ROBOTICS & PERCEPTION GROUP

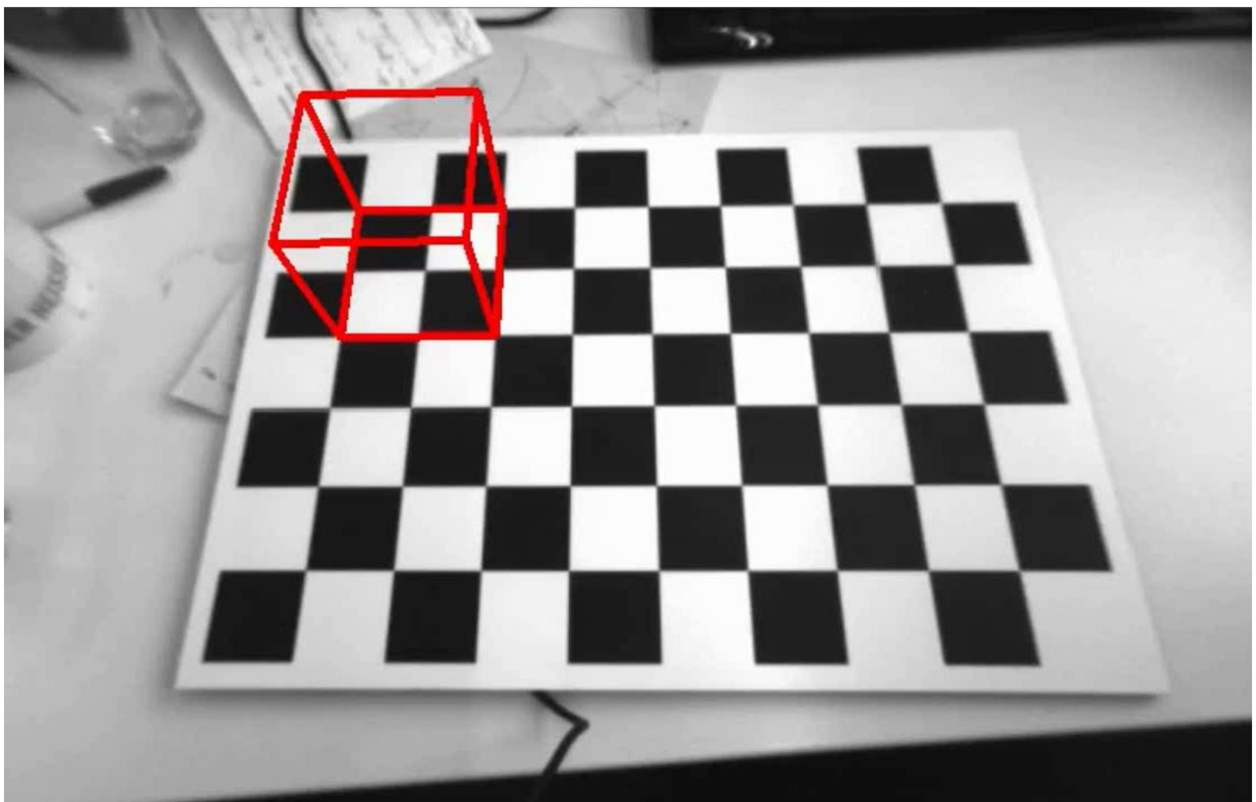
Universität Zürich[UZH]

Lecture 03
Image Formation 2

Davide Scaramuzza

Lab Exercise 1 - Today afternoon

- Room ETH HG E 33.1 from 14:15 to 16:00
- Work description: implement an augmented reality wireframe cube
  - Practice the perspective projection



Course Schedule update

For updates, slides, and additional material: http://rpg.ifi.uzh.ch/teaching.html

Date	Time	Description of the lecture/exercise	Lecturer
22.09.2016	10:15 - 12:00	01 – Introduction	Scaramuzza
29.09.2016	10:15 - 12:00	02 - Image Formation 1: perspective projection and camera models	Scaramuzza
06.10.2016	10:15 - 12:00	03 - Image Formation 2: camera calibration algorithms	Scaramuzza
	14:15 – 16:00	Lab Exercise 1: Augmented reality wireframe cube	Titus Cieslewski/Henri Rebecq
13.10.2016	10:15 - 12:00 14:15 – 16:00	04 - Filtering & Edge detection Lab Exercise 2: PnP problem	Scaramuzza Titus Cieslewski/Henri Rebecq
20.10.2016	10:15 - 12:00	05 - Point Feature Detectors 1: Harris detector	Scaramuzza
	14:15 – 16:00	Lab Exercise 3: Harris detector + descriptor + matching	Titus Cieslewski/Henri Rebecq
27.10.2016	10:15 - 12:00	06 - Point Feature Detectors 2: SIFT, BRIEF, BRISK	Scaramuzza
3.11.2016	10:15 - 12:00 14:15 – 16:00	07 - Multiple-view geometry 1 Lab Exercise 4: Stereo vision: rectification, epipolar matching, disparity, triangulation	Scaramuzza Titus Cieslewski/Henri Rebecq
10.11.2016	10:15 - 12:00 14:15 – 16:00	08 - Multiple-view geometry 2 Exercise 5: Eight-point algorithm and RANSAC	Scaramuzza Titus Cieslewski/Henri Rebecq
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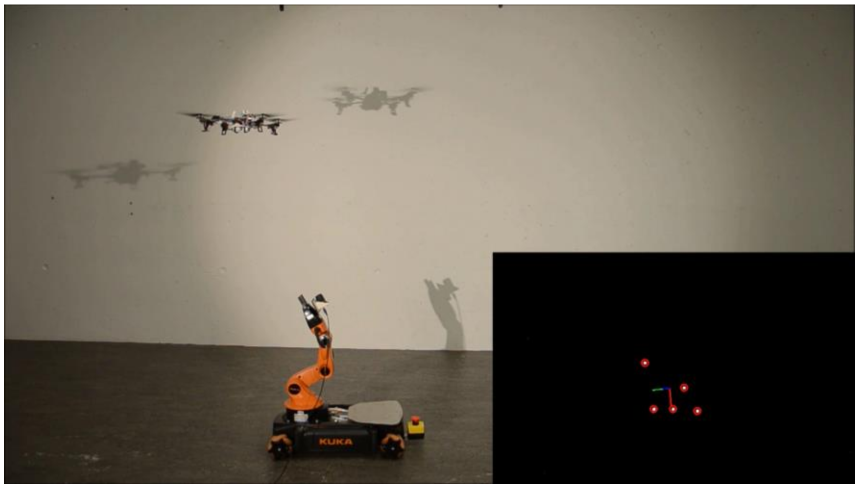
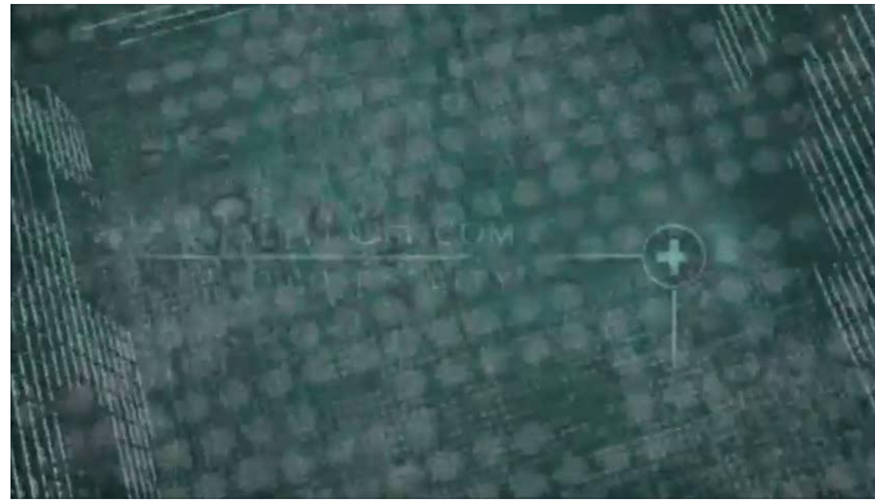
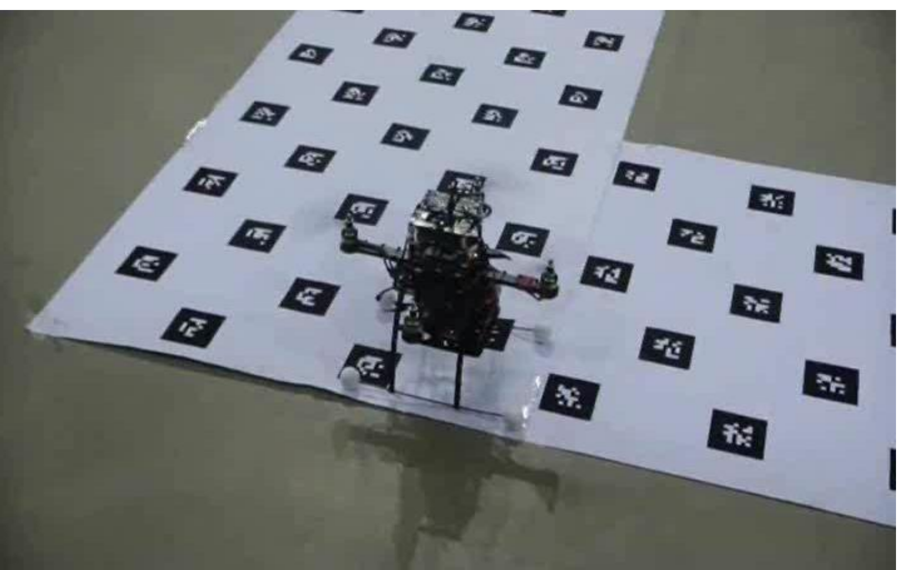
24.11.2016	10:15 - 12:00 14:15 – 16:00	10 - Dense 3D Reconstruction (Multi-view Stereo) Exercise 7: Intermediate VO Integration	Scaramuzza Titus Cieslewski/Henri Rebecq
01.12.2016	10:15 - 12:00 14:15 – 16:00	11 - Optical Flow and Tracking (Lucas-Kanade) Exercise 8: Lucas-Kanade tracker	Scaramuzza Titus Cieslewski/Henri Rebecq
08.12.2016	10:15 - 12:00	12 – Place recognition	Scaramuzza
	14:15 – 16:00	Exercise 9: Recognition with Bag of Words	Titus Cieslewski/Henri Rebecq
	10:15 - 12:00	13 – Visual inertial fusion	Scaramuzza
15.12.2016	14:15 – 16:00	Exercise 10: Pose graph optimization and Bundle adjustment	Titus Cieslewski/Henri Rebecq
22.12.2016	10:15 - 12:00 14:15 – 16:00	14 - Event based vision + lab visit and live demonstrations Exercise 11: final VO integration	Scaramuzza Titus Cieslewski/Henri Rebecq

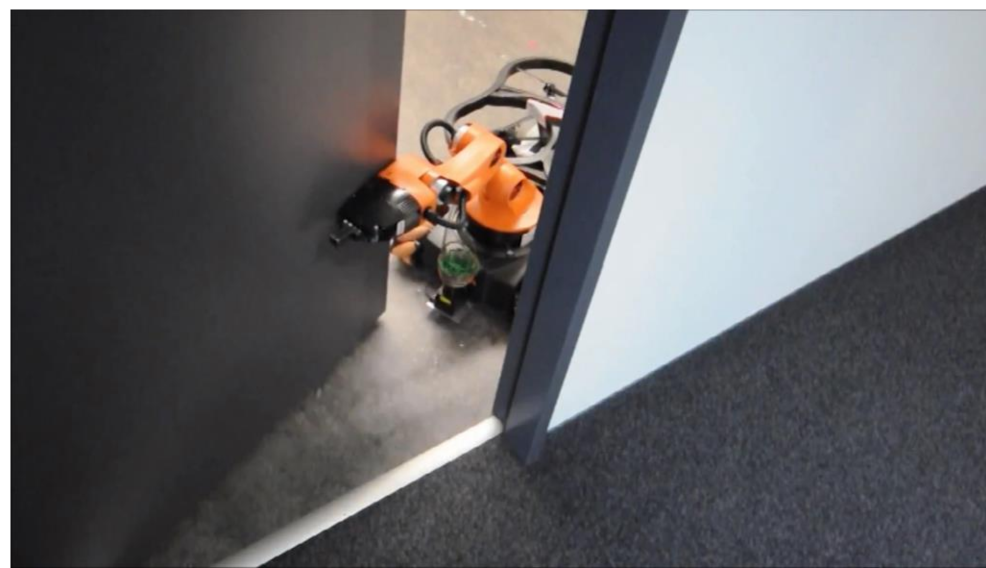
Goal of today’s lecure

- Study the algorithms behind robot-position control and augmented reality







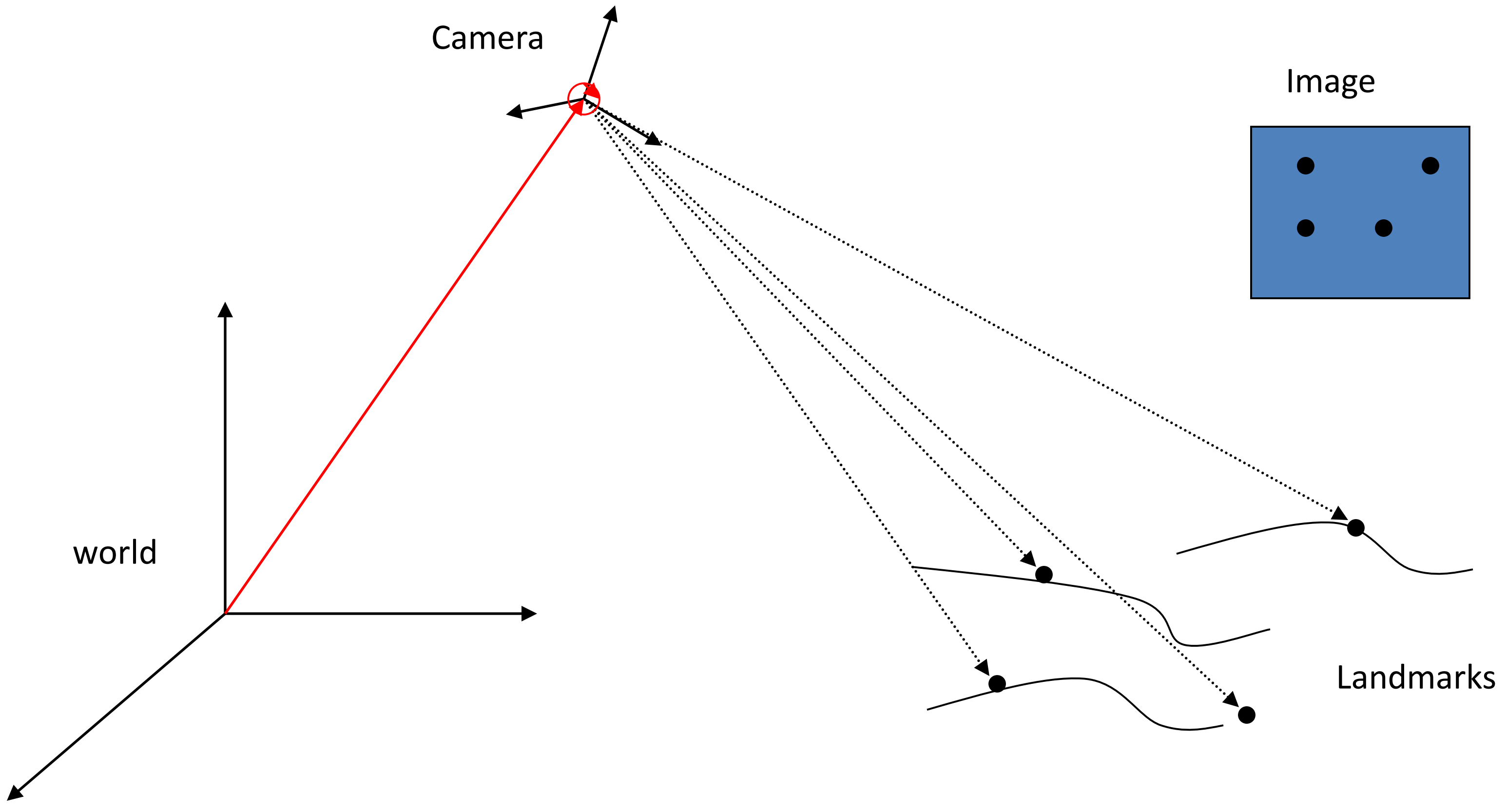


Outline of this lecture

- Camera calibration
  - Non-linear algorithms: P3P and PnP for calibrated cameras
    - From general 3D objects
  - Linear algorithms (DLT) for uncalibrated cameras
    - From 3D objects
    - From planar grids
- Non conventional camera models

Pose determination from n Points (PnP) Problem

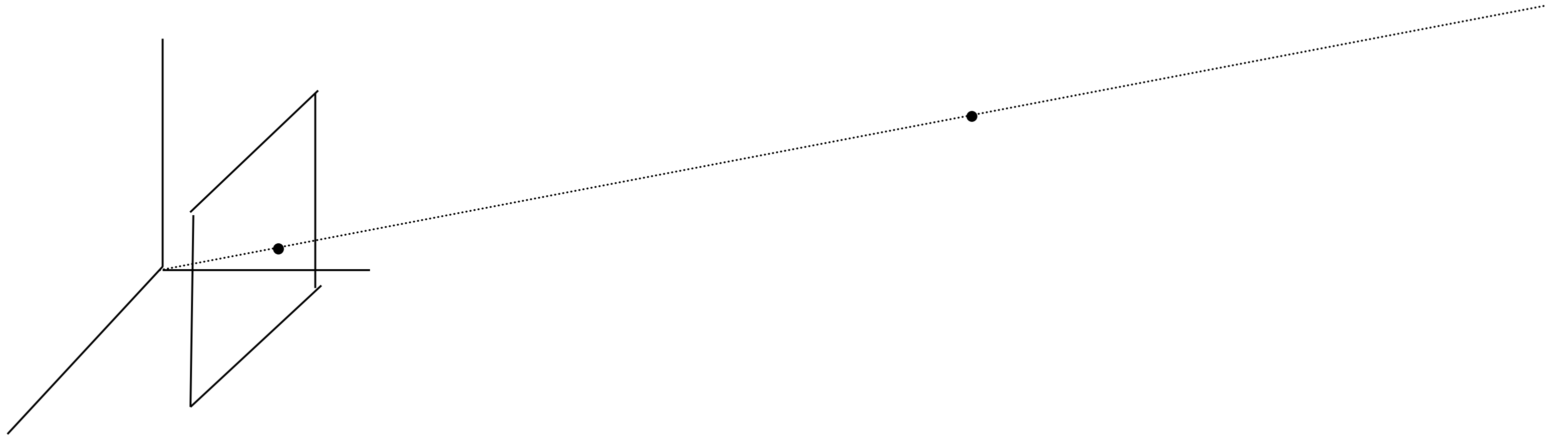
- Assumption: camera intrinsic parameters are known
- Given known 3D landmarks in the world frame and given their image correspondences in the camera frame, determine the 6DOF pose of the camera in the world frame (including the intrinsinc parameters if uncalibrated)



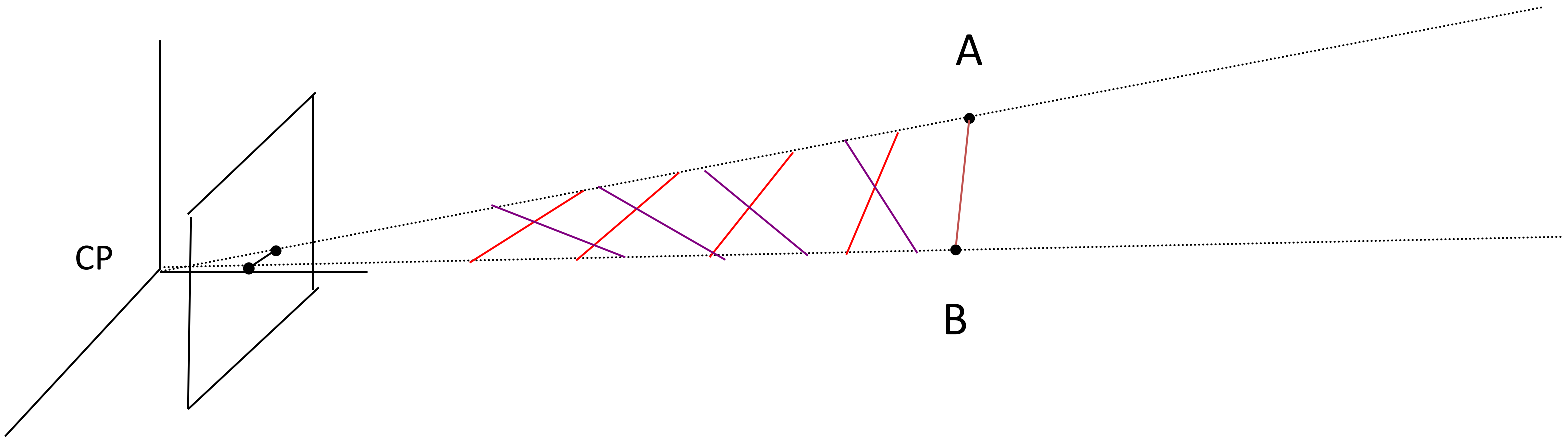
How Many Points are Enough?

- 1 Point: infinitely many solutions.
- 2 Points: infinitely many solutions, but bounded.
- 3 Points:
  - (no 3 collinear) finitely many solutions (up to 4).
- 4 Points:
  - Unique solution

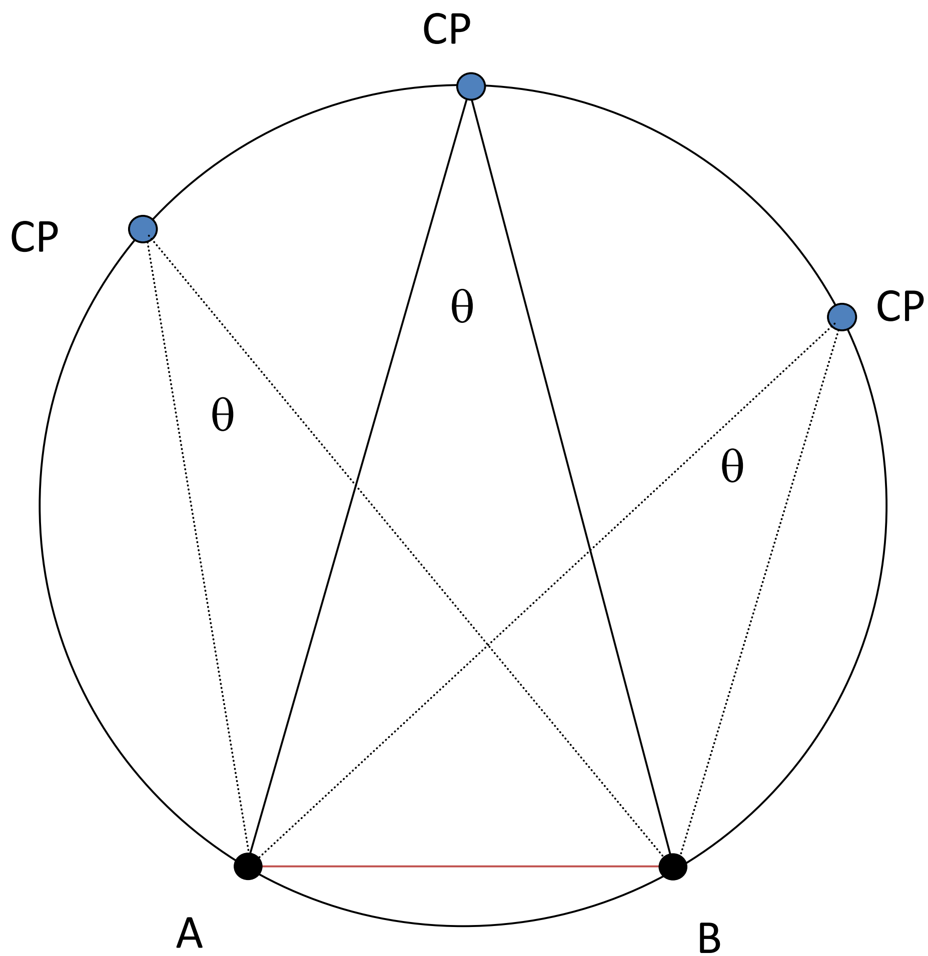
1 Point



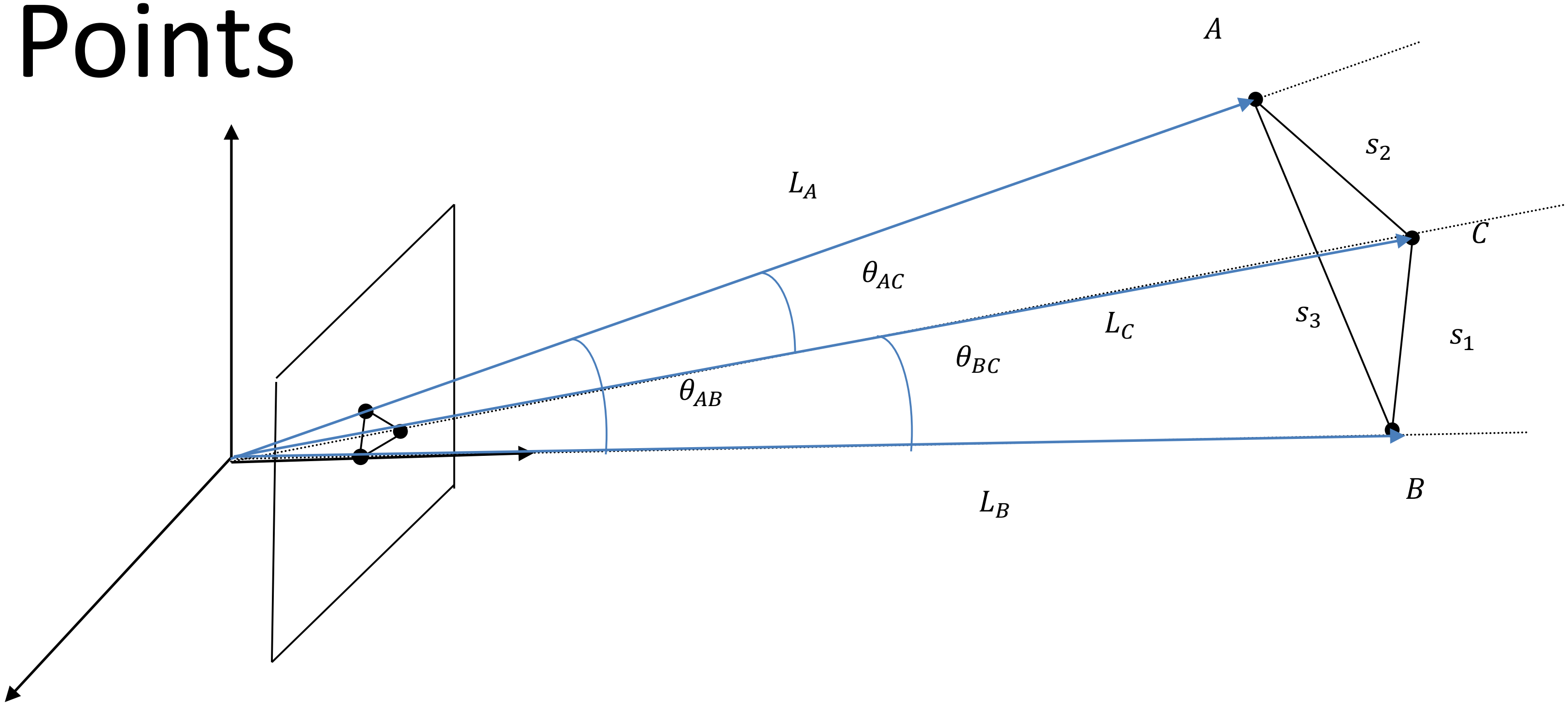
2 Points



Inscribed Angles are Equal



3 Points



$$s_1^2 = L_A^2 + L_B^2 - L_A L_B \cos\theta_{AB}$$

$$s_2^2 = L_B^2 + L_C^2 - L_B L_C \cos\theta_{BC}$$

$$s_3^2 = L_A^2 + L_C^2 - L_A L_C \cos\theta_{AC}$$

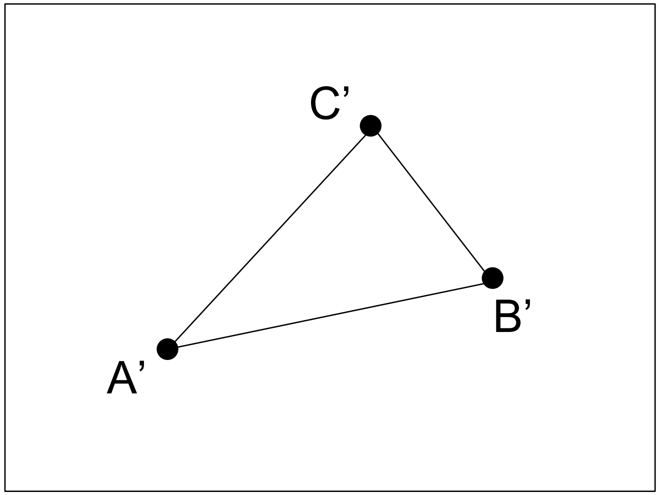


Image Plane

Algebraic Approach: reduce to 4th order equation

(Fischler and Bolles, 1981)

$$s_1^2 = L_A^2 + L_B^2 - L_A L_B \cos\theta_{AB}$$

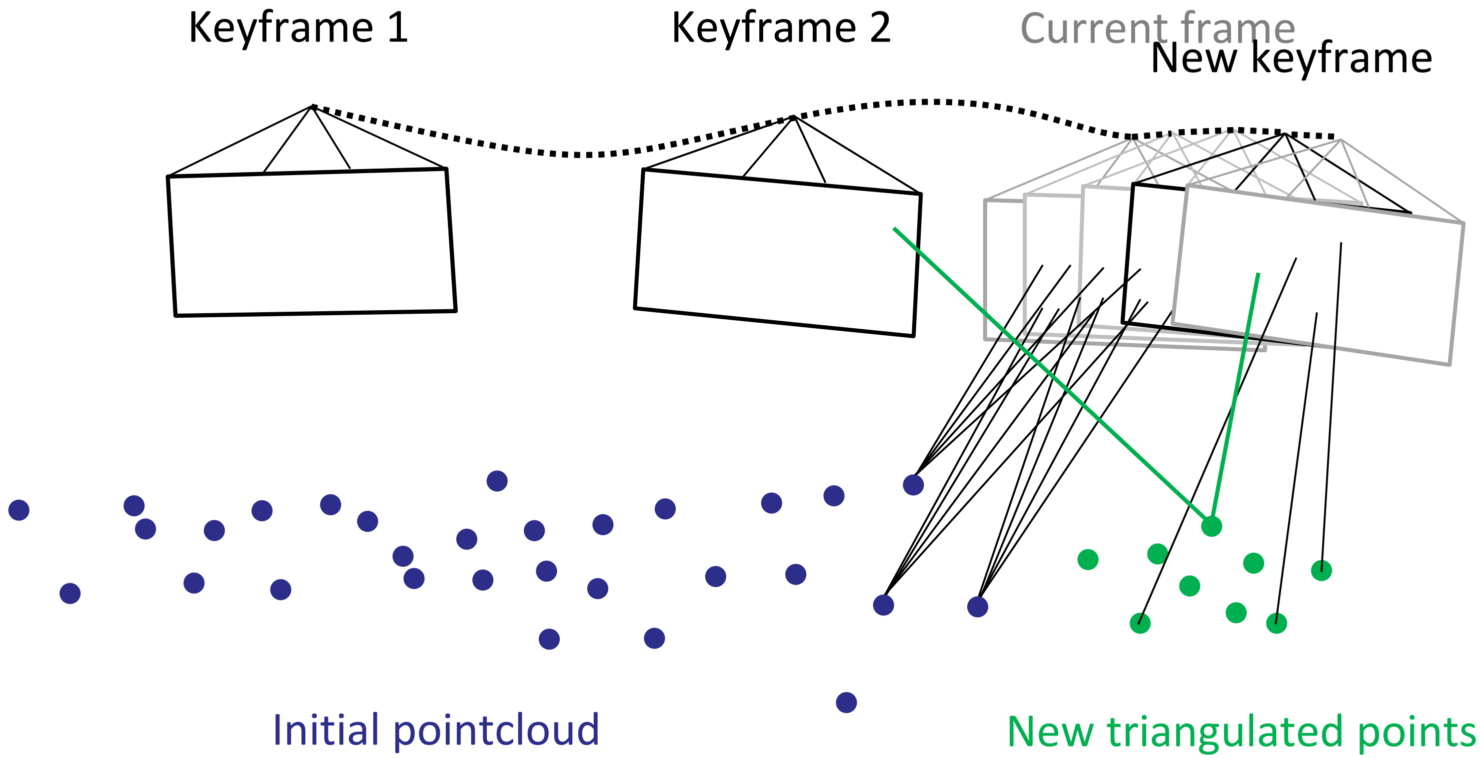
$$s_2^2 = L_B^2 + L_C^2 - L_B L_C \cos\theta_{BC}$$

$$s_3^2 = L_A^2 + L_C^2 - L_A L_C \cos\theta_{AC}$$

$$G_0 + G_1 x + G_2 x^2 + G_3 x^3 + G_4 x^4 = 0$$

- With 3 points, it generates up to 4 valid solutions.
- A 4th point can be used to disambiguate the solutions.
- Can be extended to n points; unique solution

Visual Odometry Application: camera pose estimation from known 3D-2D correspondences



AR Application: Microsoft HoloLens

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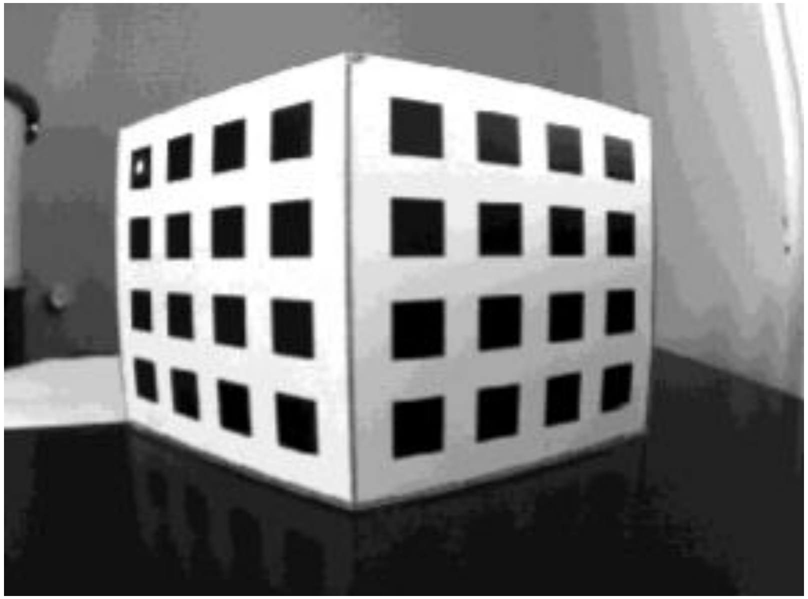
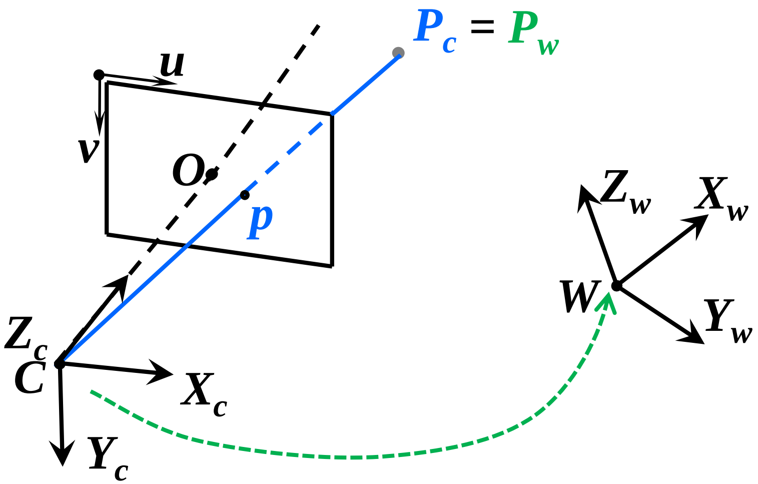
Camera calibration

- Calibration is the process to determine the **intrinsic and extrinsic** parameters of the camera model
- A method proposed in 1987 by Tsai consists of measuring the 3D position of $n \geq 6$ control points on a three-dimensional calibration target and the 2D coordinates of their projection in the image. This problem is also called "**Resection**", or "**Perspective from n Points**", or "**Camera pose from 3D-to-2D correspondences**", and is one of the most widely used algorithms in Computer Vision and Robotics
- Solution: The intrinsic and extrinsic parameters are computed directly from the perspective projection equation; let's see how!



3D position of control points is assigned in a reference frame specified by the user



Camera calibration: Direct Linear Transform (DLT)

Our goal is to compute K, R, and T that satisfy the perspective projection equation (we neglect the radial distortion)

$$\tilde{p} = \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K\left[R \,\middle|\, T\right] \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \quad \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} \alpha_u r_{11} + u_0 r_{31} & \alpha_u r_{12} + u_0 r_{32} & \alpha_u r_{13} + u_0 r_{33} & \alpha_u t_1 + u_0 t_3 \\ \alpha_v r_{21} + v_0 r_{31} & \alpha_v r_{22} + v_0 r_{32} & \alpha_v r_{23} + v_0 r_{33} & \alpha_v t_2 + v_0 t_3 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Camera calibration: Direct Linear Transform (DLT)

Our goal is to compute K, R, and T that satisfy the perspective projection equation (we neglect the radial distortion)

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = M \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

where m_i^T is the i-*th* row of M

Camera calibration: Direct Linear Transform (DLT)

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} m_1^T \\ m_2^T \\ m_3^T \end{bmatrix} \cdot \underbrace{\begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}}_{P}$$

Conversion back from homogeneous coordinates to pixel coordinates leads to:

$$u = \frac{\tilde{u}}{\tilde{w}} = \frac{m_1^T \cdot P}{m_3^T \cdot P} \qquad\qquad (m_1^T - u_i m_3^T) \cdot P_i = 0$$
$$\Rightarrow$$
$$v = \frac{\tilde{v}}{\tilde{w}} = \frac{m_2^T \cdot P}{m_3^T \cdot P} \qquad\qquad (m_2^T - v_i m_3^T) \cdot P_i = 0$$

Camera calibration: Direct Linear Transform (DLT)

By re-arranging the terms, we obtain

$$\begin{array}{l}(m_1^T - u_i m_3^T) \cdot P_i = 0 \\ (m_2^T - v_i m_3^T) \cdot P_i = 0\end{array} \Rightarrow \begin{pmatrix} P_1^T & 0^T & -u_1 P_1^T \\ 0^T & P_1^T & -v_1 P_1^T \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

For n points, we can stack all these equations into a big matrix:

$$\begin{pmatrix} P_1^T & 0^T & -u_1 P_1^T \\ 0^T & P_1^T & -v_1 P_1^T \\ \cdots & \cdots & \cdots \\ P_n^T & 0^T & -u_n P_n^T \\ 0^T & P_n^T & -v_n P_n^T \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

Camera calibration: Direct Linear Transform (DLT)

By re-arranging the terms, we obtain

$$\begin{matrix}(m_1^T - u_i m_3^T)\cdot P_i = 0 \\ (m_2^T - v_i m_3^T)\cdot P_i = 0\end{matrix} \Rightarrow \begin{pmatrix} P_1^T & 0^T & -u_1 P_1^T \\ 0^T & P_1^T & -v_1 P_1^T \end{pmatrix}\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

For n points, we can stack all these equations into a big matrix:

$$\underbrace{\begin{pmatrix} X_w^1 & Y_w^1 & Z_w^1 & 1 & 0 & 0 & 0 & 0 & -u_1 X_w^1 & -u_1 Y_w^1 & -u_1 Z_w^1 & -u_1 \\ 0 & 0 & 0 & 0 & X_w^1 & Y_w^1 & Z_w^1 & 1 & -v_1 X_w^1 & -v_1 Y_w^1 & -v_1 Z_w^1 & -v_1 \\ & & & & & & \cdots & \cdots & \cdots & & & \\ X_w^n & Y_w^n & Z_w^n & 1 & 0 & 0 & 0 & 0 & -u_n X_w^n & -u_n Y_w^n & -u_n Z_w^n & -u_n \\ 0 & 0 & 0 & 0 & X_w^n & Y_w^n & Z_w^n & 1 & -v_n X_w^n & -v_n Y_w^n & -v_n Z_w^n & -v_n \end{pmatrix}}_{\text{Q (this matrix is } \textbf{known}\text{)}} \underbrace{\begin{pmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{pmatrix}}_{\text{M (this matrix is } \textbf{unknown}\text{)}} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \Rightarrow \mathrm{Q}\cdot\mathrm{M} = 0$$

Camera calibration: Direct Linear Transform (DLT)

$$\mathrm{Q} \cdot \mathrm{M} = 0$$

Minimal solution

- $Q_{(2n\times12)}$ should have rank 11 to have a unique (up to a scale) non-trivial solution M
- Each 3D-to-2D point correspondence provides 2 independent equations
- Thus, $5+\frac{1}{2}$ point correspondences are needed (in practice **6 point** correspondences!)

Over-determined solution

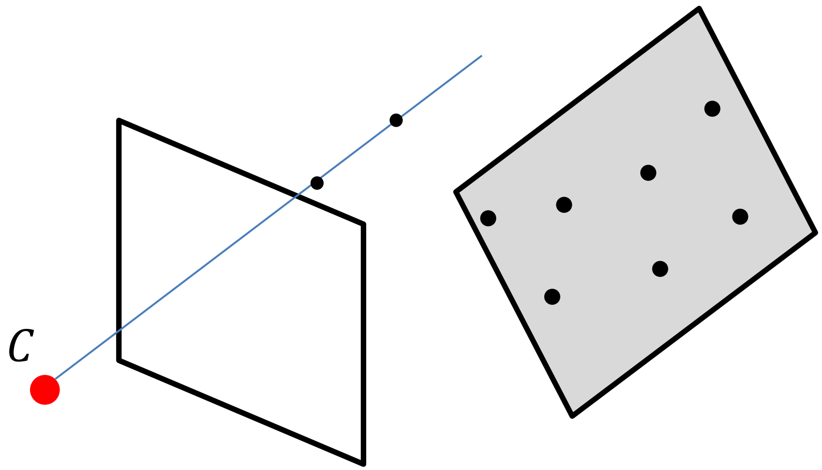
- $n \geq 6$ points
- A solution is to minimize $||QM||^2$ subject to the constraint $||M||^2 = 1$.
  It can be solved through Singular Value Decomposition (SVD). The solution is the eigenvector corresponding to the smallest eigenvalue of the matrix $Q^T Q$ (because it is the unit vector x that minimizes $||Qx||^2 = x^T Q^T Q x$).
- Matlab instructions:
  - `[U,S,V] = svd(Q);`
  - `M = V(:,12);`

Camera calibration: Direct Linear Transform (DLT)

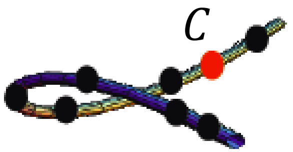
$$\mathrm{Q} \cdot \mathrm{M} = 0$$

Degenerate configurations

1. Points lying on a **plane** and/or along a single **line** passing through the **projection center**



2. Camera and points on a twisted cubic (i.e., smooth curve in 3D space of degree 3)



Camera calibration: Direct Linear Transform (DLT)

- Once we have the M matrix, we can recover the intrinsic and extrinsic parameters by remembering that

$$\mathrm{M} = \mathrm{K}(\mathrm{R} \,|\, \mathrm{T})$$

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix}$$

Camera calibration: Direct Linear Transform (DLT)

- Once we have the M matrix, we can recover the intrinsic and extrinsic parameters by remembering that

$$\mathrm{M} = \mathrm{K}(\mathrm{R} \mid \mathrm{T})$$

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} = \begin{bmatrix} \alpha r_{11} + u_0 r_{31} & \alpha r_{12} + u_0 r_{32} & \alpha r_{13} + u_0 r_{33} & \alpha t_1 + u_0 t_3 \\ \alpha r_{21} + v_0 r_{31} & \alpha r_{22} + v_0 r_{32} & \alpha r_{23} + v_0 r_{33} & \alpha t_2 + v_0 t_3 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix}$$

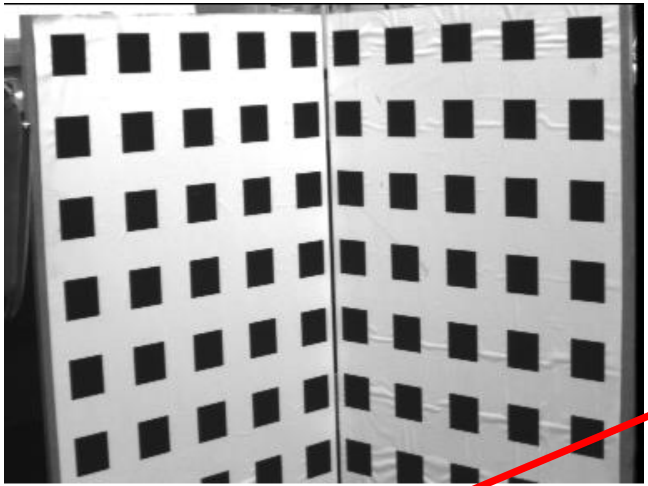
- However, notice that we are not enforcing the constraint that R is orthogonal, i.e., $R \cdot R^T = I$

- To do this, we can use the so-called QR factorization of M, which decomposes M into a R (orthogonal), T, and an upper triangular matrix (i.e., K)

Tsai's (1987) Calibration example

1. Edge detection
2. Straight line fitting to the detected edges
3. Intersecting the lines to obtain the images corners (corner accuracy <0.1 pixels!)
4. Use more than 6 points (ideally more than 20)

Why is this ratio not 1?

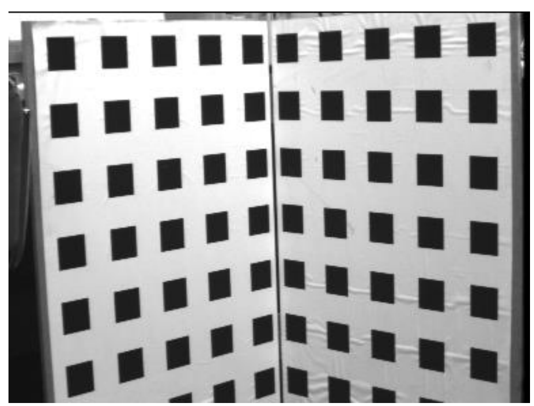


What are the «skew» and «residuals»?

f_y	f_x/f_y	skew	x_0	y_0	residual
1673.3	1.0063	1.39	379.96	305.78	0.365

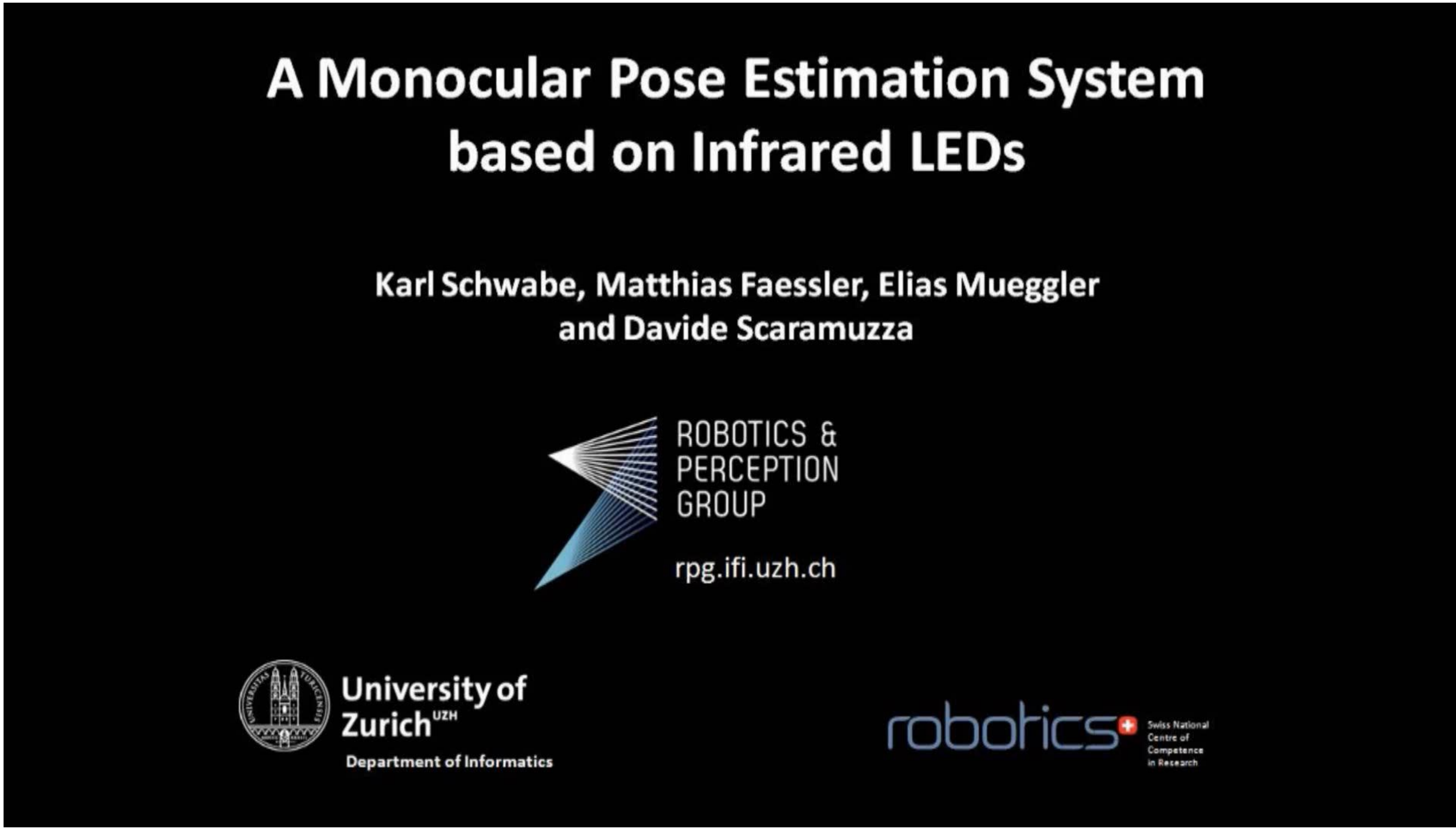
Tsai's (1987) Calibration example

- The original Tsai calibration (1987) used to consider two different focal lengths α_u, α_v (which means that the pixels are not squared) and a skew factor ($K_{12} \neq 0$, which means the pixes are parallelograms instead of rectangles) to account for possible misalignments between image plane and lens
- Most today's cameras are well manufactured, thus, we can assume $\frac{\alpha_u}{\alpha_v} = 1$ and $K_{12} = 0$
- What is the residual? The residual is the *average* "reprojection error". The reprojection error is computed as the distance (in pixels) between the observed pixel point and the camera-reprojected 3D point. The reprojection error gives as a quantitative measure of the accuracy of the calibration (ideally it should be zero).



f_y	f_x/f_y	skew	x_0	y_0	residual
1673.3	1.0063	1.39	379.96	305.78	0.365

DLT algorithm applied to mutual robot localization



In this case, the camera has been pre-calibrated (i.e., K is known). Can you think of how the DLT algorithm could be modified so that only R and T need to determined and not K?

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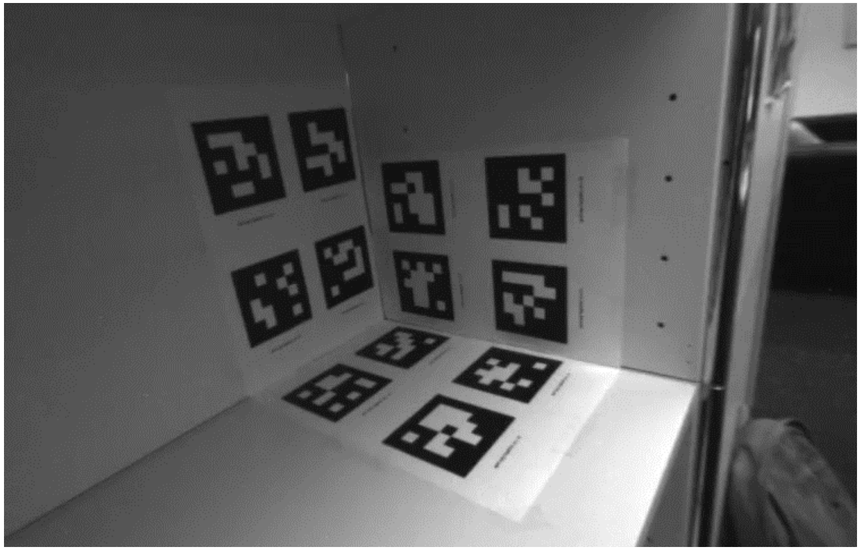
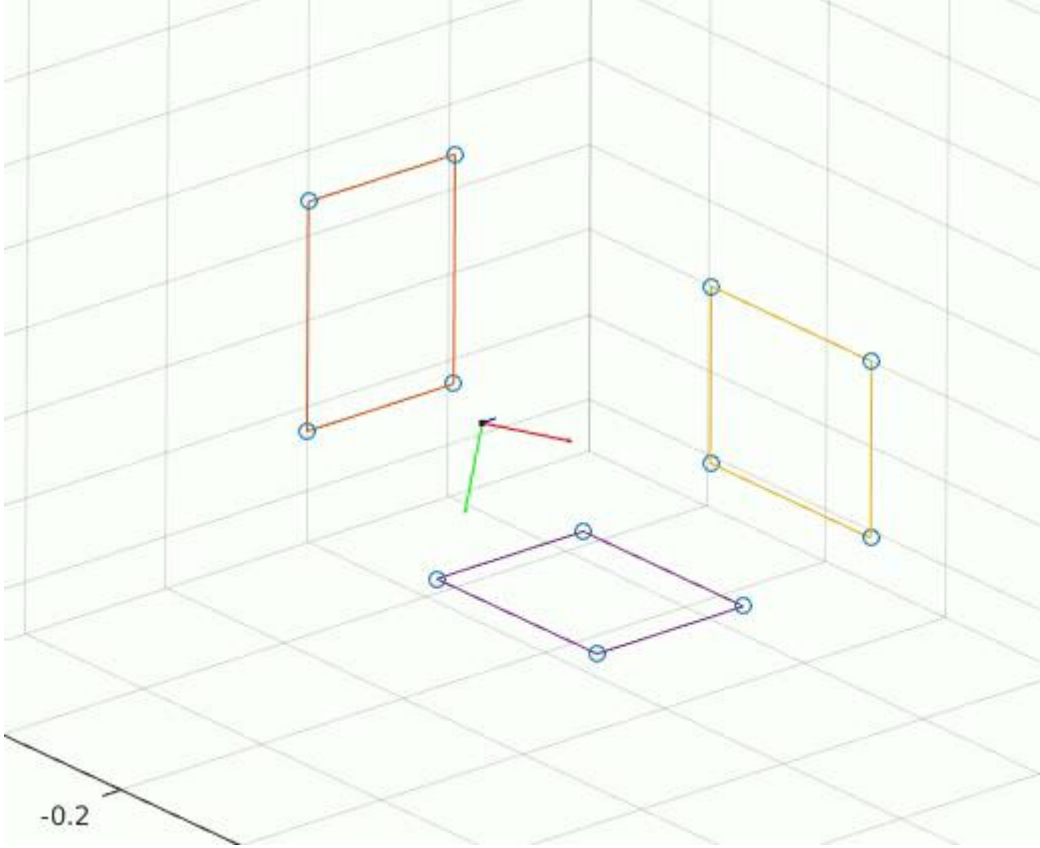
24.11.2016	10:15 - 12:00 14:15 – 16:00	10 - Dense 3D Reconstruction (Multi-view Stereo) Exercise 7: Intermediate VO Integration	Scaramuzza Titus Cieslewski/Henri Rebecq
01.12.2016	10:15 - 12:00 14:15 – 16:00	11 - Optical Flow and Tracking (Lucas-Kanade) Exercise 8: Lucas-Kanade tracker	Scaramuzza Titus Cieslewski/Henri Rebecq
08.12.2016	10:15 - 12:00	12 – Place recognition	Scaramuzza
	14:15 – 16:00	Exercise 9: Recognition with Bag of Words	Titus Cieslewski/Henri Rebecq
	10:15 - 12:00	13 – Visual inertial fusion	Scaramuzza
15.12.2016	14:15 – 16:00	Exercise 10: Pose graph optimization and Bundle adjustment	Titus Cieslewski/Henri Rebecq
22.12.2016	10:15 - 12:00 14:15 – 16:00	14 - Event based vision + lab visit and live demonstrations Exercise 11: final VO integration	Scaramuzza Titus Cieslewski/Henri Rebecq

Lab Exercise 2 - Today afternoon

- Room ETH HG E 33.1 from 14:15 to 16:00
- Work description: your first camera motion estimator using DLT



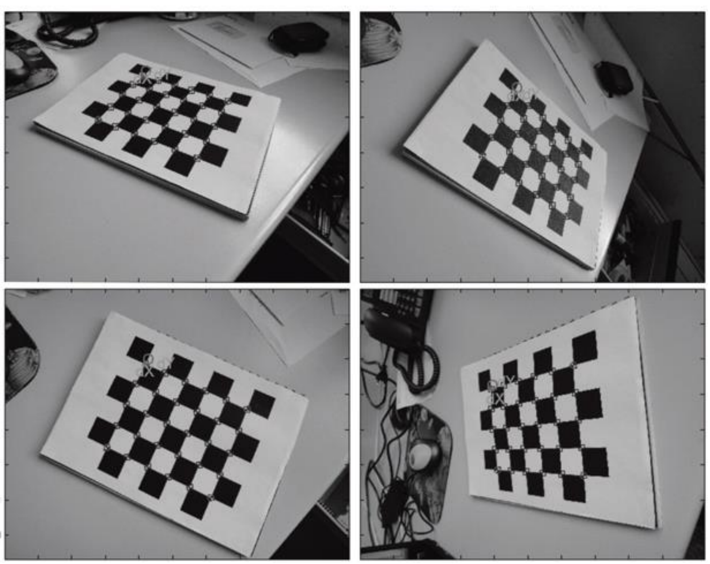


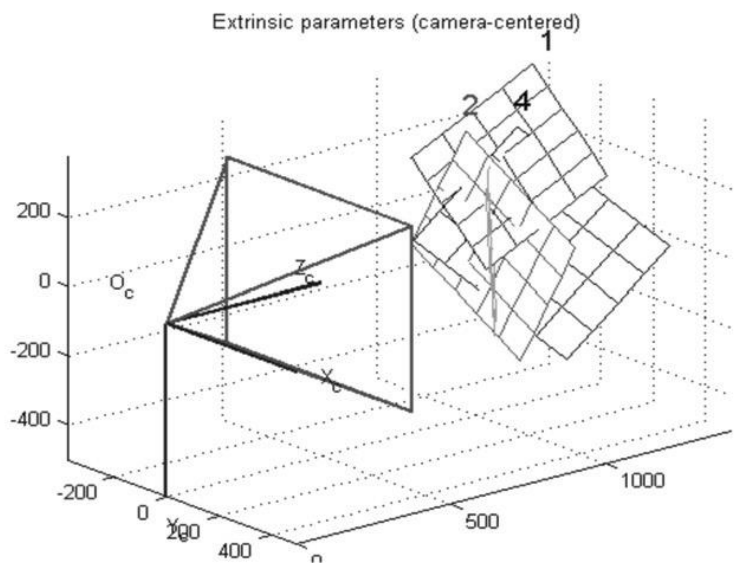
Outline of this lecture

- Camera calibration
  - Non-linear algorithms: P3P and PnP for calibrated cameras
    - From general 3D objects
  - Linear algorithms (DLT) for uncalibrated cameras
    - From 3D objects
    - From planar grids
- Non conventional camera models

Camera calibration from planar grids: homographies

- Tsai calibration is based on DLT algorithm, which requires points not to lie on the same plane
- An alternative method (today's standar camera calibration method) consists of using a planar grid (e.g., a chessboard) and a few images of this shown at different orientations
- This method was invented by Zhang (1999)





Camera calibration from planar grids: homographies

- Our goal is to compute K, R, and T, that satisfy the perspective projection equation (we neglect the radial distortion)
- Since the points lie on a plane, we have $Z_w = 0$

$$\tilde{p} = \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K\left[R \mid T\right] \cdot \begin{bmatrix} X_w \\ Y_w \\ 0 \\ 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

Camera calibration from planar grids: homographies

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$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = H \cdot \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

This matrix is called Homography

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} h_1^T \\ h_2^T \\ h_3^T \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

where h_i^T is the i-*th* row of H

Camera calibration from planar grids: homographies

$$\Rightarrow \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} h_1^T \\ h_2^T \\ h_3^T \end{bmatrix} \cdot \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

Conversion back from homogeneous coordinates to pixel coordinates leads to:

$$u = \frac{\tilde{u}}{\tilde{w}} = \frac{h_1^T \cdot P}{h_3^T \cdot P} \qquad\qquad (h_1^T - u_i h_3^T) \cdot P_i = 0$$

$$\Rightarrow$$

$$v = \frac{\tilde{v}}{\tilde{w}} = \frac{h_2^T \cdot P}{h_3^T \cdot P} \qquad\qquad (h_2^T - v_i h_3^T) \cdot P_i = 0$$

where P = $(X_w, Y_w, 1)^T$

Camera calibration from planar grids: homographies

By re-arranging the terms, we obtain

$$\begin{matrix}(h_1^T - u_i h_3^T)\cdot P_i = 0 \\ (h_2^T - v_i h_3^T)\cdot P_i = 0\end{matrix} \Rightarrow \begin{matrix} P_i^T \cdot h_1 + 0\cdot h_2^T - u_i P_i^T \cdot h_3^T = 0 \\ 0\cdot h_1^T + P_i^T \cdot h_2^T - v_i P_i^T \cdot h_3^T = 0\end{matrix} \Rightarrow \begin{pmatrix} P_i^T & 0^T & -u_1 P_i^T \\ 0^T & P_i^T & -v_1 P_i^T \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

For n points, we can stack all these equations into a big matrix:

$$\underbrace{\begin{pmatrix} P_1^T & 0^T & -u_1 P_1^T \\ 0^T & P_1^T & -v_1 P_1^T \\ \cdots & \cdots & \cdots \\ P_n^T & 0^T & -u_n P_n^T \\ 0^T & P_n^T & -v_n P_n^T \end{pmatrix}}_{\text{Q (this matrix is \textbf{known})}} \underbrace{\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}}_{\text{H (this matrix is \textbf{unknown})}} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \Rightarrow \mathrm{Q}\cdot\mathrm{H} = 0$$

Camera calibration from planar grids: homographies

$$Q \cdot H = 0$$

Minimal solution

- $Q_{(2n\times 9)}$ should have rank 8 to have a unique (up to a scale) non-trivial solution H
- Each point correspondence provides 2 independent equations
- Thus, a minimum of **4 non-collinear points** is required

Over-determined solution

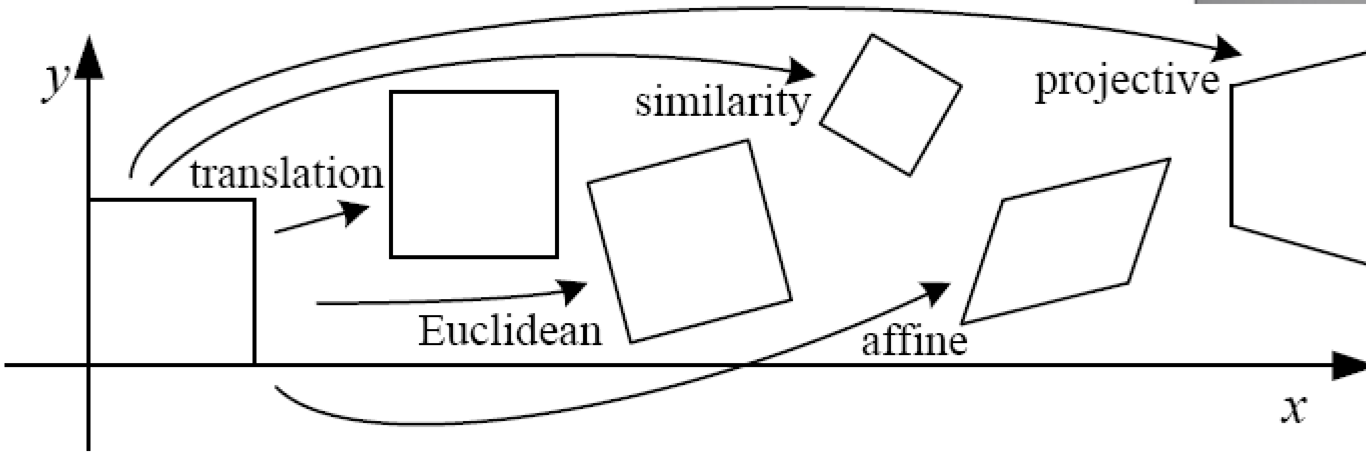
- $n \geq 4$ points
- It can be solved through Singular Value Decomposition (SVD)

Solving for K, R and T

- H can be decomposed by recalling that

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}$$

Types of 2D Transformations

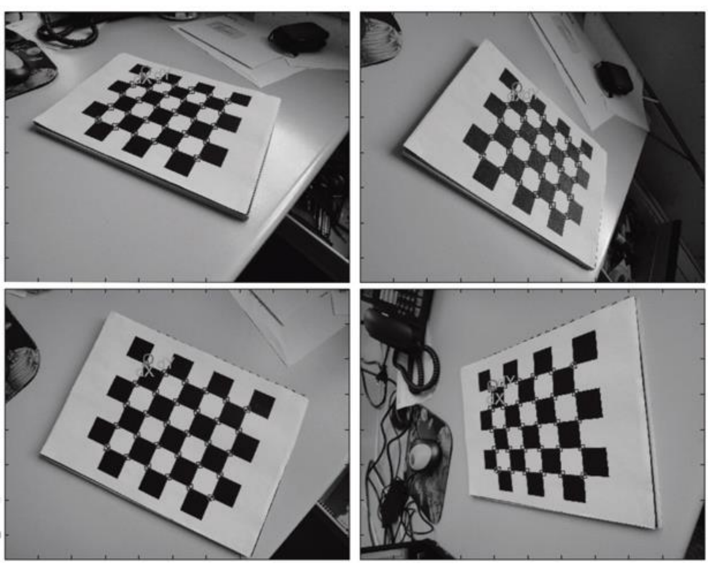


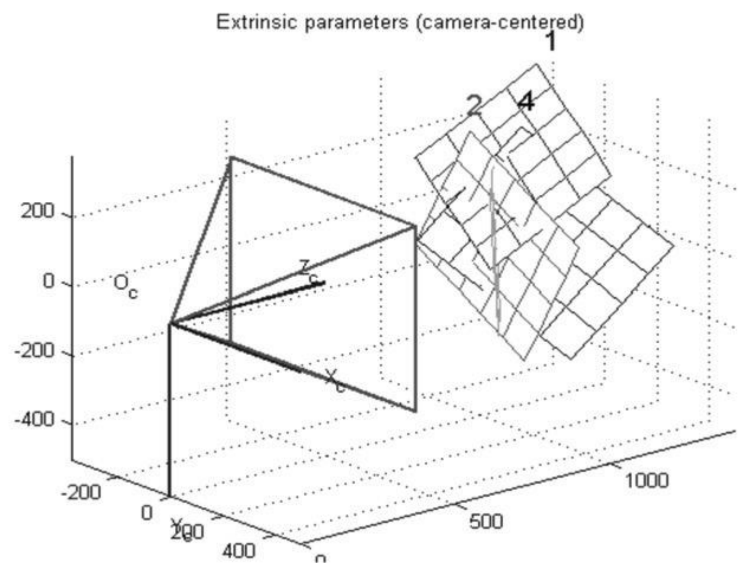
Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\left[\, I \mid t \,\right]_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\left[\, R \mid t \,\right]_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\left[\, sR \mid t \,\right]_{2\times 3}$	4	angles $+\cdots$	
affine	$\left[\, A \,\right]_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\left[\, \tilde{H} \,\right]_{3\times 3}$	8	straight lines	

This transformation is called Homography

Camera calibration from planar grids: homographies

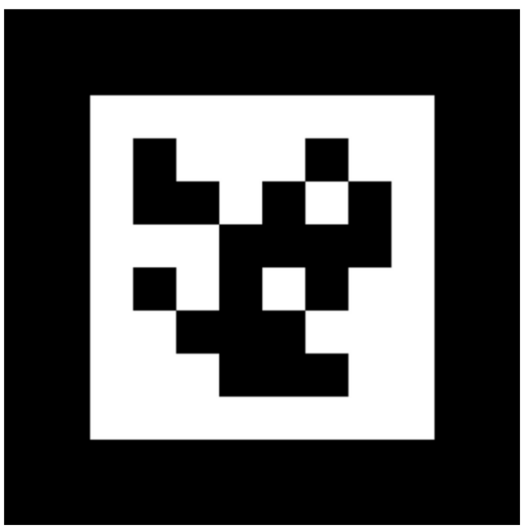
- Demo of Camera Calibration Toolbox for Matlab (world's standard toolbox for calibrating perspective cameras): http://www.vision.caltech.edu/bouguetj/calib_doc/





Application of calibration from planar grids

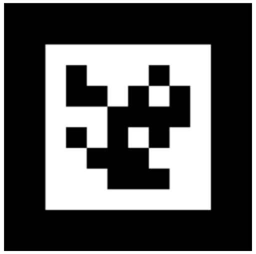
- Today, there are thousands of application of this algorithm:
  - Augmented reality

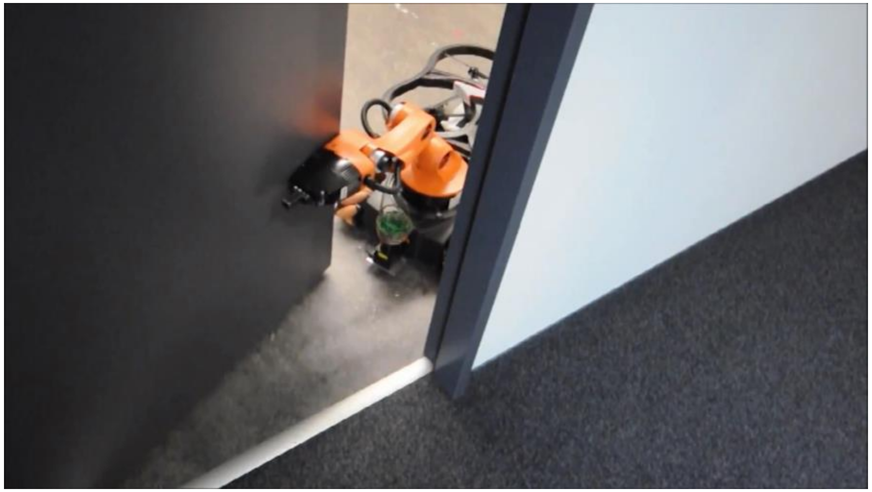


AR Tags: http://april.eecs.umich.edu/wiki/index.php/April_Tags

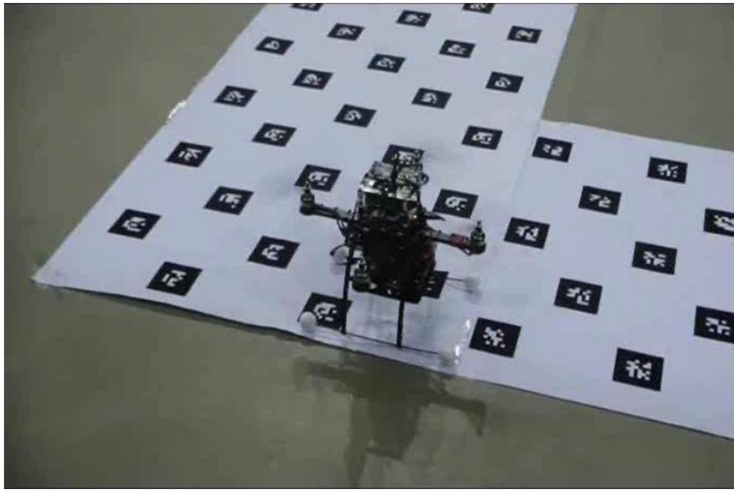
Application of calibration from planar grids

- Today, there are thousands of application of this algorithm:
  - Augmented reality
  - Robotics (beacon-based localization)
- Do we need to know the metric size of the tag?
  - For Augmented Reality?
  - For Robotics?





RPG (us) 2013

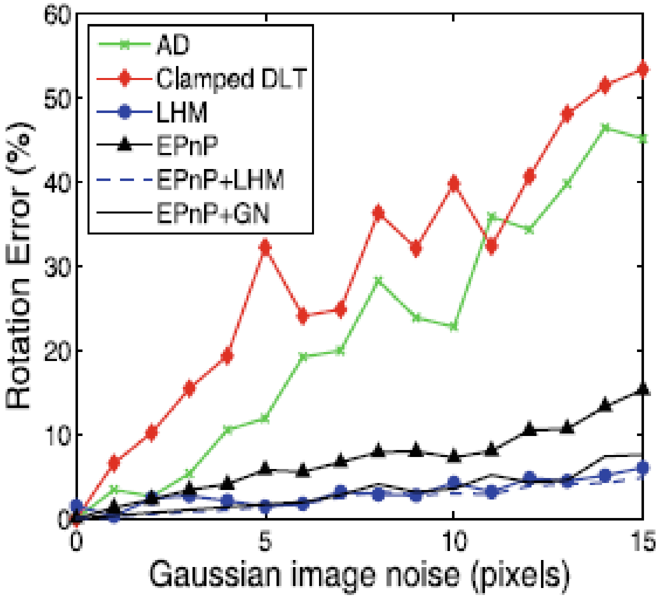


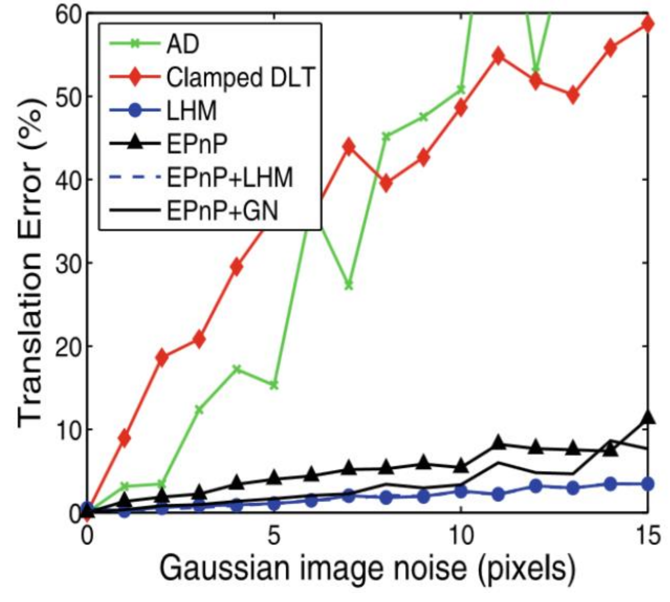
ETH, Pollefeys group, 2010

AR Tags: http://april.eecs.umich.edu/wiki/index.php/April_Tags

DLT vs PnP: Accuracy vs noise

If the camera is calibrated, only R and T need to be determined. In this case, should we use DLT (linear system of equations) or PnP (non linear)?

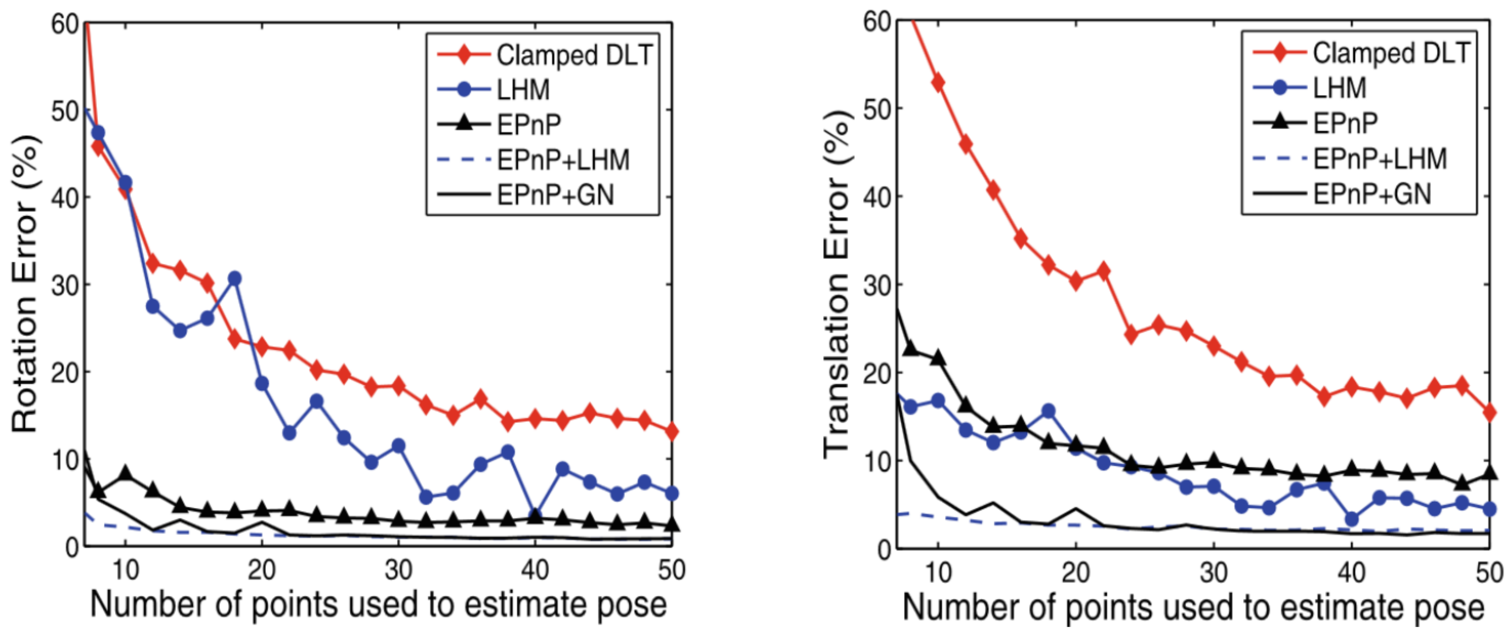




Lepetit, Moreno Noguer, Fua, EP*n*P: An Accurate *O(n)* Solution to the P*n*P Problem, IJCV'09

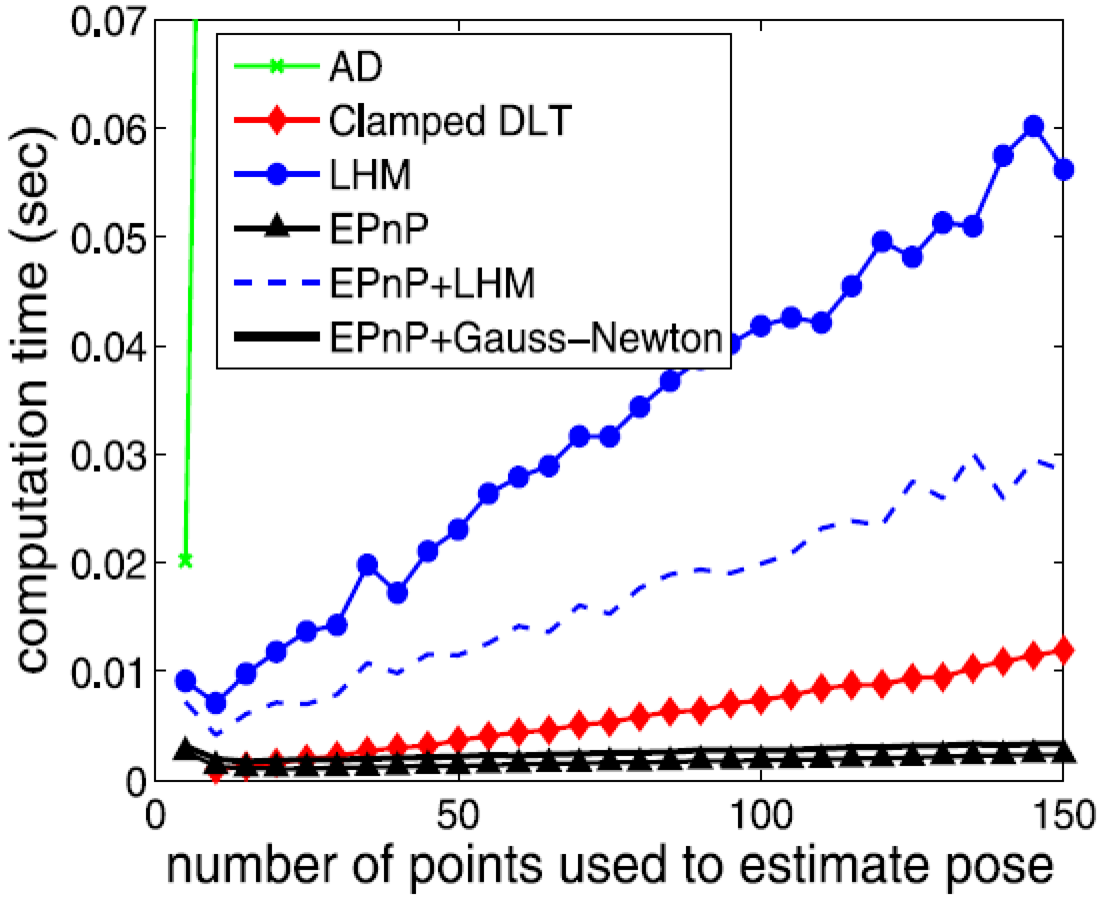
DLT vs PnP: Accuracy vs number of points

If the camera is calibrated, only R and T need to be determined. In this case, should we use DLT (linear system of equations) or PnP (non linear)?



Lepetit, Moreno Noguer, Fua, EP*n*P: An Accurate *O(n)* Solution to the P*n*P Problem, IJCV'09

DLT vs PnP: Timing



Lepetit, Moreno Noguer, Fua, EP*n*P: An Accurate *O(n)* Solution to the P*n*P Problem, IJCV'09

Concepts to remember

- Camera calibration
  - DLT algorithm
  - Calibration from planar grids
- Readings:
  - Chapter 2.1 of Szeliski book

Outline of this lecture

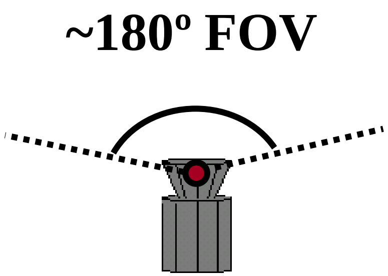
- Camera calibration
  - From 3D objects
  - From planar grids
- Non conventional camera models

Omnidirectional Cameras

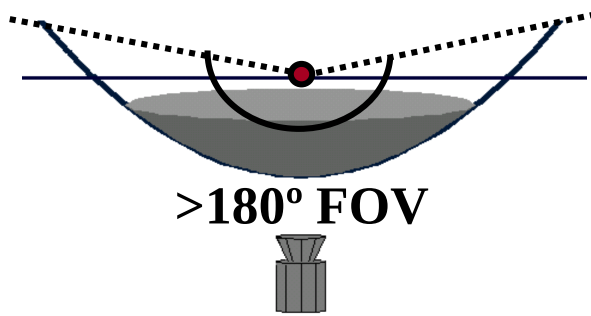
Rome, St. Peter’s square

Overview on Omnidirectional Cameras

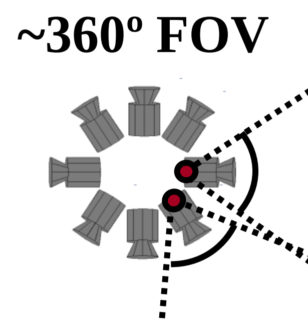
Omnidirectional sensors come in many varieties, but by definition must have a wide field-of-view.



Wide FOV dioptric cameras (e.g. fisheye)



Catadioptric cameras (e.g. cameras and mirror systems)



Polydioptric cameras (e.g. multiple overlapping cameras)

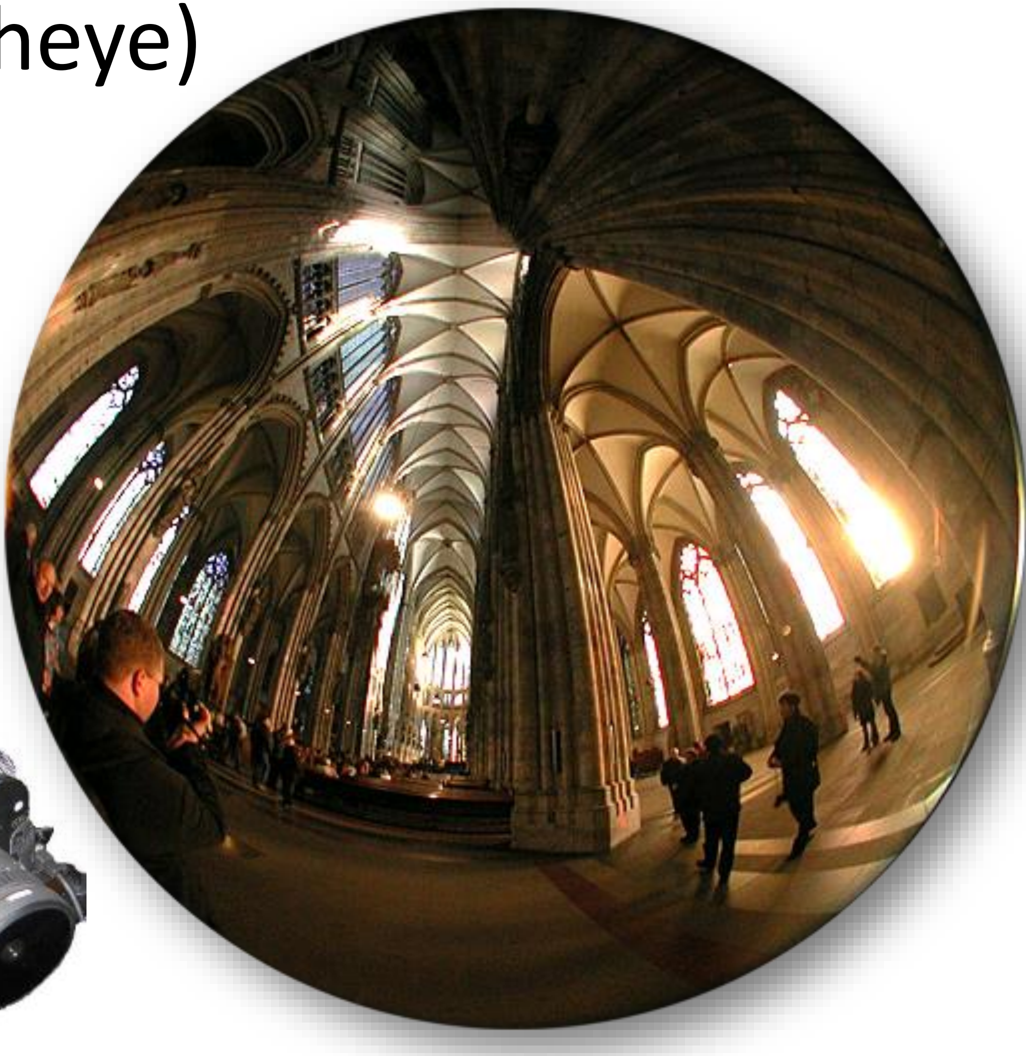
Dioptric

Catadioptric

Polydioptric

Catadioptric Cameras

Dioptric Cameras (fisheye)



Nikon Coolpix
FC-E9 Lens
360º×183º

Canon EOS-1
Sigma Lens
360º×180º

Example:

Same scene viewed by three different camera models:

Perspective

Fisheye

Catadioptric

http://rpg.ifi.uzh.ch/fov.html

Z. Zhang et al. (RPG), Benefit of Large Field-of-View Cameras for Visual Odometry, ICRA 2016

Applications

Applications

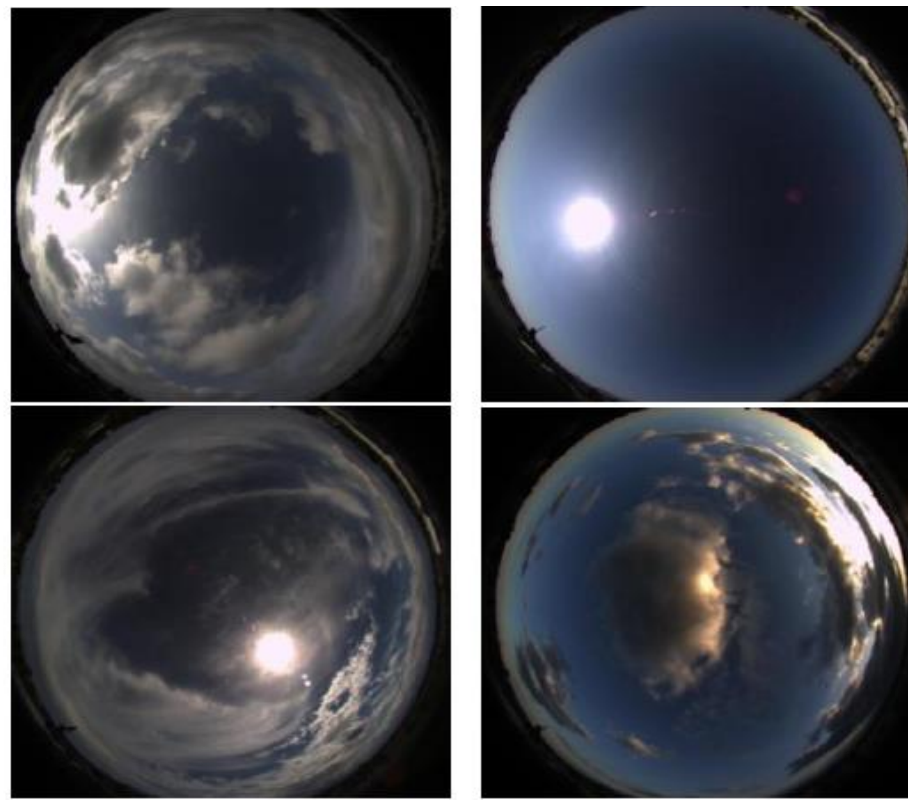
- Daimler, Bosch: for car driving assistance systems

(Courtesy of Daimler)

Applications

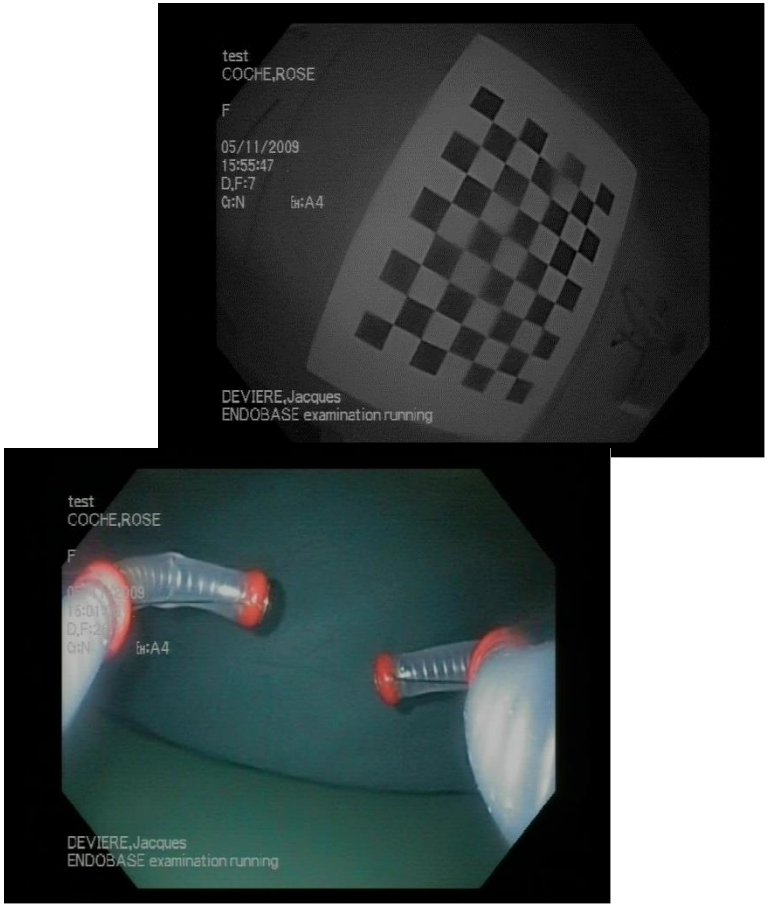
- Daimler, Bosch: for car driving assistance systems

- Meteorology: for sky observation



Applications

- Daimler, Bosch: for car driving assistance systems
- Meteorology: for sky observation
- Endoscopic Imagery: distortion removal (for the surgeon)



(Courtesy of Endo Tools Therapeutics, Brussels)

Applications

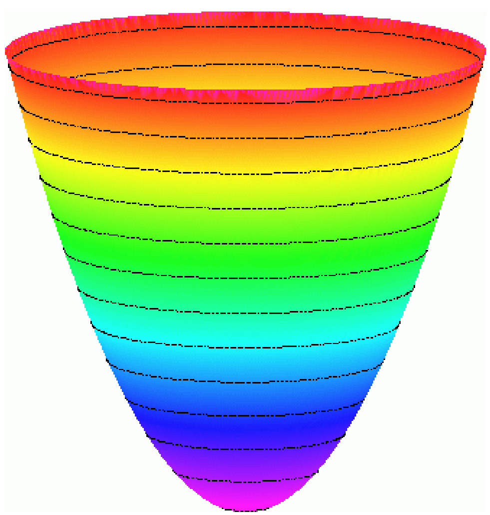
- Daimler, Bosch: for car driving assistance systems
- Meteorology: for sky observation
- Endoscopic Imagery: distortion removal (for the surgeon)
- RoboCup domain

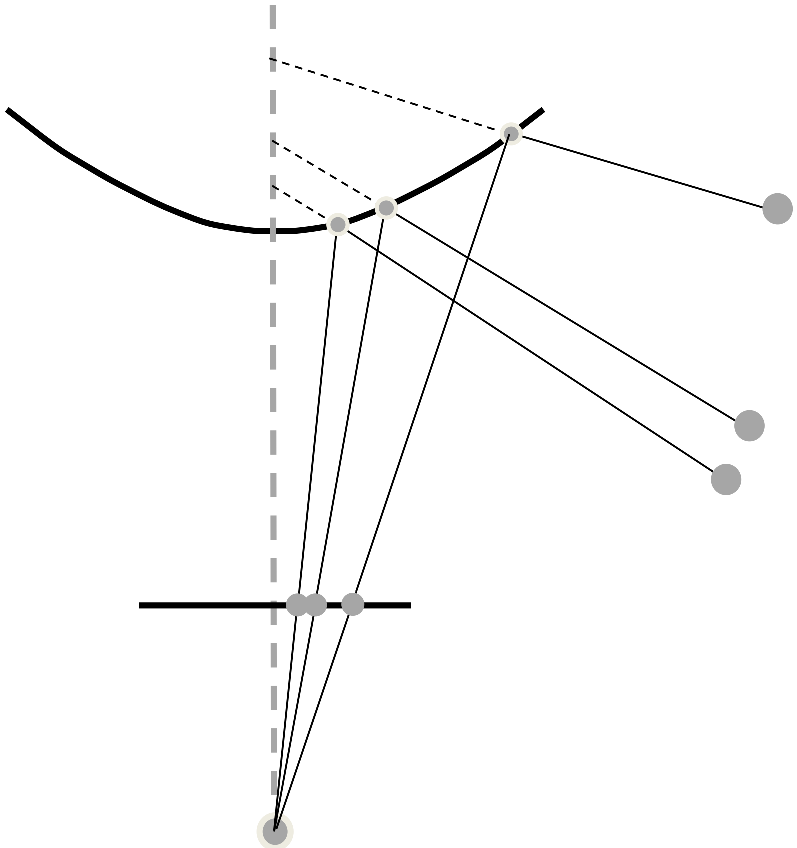
Applications

- Daimler, Bosch: for car driving assistance systems
- Meteorology: for sky observation
- Endoscopic Imagery: distortion removal (for the surgeon)
- RoboCup domain
- Google Street View

Catadioptric cameras

- mirror
- perspective camera

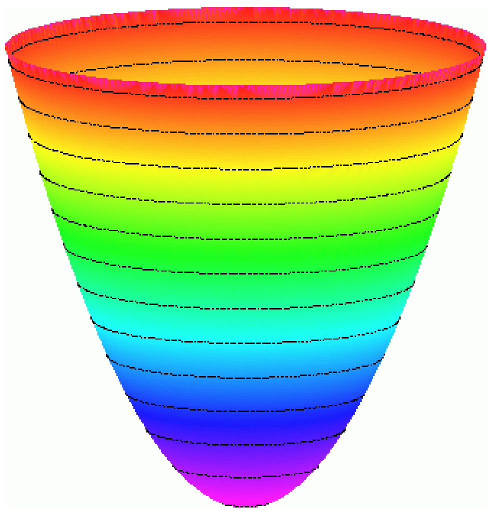


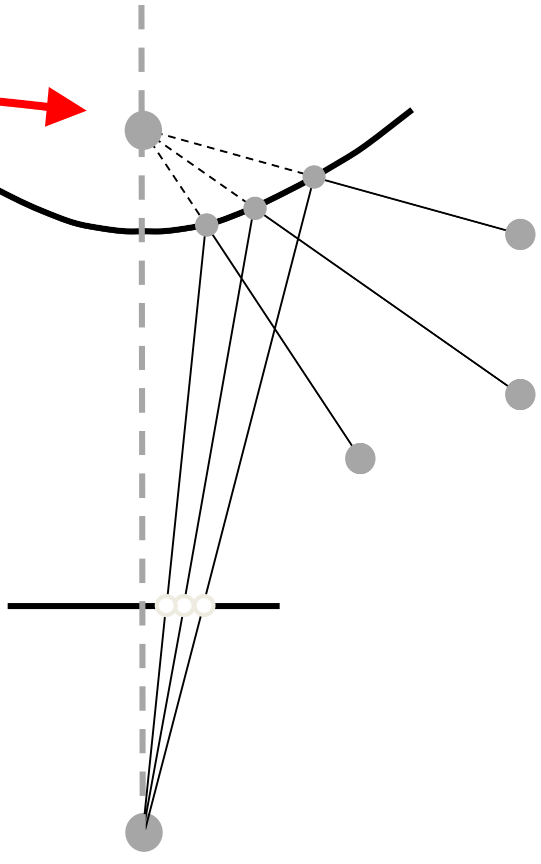


Catadioptric cameras

Central catadioptric cameras

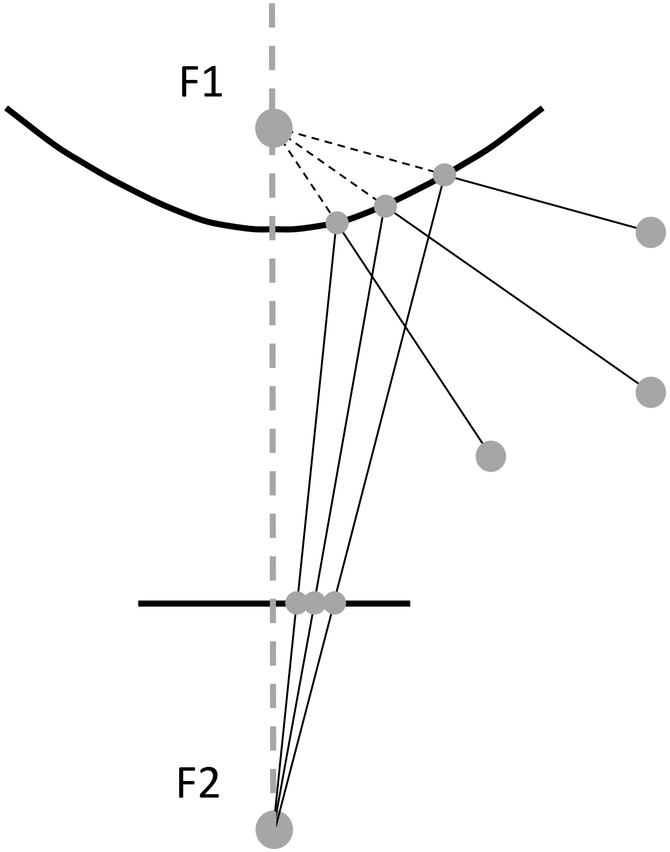
- mirror **(surface of revolution of a conic)**
- camera
- single effective viewpoint

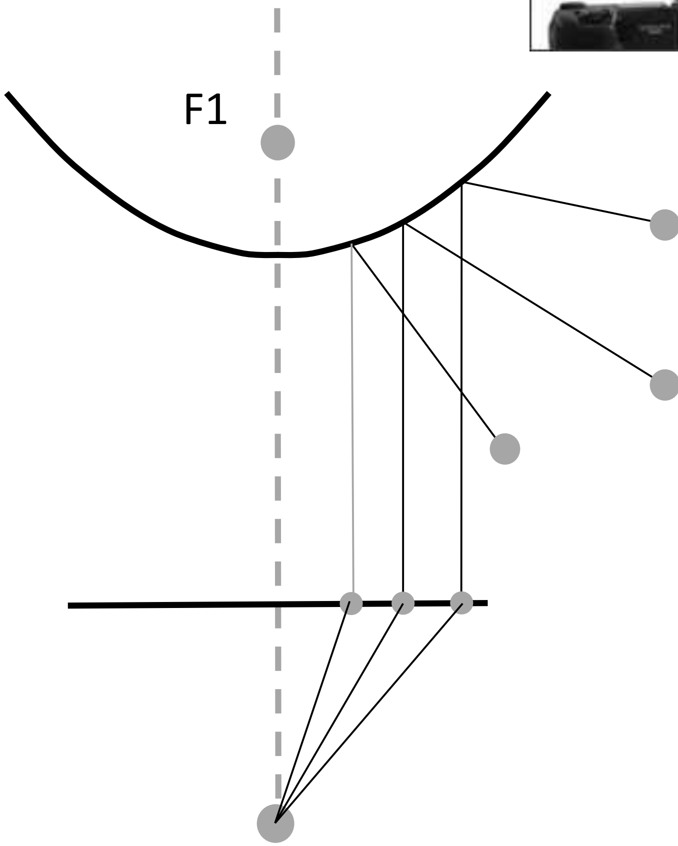




Catadioptric cameras

- hyperbola + perspective camera
- parabola + orthographic lens



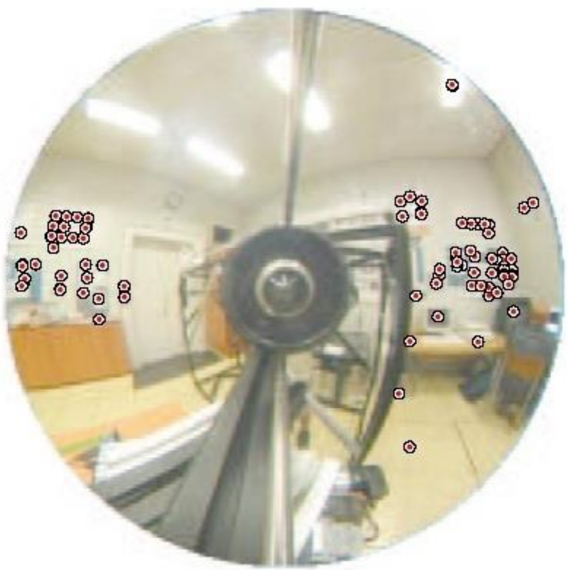


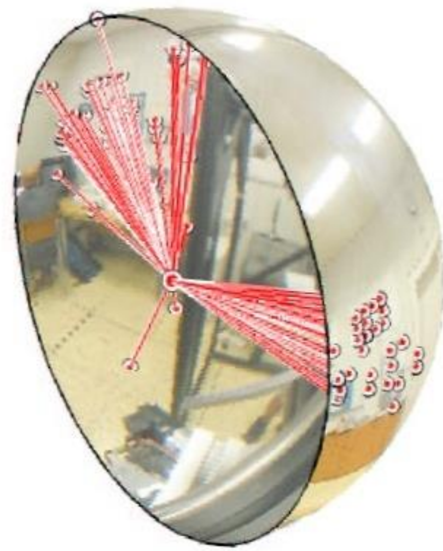
Why is it important that the camera be central (i.e., have a single effective viewpoint)?

- We can unwrap parts or all omnidirectional image into a perspective one

Why is it important that the camera be central (i.e., have a single effective viewpoint)?

- We can unwrap parts or all omnidirectional image into a perspective one
- We can transform image points normalized vectors in the unit sphere
- We can apply standard algorithms valid for perspective geometry.


Points

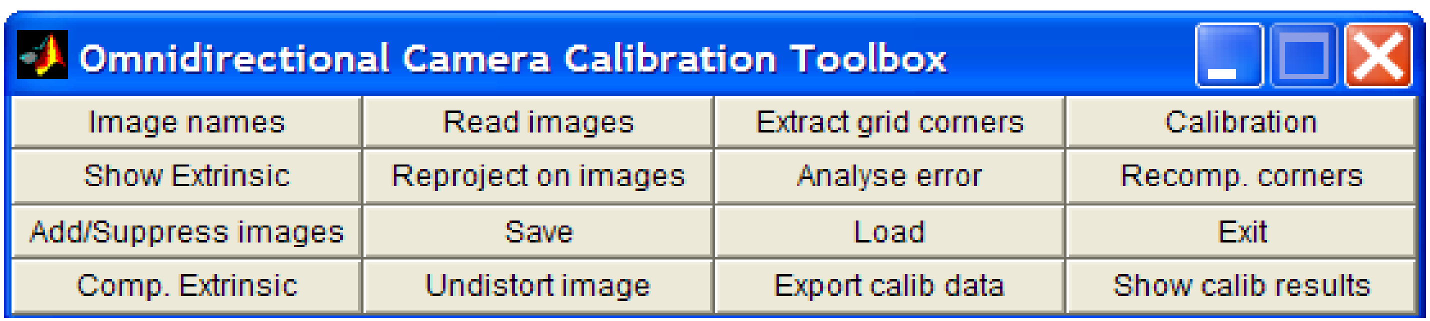

Rays

Omnidirectional camera calibration toolbox for Matlab (Scaramuzza, 2006)

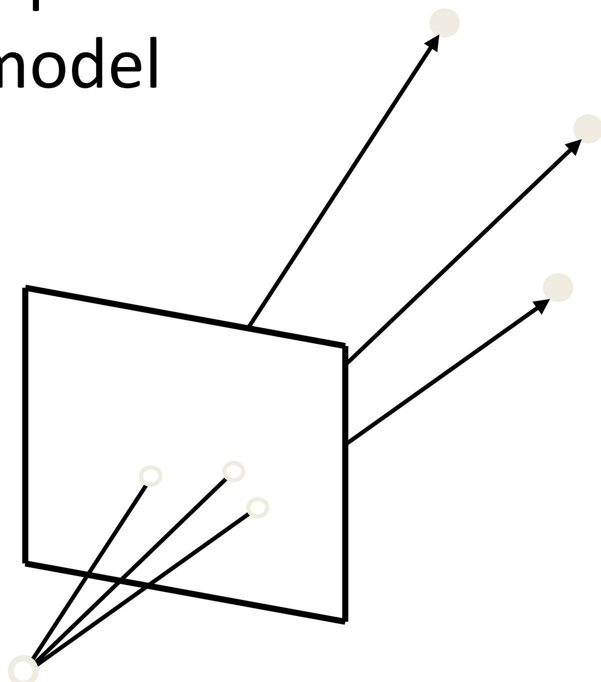
- World's standard toolbox for calibrating omnidirectional cameras (used at NASA, Daimler, IDS, Volkswagen, Audi, VW, Volvo, …)

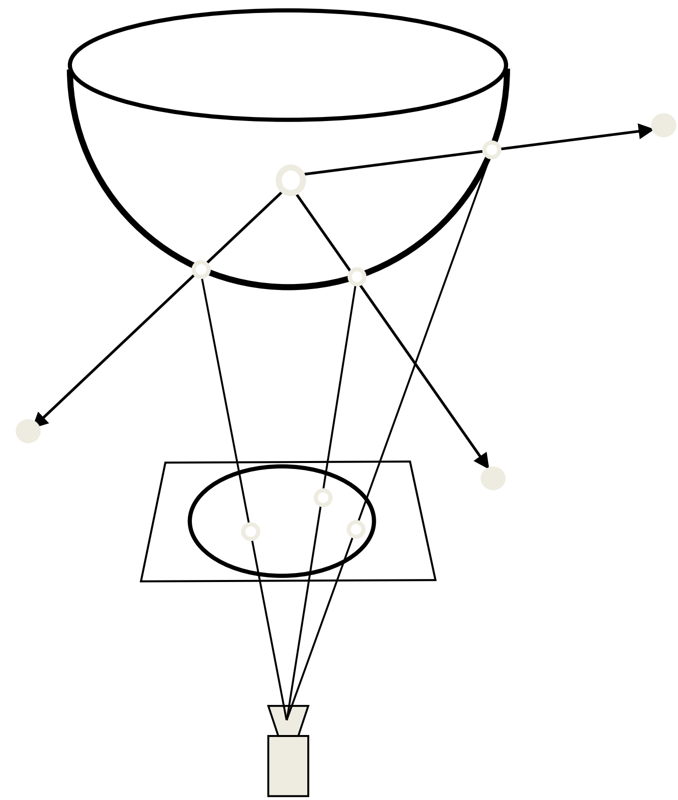
- Main applications are in robotics, endoscopy, video-surveillance, sky observation, automotive (Audi, VW, Volvo, …)

https://sites.google.com/site/scarabotix/ocamcalib-toolbox



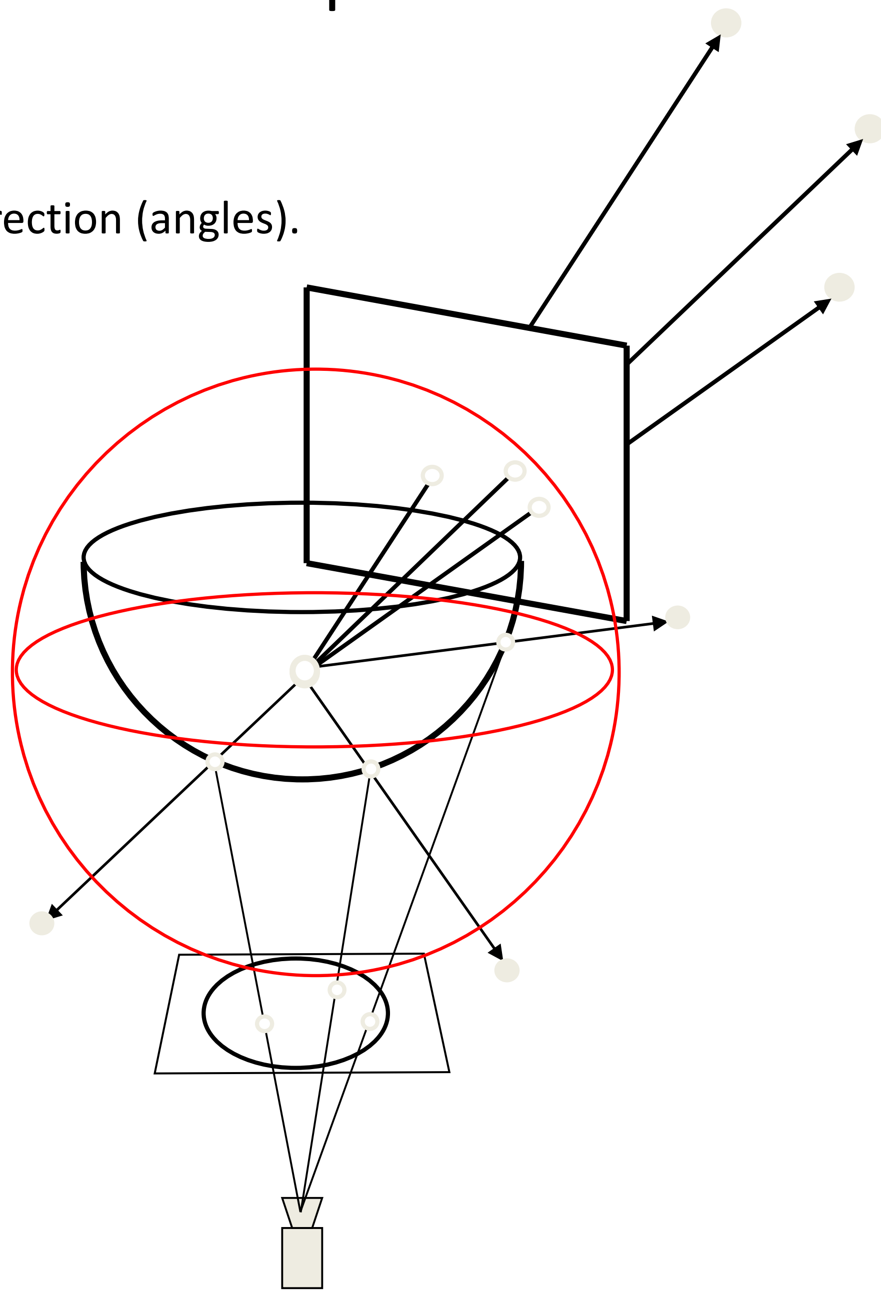
Equivalence between Perspective and Omnidirectional model





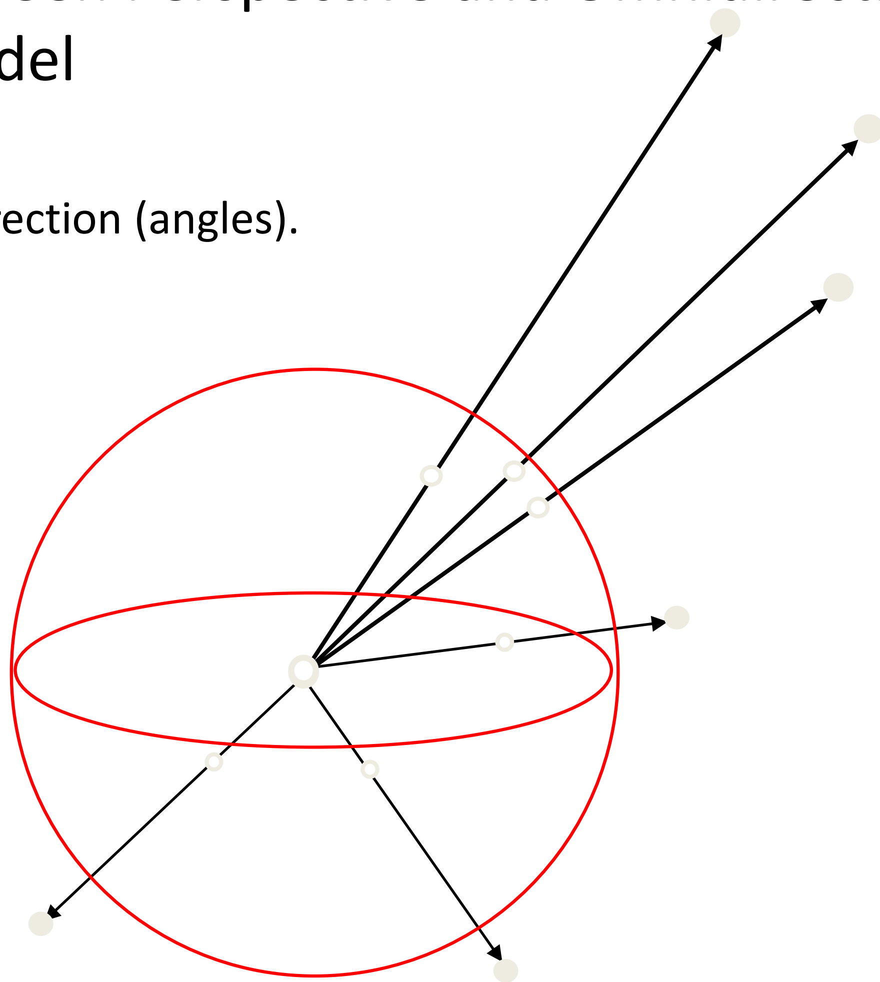
Equivalence between Perspective and Omnidirectional model

Measures the ray direction (angles).



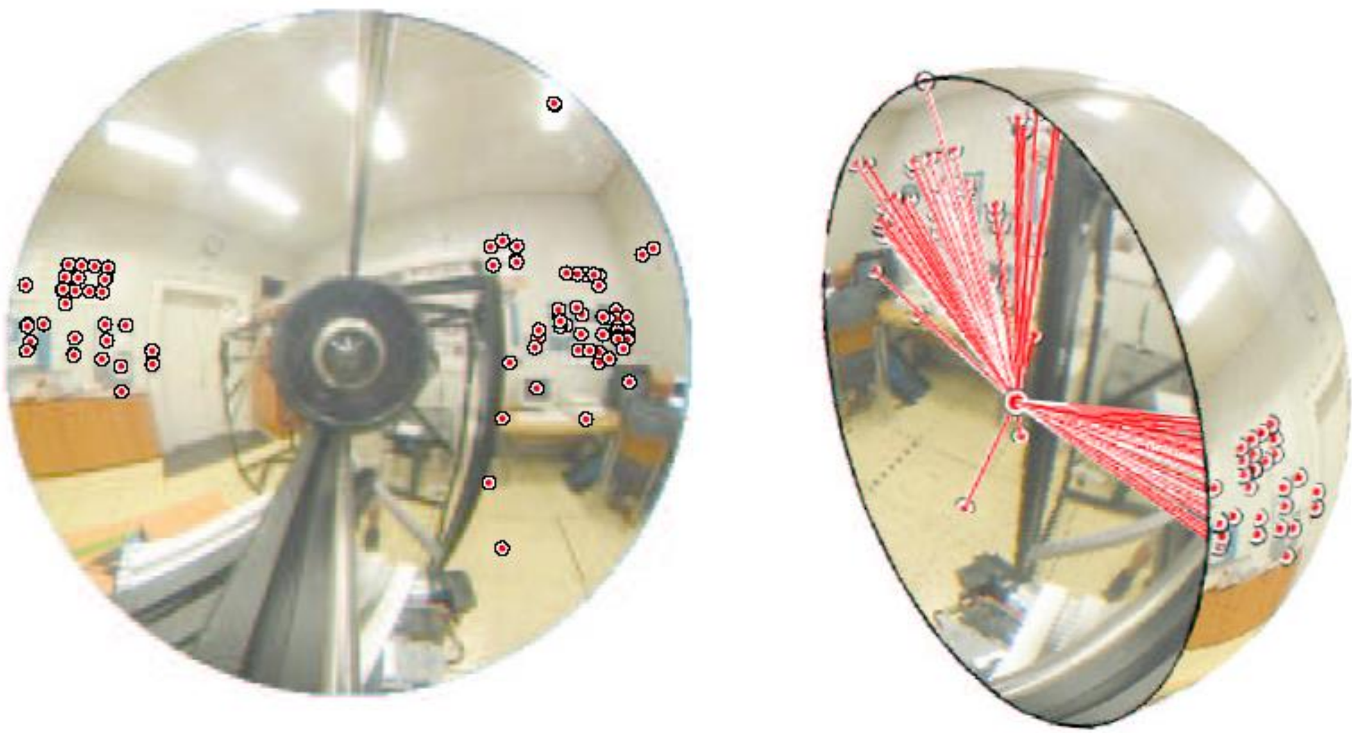
Equivalence between Perspective and Omnidirectional model: the Spherical Model

Measures the ray direction (angles).



Representation of image points on the unit sphere

Always possible after the camera has been calibrated!



Points   Rays

Summary (things to remember)

- P3P and PnP problems
- DLT algorithm
- Calibration from planar grid (Homography algorithm)
- Omnidirectional cameras
  - Central and non central projection
  - Dioptric
  - Catadioptric (working principle of conic mirrors)
- Unified (spherical) model for perspective and omnidirectional cameras