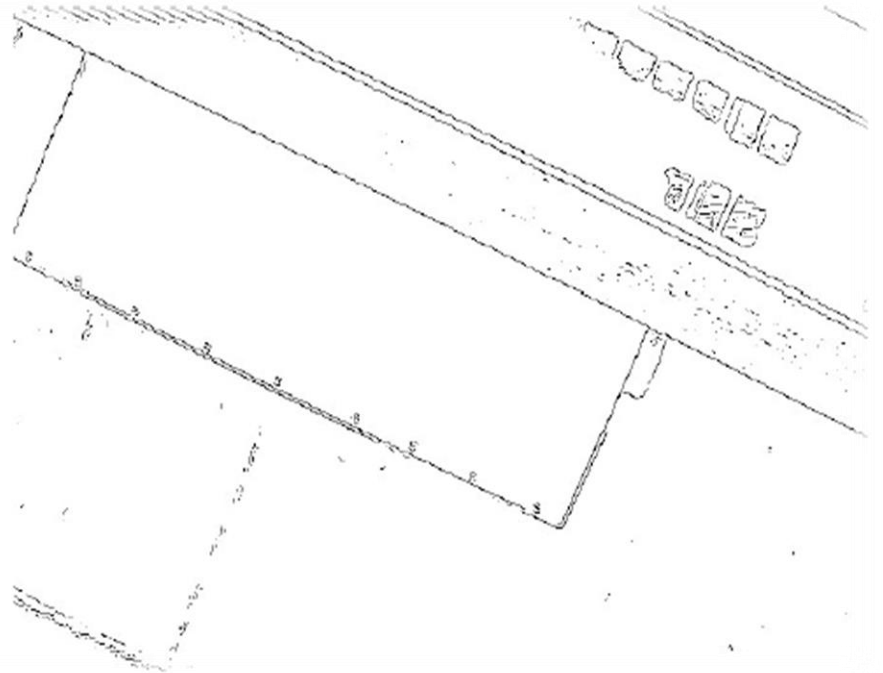
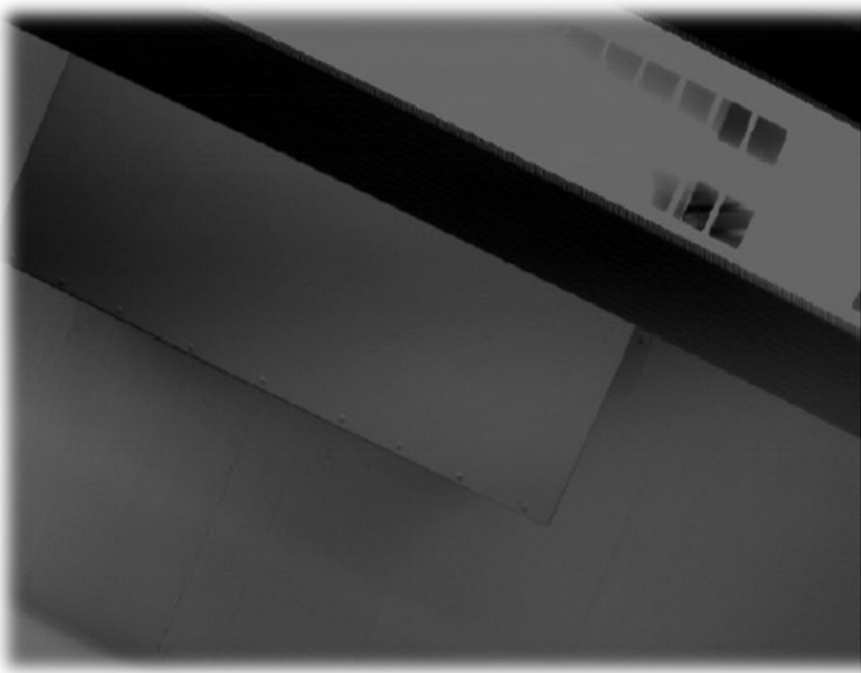


... but it removes small brightness variations.

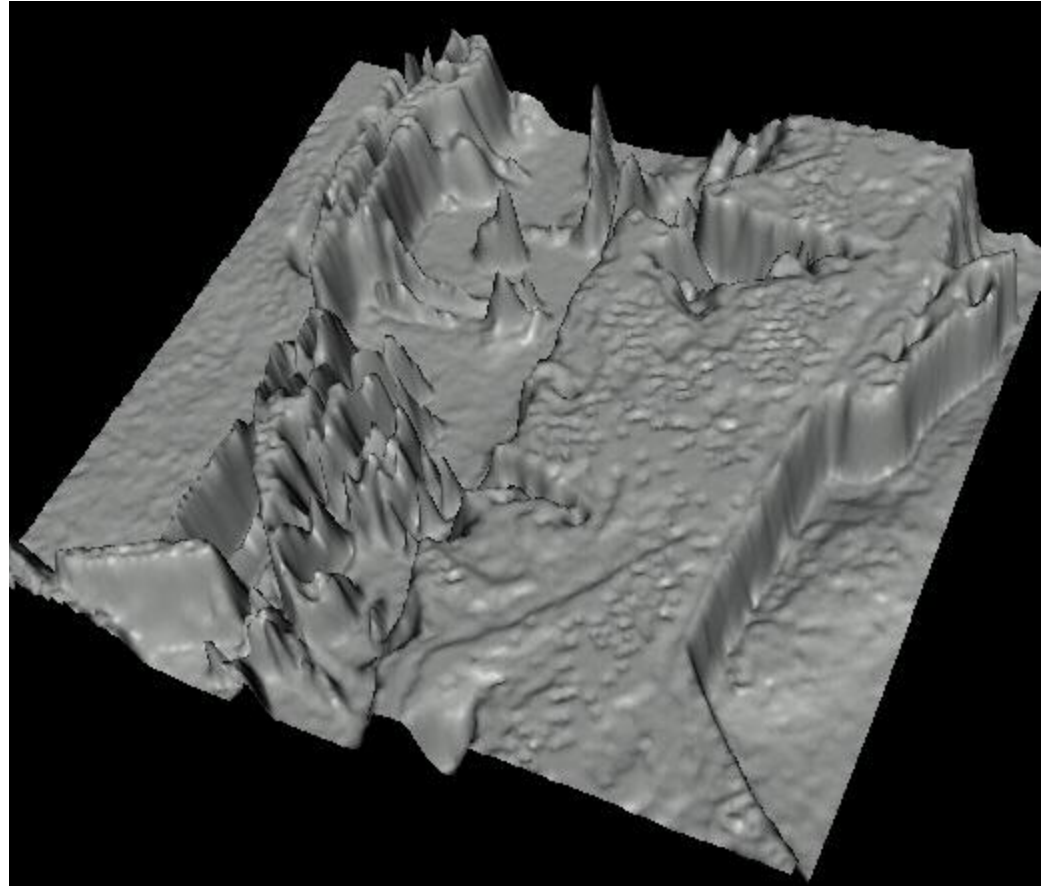
High-pass filtering
(edge detection)

Edge detection

- Ultimate goal of edge detection: an idealized line drawing.
- Edge contours in the image correspond to important scene contours.



Images as functions $f(x, y)$

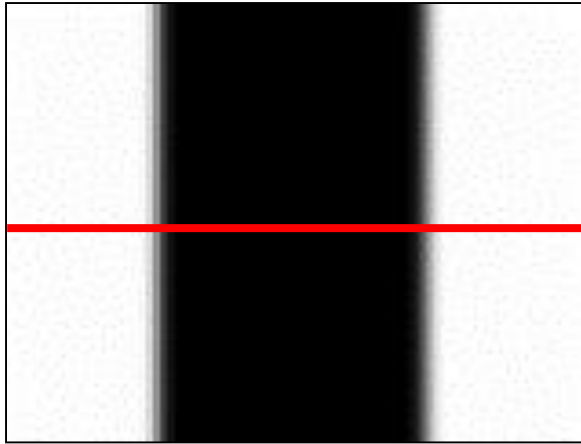


- Edges look like steep cliffs

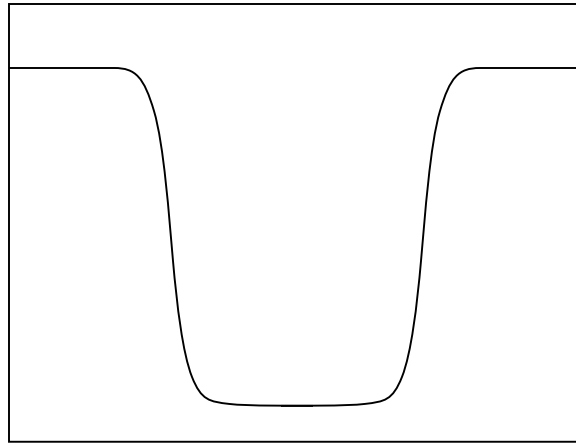
Derivatives and edges

An edge is a place of rapid change in the image intensity function.

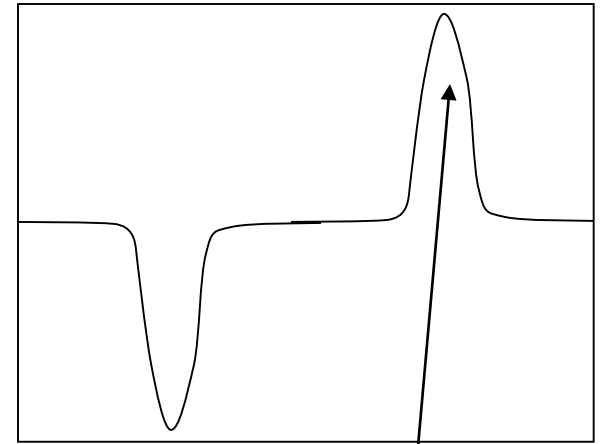
image



intensity function
(along horizontal scanline)



first derivative



edges correspond to
extrema of derivative

Differentiation and convolution

For 2D function, $f(x,y)$, the partial derivative is:

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

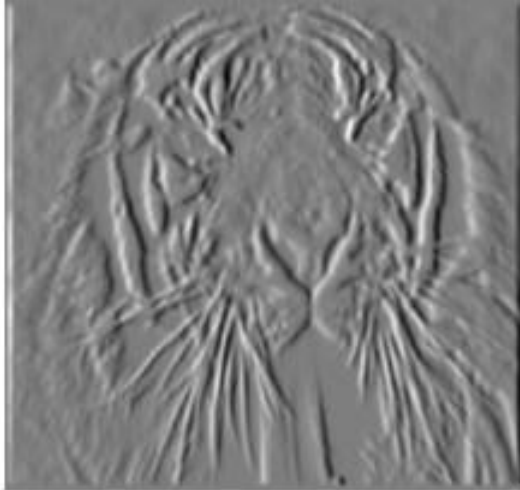
$$\frac{\partial f(x, y)}{\partial x} \approx \frac{f(x+1, y) - f(x, y)}{1}$$

To implement above as convolution, what would be the associated filter?

Partial derivatives of an image

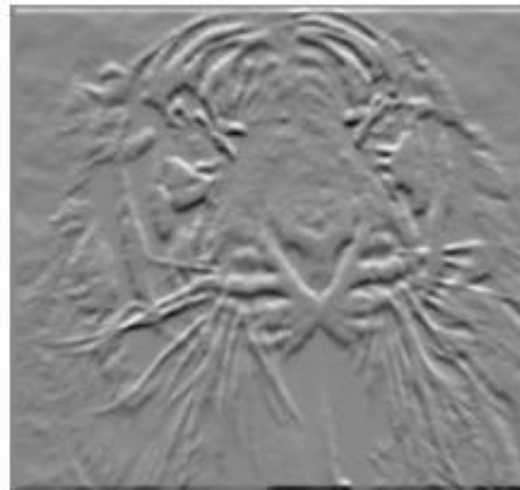


$$\frac{\partial f(x, y)}{\partial x}$$



| | |
|----|---|
| -1 | 1 |
|----|---|

$$\frac{\partial f(x, y)}{\partial y}$$



| |
|----|
| -1 |
| 1 |

Alternative Finite-difference filters

Prewitt filter $\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix} * \mathbf{A}$ and $\mathbf{G}_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{bmatrix} * \mathbf{A}$

Sobel filter $\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} * \mathbf{A}$ and $\mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix} * \mathbf{A}$

Sample Matlab code

```
>> im = imread('lion.jpg')
>> My = fspecial('sobel');
>> outim = imfilter(double(im), My);
>> imagesc(outim);
>> colormap gray;
```

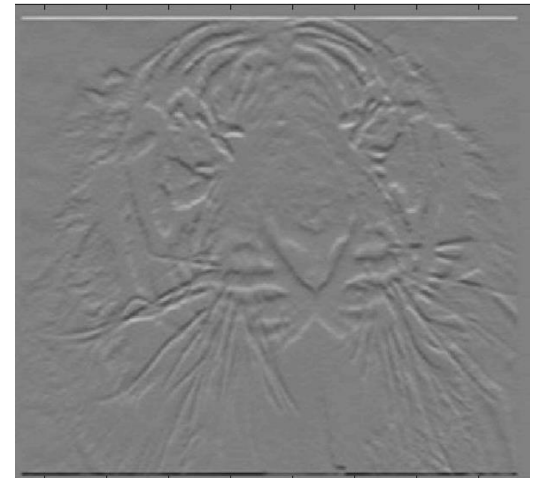
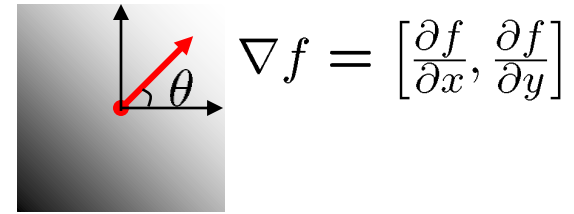
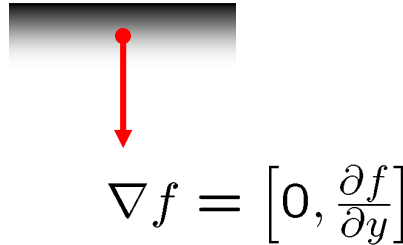
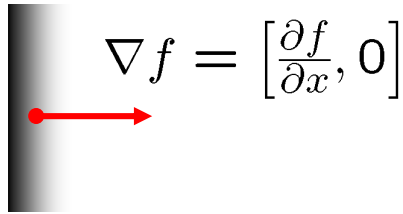


Image gradient

The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

The gradient points in the direction of fastest intensity change



The gradient direction (orientation of edge normal) is given by:

$$\theta = \tan^{-1} \left(\frac{\partial f / \partial y}{\partial f / \partial x} \right)$$

The *edge strength* is given by the gradient magnitude

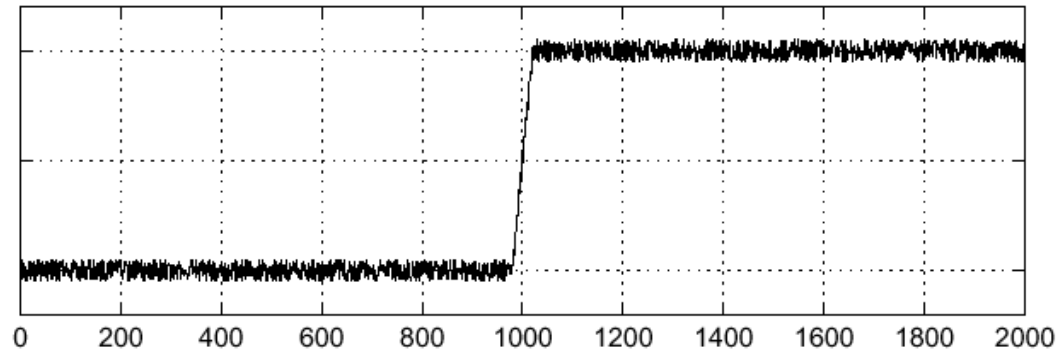
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2}$$

Effects of noise

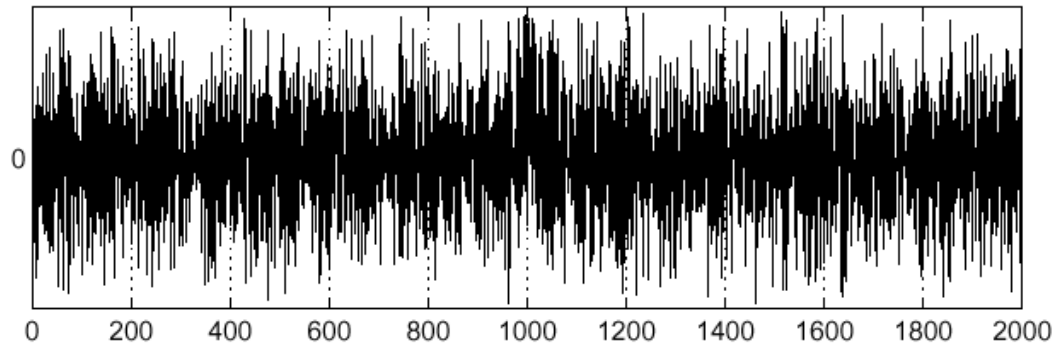
Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal

$f(x)$

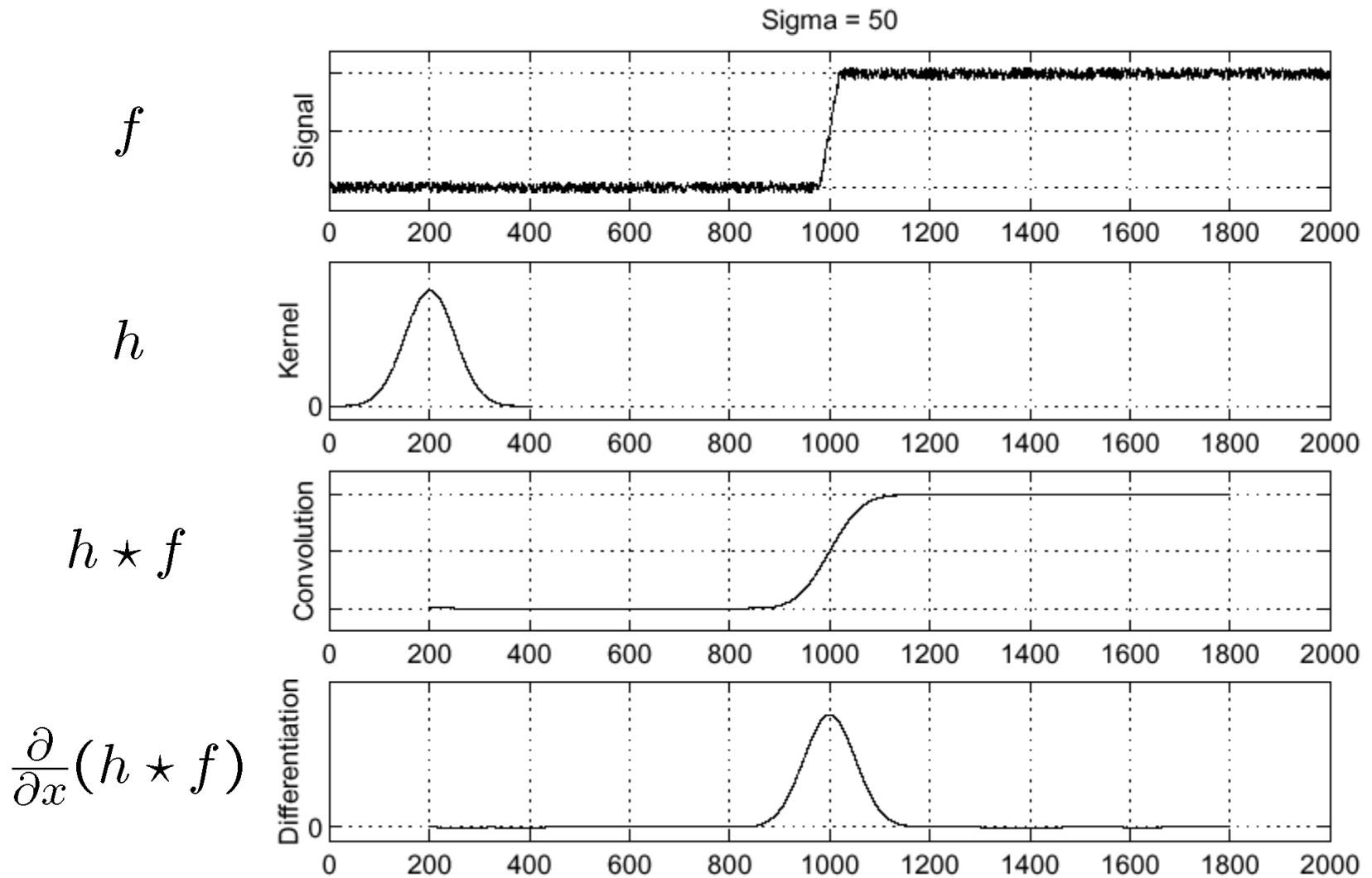


$\frac{d}{dx}f(x)$



Where is the edge?

Solution: smooth first



Where is the edge?

Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

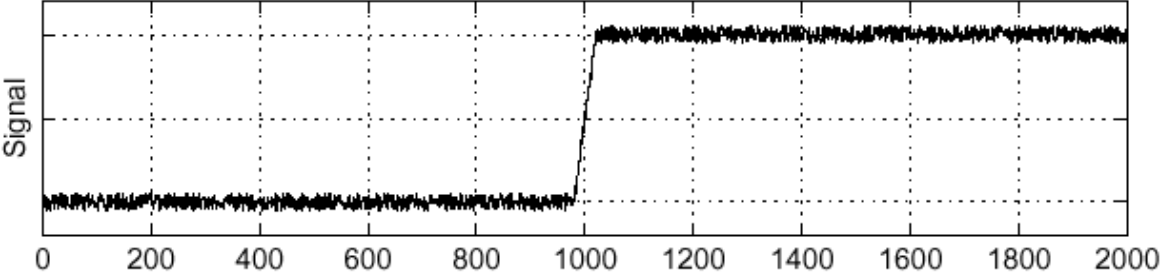
Alternative: combined derivative and smoothing filter

$$\frac{\partial}{\partial x}(h \star f) = \left(\frac{\partial}{\partial x}h\right) \star f$$

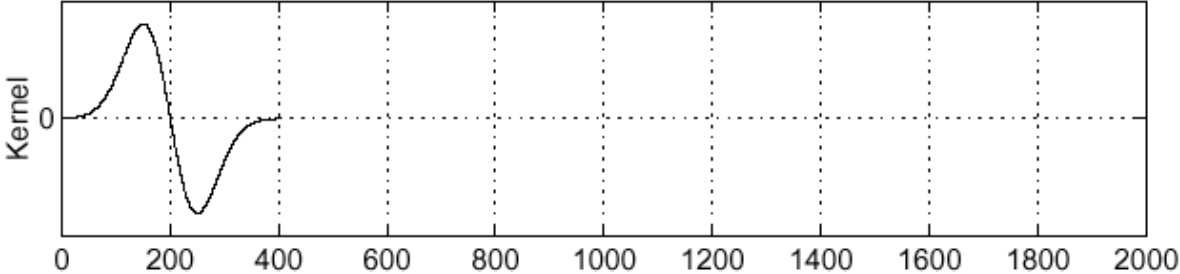
Differentiation property of convolution.

Sigma = 50

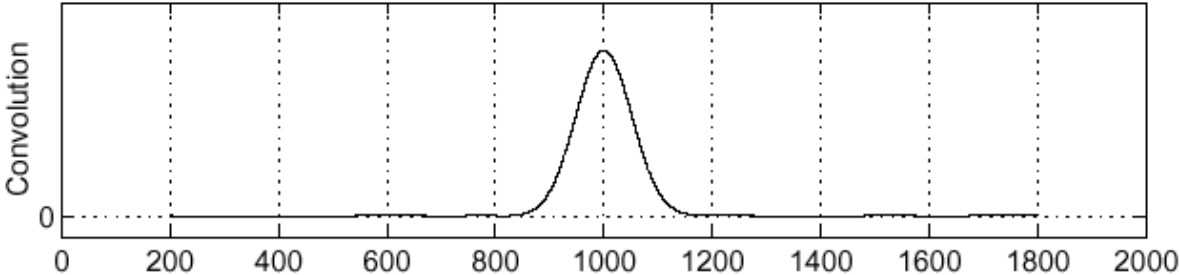
f



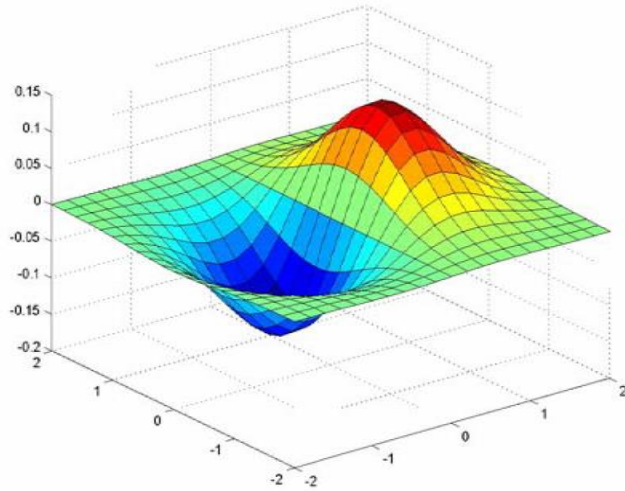
$\frac{\partial}{\partial x}h$



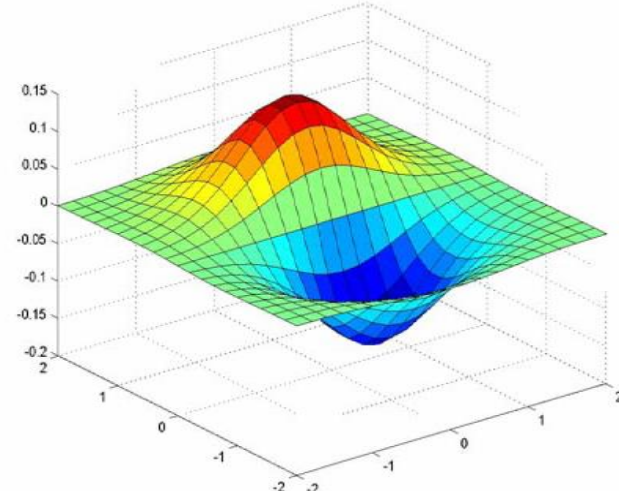
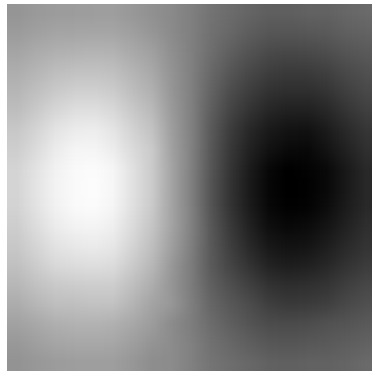
$\left(\frac{\partial}{\partial x}h\right) \star f$



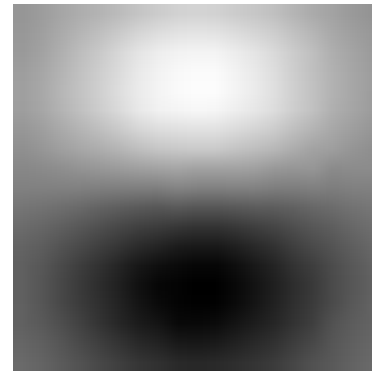
Derivative of Gaussian filters



x-direction



y-direction



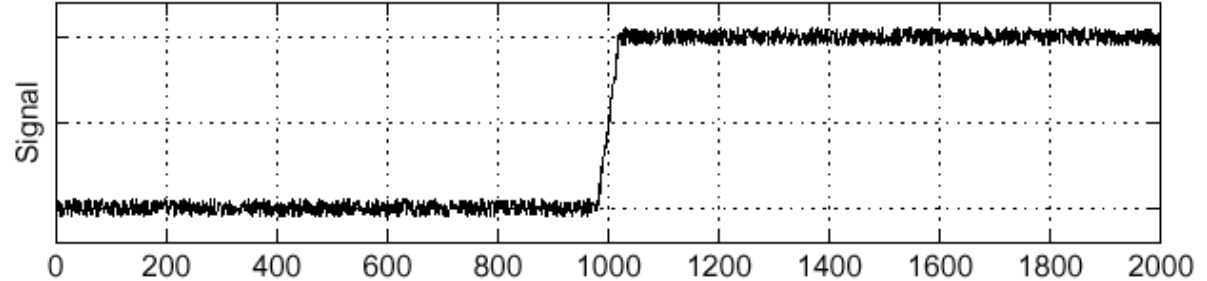
Laplacian of Gaussian

Consider

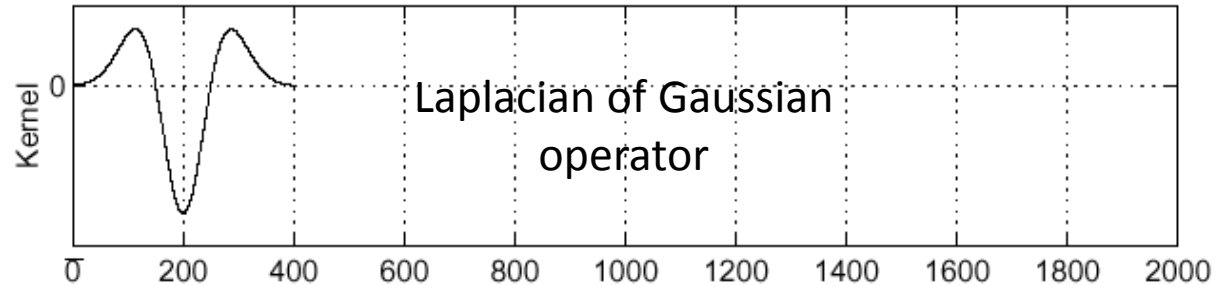
$$\frac{\partial^2}{\partial x^2}(h \star f)$$

Sigma = 50

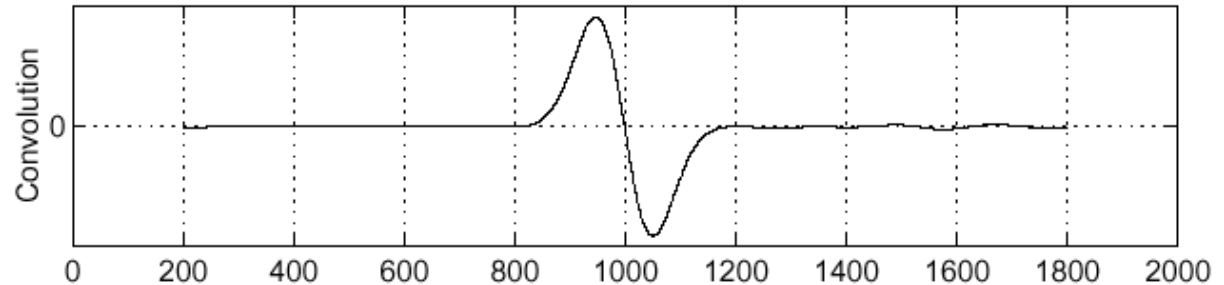
f



$$\frac{\partial^2}{\partial x^2}h$$



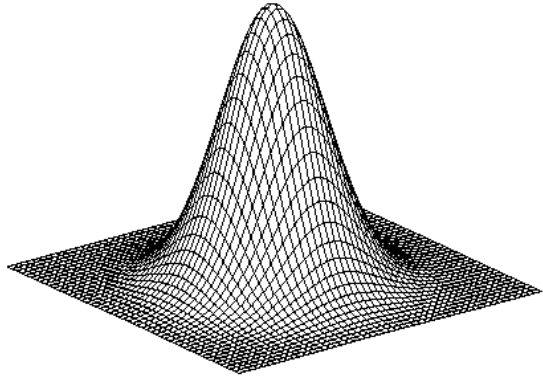
$$\left(\frac{\partial^2}{\partial x^2}h\right) \star f$$



Where is the edge?

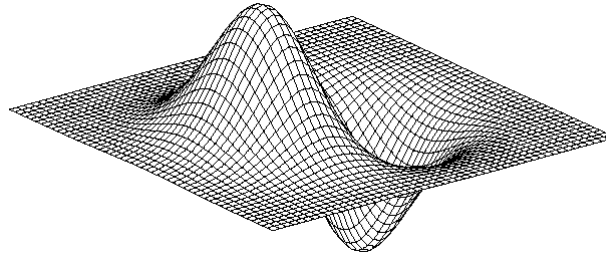
Zero-crossings of bottom graph

2D edge detection filters



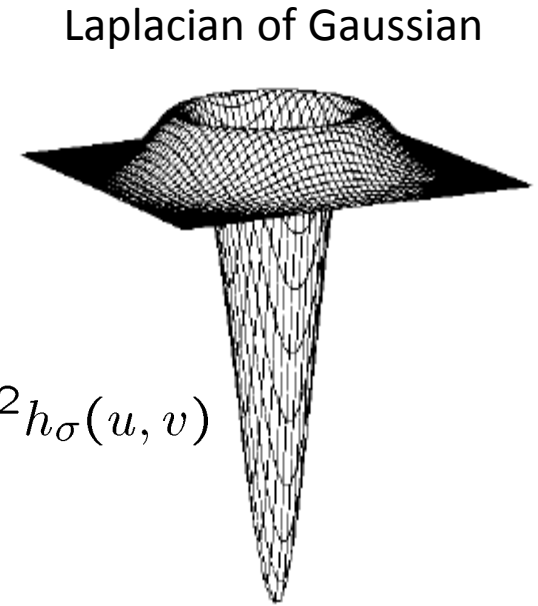
Gaussian

$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$



$$\nabla^2 h_{\sigma}(u, v)$$

- ∇^2 is the Laplacian operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Summary on (linear) filters

- Smoothing

- Filter has positive values (also called coefficients)
- Sum to 1 \rightarrow preserve brightness of constant regions
- Amount of smoothing is proportional to mask size
- Remove “high-frequency” components; “low-pass” filter

- Derivatives

- Opposite signs used to get high response in regions of high contrast
- Sum to 0 \rightarrow no response in constant regions
- High absolute value at points of high contrast

The Canny edge-detection algorithm (1986)

- Compute gradient of smoothed image in both directions
- Discard pixels whose gradient magnitude is below a certain threshold
- **Non-maximal suppression:** identify local maxima along gradient direction

The Canny edge-detection algorithm (1986)



Take a grayscale image. If not grayscale (i.g., RGB), convert it into a grayscale by replacing each pixel by the mean value of its R, G, B components.

Original image (Lenna image: <https://en.wikipedia.org/wiki/Lenna>)

The Canny edge-detection algorithm (1986)



Take a grayscale image. If not grayscale (i.g., RGB), convert it into a grayscale by replacing each pixel by the mean value of its R, G, B components.

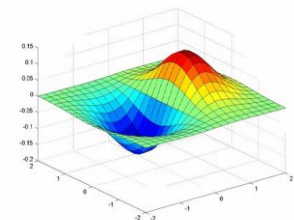
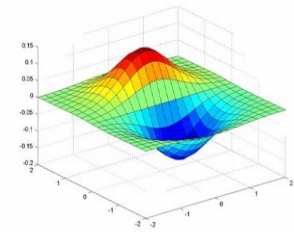
Original image (Lenna image: <https://en.wikipedia.org/wiki/Lenna>)

The Canny edge-detection algorithm (1986)



Convolve the image with x and y derivatives of Gaussian filter

$$\nabla f = \nabla(G_\sigma * I)$$



$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} : \text{Edge strength}$$

The Canny edge-detection algorithm (1986)



Threshold it (i.e., set to 0 all pixels whose value is below a given threshold)

Thresholding $|\nabla f|$

The Canny edge-detection algorithm (1986)



Take local maximum
along gradient direction

Thinning: non-maxima suppression (local-maxima detection)
along edge direction

Summary (things to remember)

- Image filtering (definition, motivation, applications)
- Moving average
- Linear filters and formulation: box filter, Gaussian filter
- Boundary issues
- Non-linear filters
 - Median filter and its applications
- Edge detection
 - Derivating filters (Prewitt, Sobel)
 - Combined derivative and smoothing filters (deriv. of Gaussian)
 - Laplacian of Gaussian
 - Canny edge detector
- Book chapters 3.2, pages 108-109, 386-387, 4.2.1, 11.3.1