ROBOTICS & PERCEPTION GROUP

Universität Zürich[UZH]

Lecture 02
Image Formation

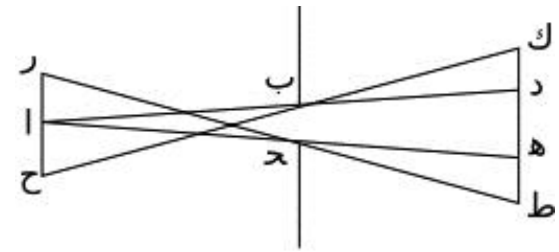
Davide Scaramuzza

Outline of this lecture

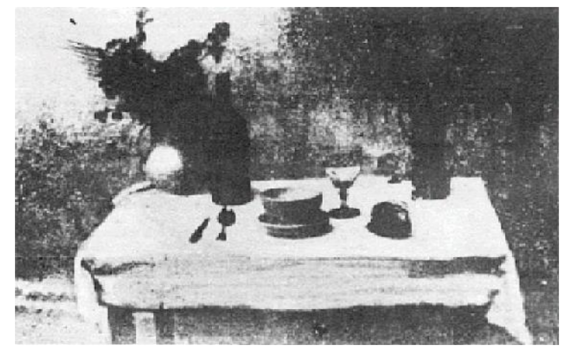
- Image Formation
- Other camera parameters
- Digital camera
- Perspective camera model
- Lens distortion
- Camera calibration
  - DLT algorithm

Historical context

- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura**: Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo**: Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film** (Eastman, 1889)
- **Cinema** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD**: Sony Mavica (1981)
- **First fully digital camera**: Kodak DCS100 (1990)



Alhacen's notes


Niepce, "La Table Servie," 1822

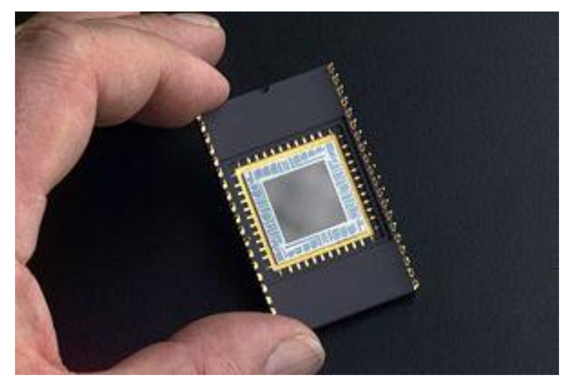
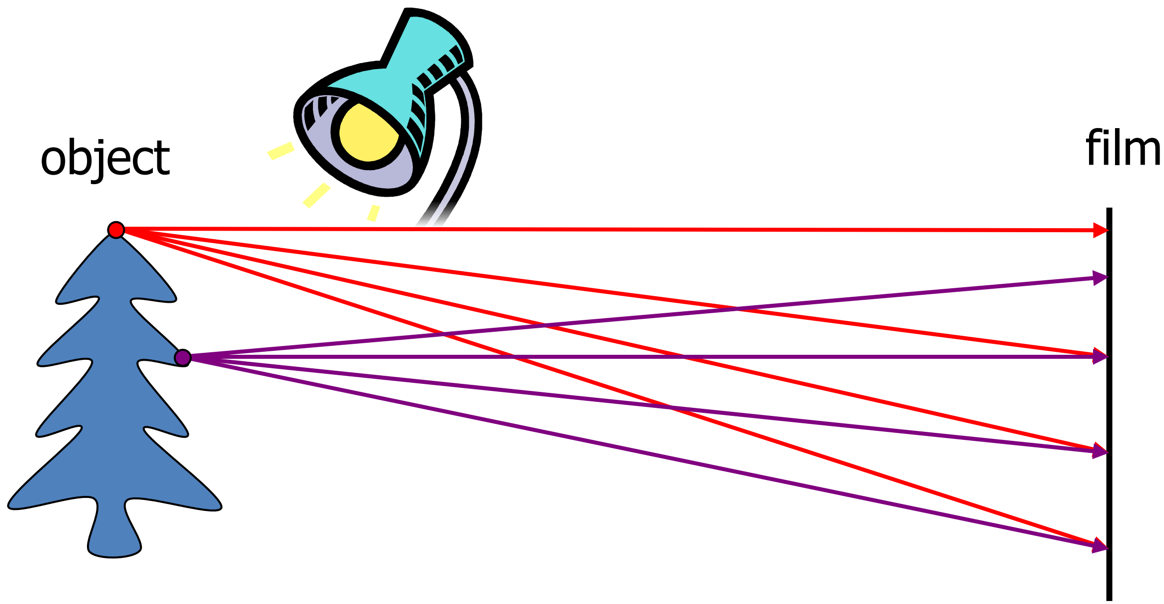

CCD chip

Image formation

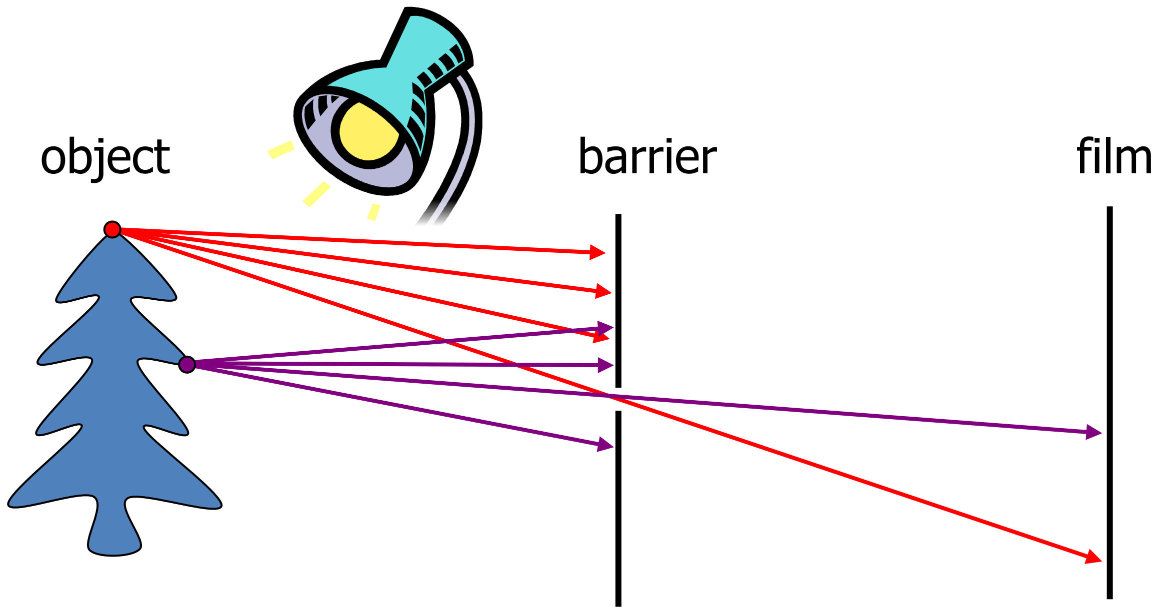
- How are objects in the world captured in an image?

How to form an image



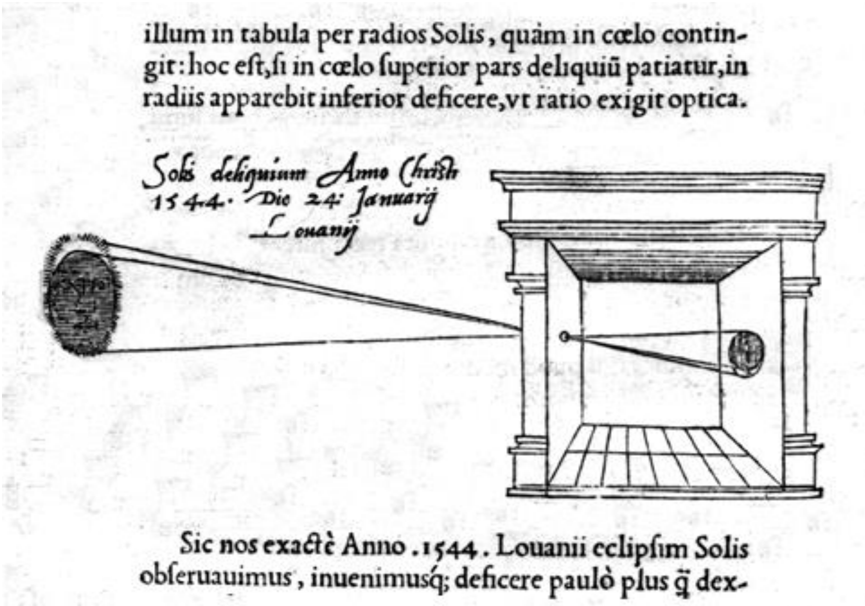
- Place a piece of film in front of an object
  ⇨ Do we get a reasonable image?

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the **aperture**

Camera obscura



In Latin, means ‘dark room’

- Basic principle known to Mozi (470-390 BC), Aristotle (384-322 BC)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)
- Image is inverted
- Depth of the room (box) is the effective focal length

"**Reinerus Gemma-Frisius**, observed an eclipse of the sun at Louvain on January 24, 1544, and later he used this illustration of the event in his book De Radio Astronomica et Geometrica, 1545. It is thought to be the first published illustration of a camera obscura..." Hammond, John H., The Camera Obscura, A Chronicle

Camera obscura at home

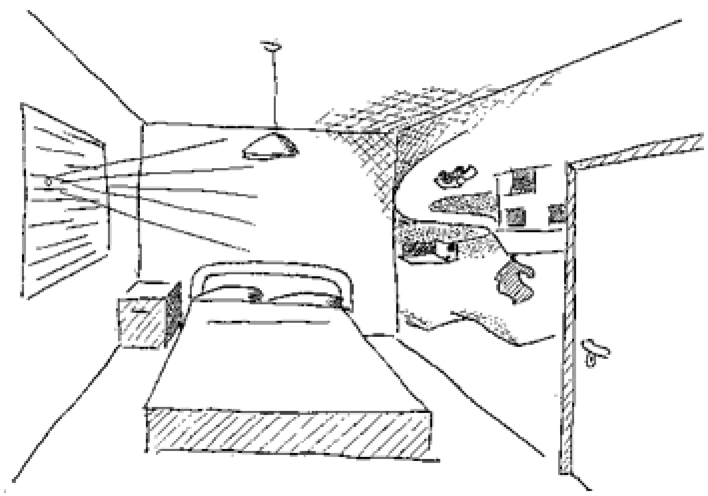


Figure 1 - A lens on the window creates the image of the external world on the opposite wall and you can see it every morning, when you wake up.

http://www.youtube.com/watch?v=B2aOs8RWntg

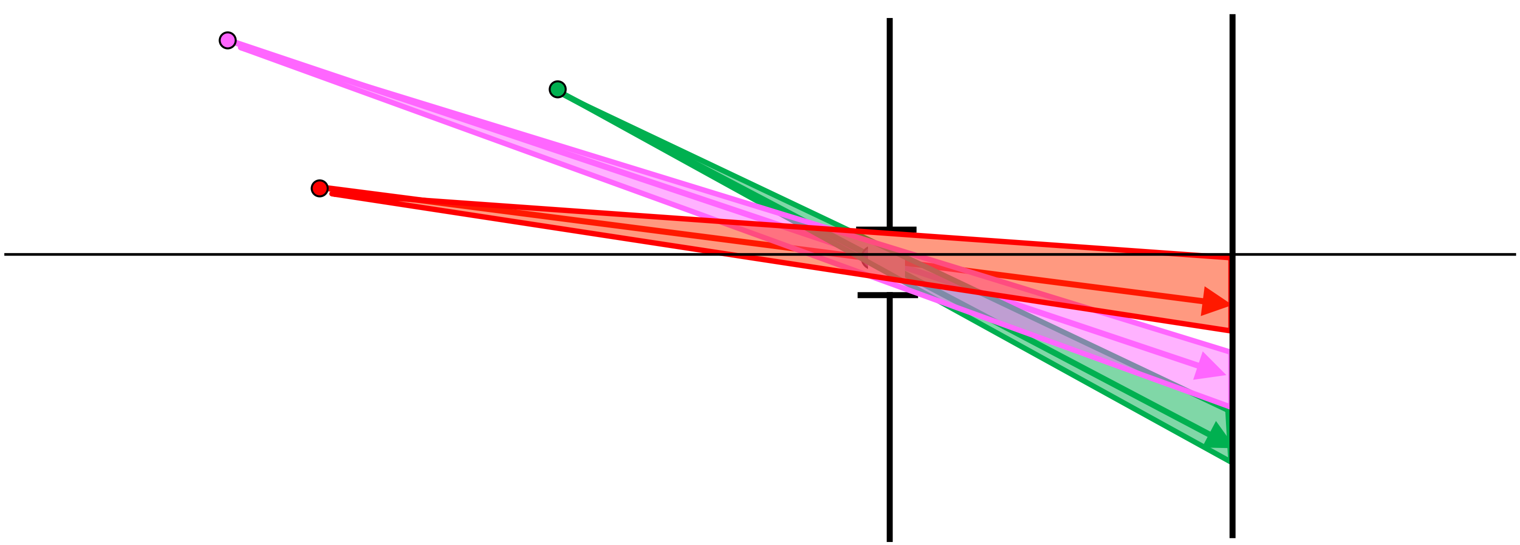
Sketch from http://www.funsci.com/fun3_en/sky/sky.htm

Home-made pinhole camera

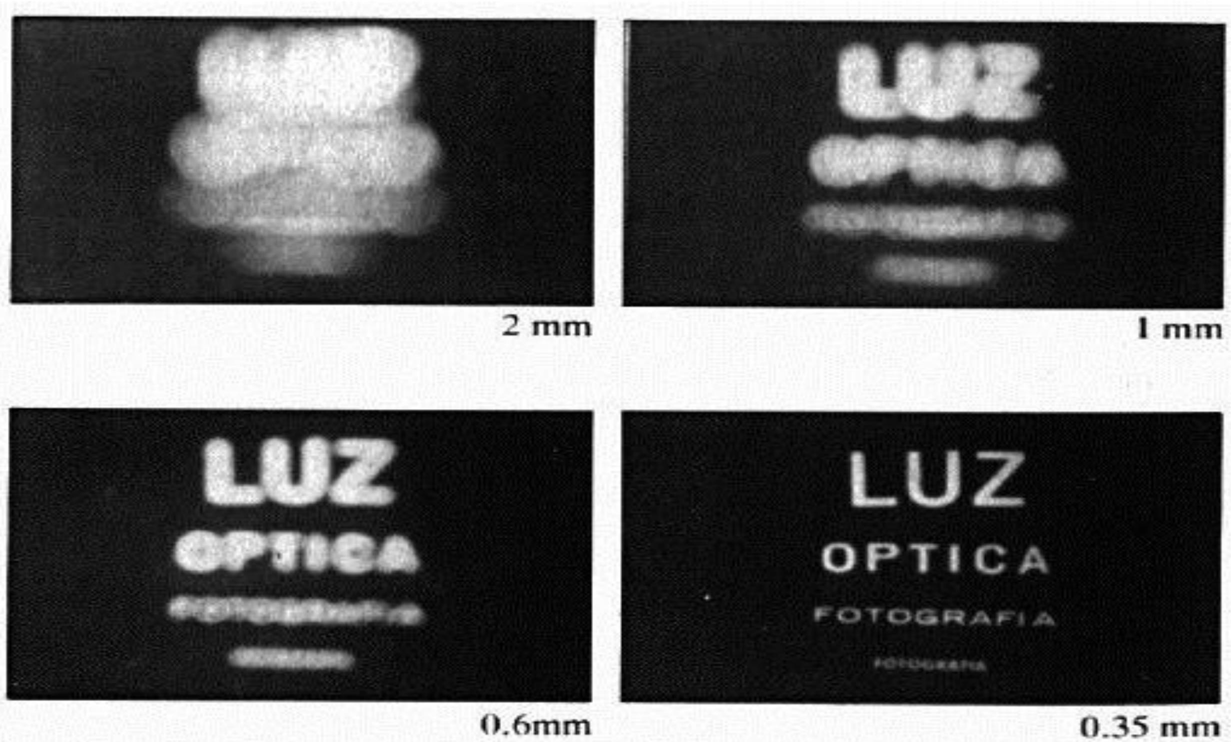
What can we do
to reduce the blur?

Effects of the Aperture Size

- *In an ideal pinhole*, only one ray of light reaches each point on the film ⇨ the image can be very dim
- Making aperture bigger makes the image blurry

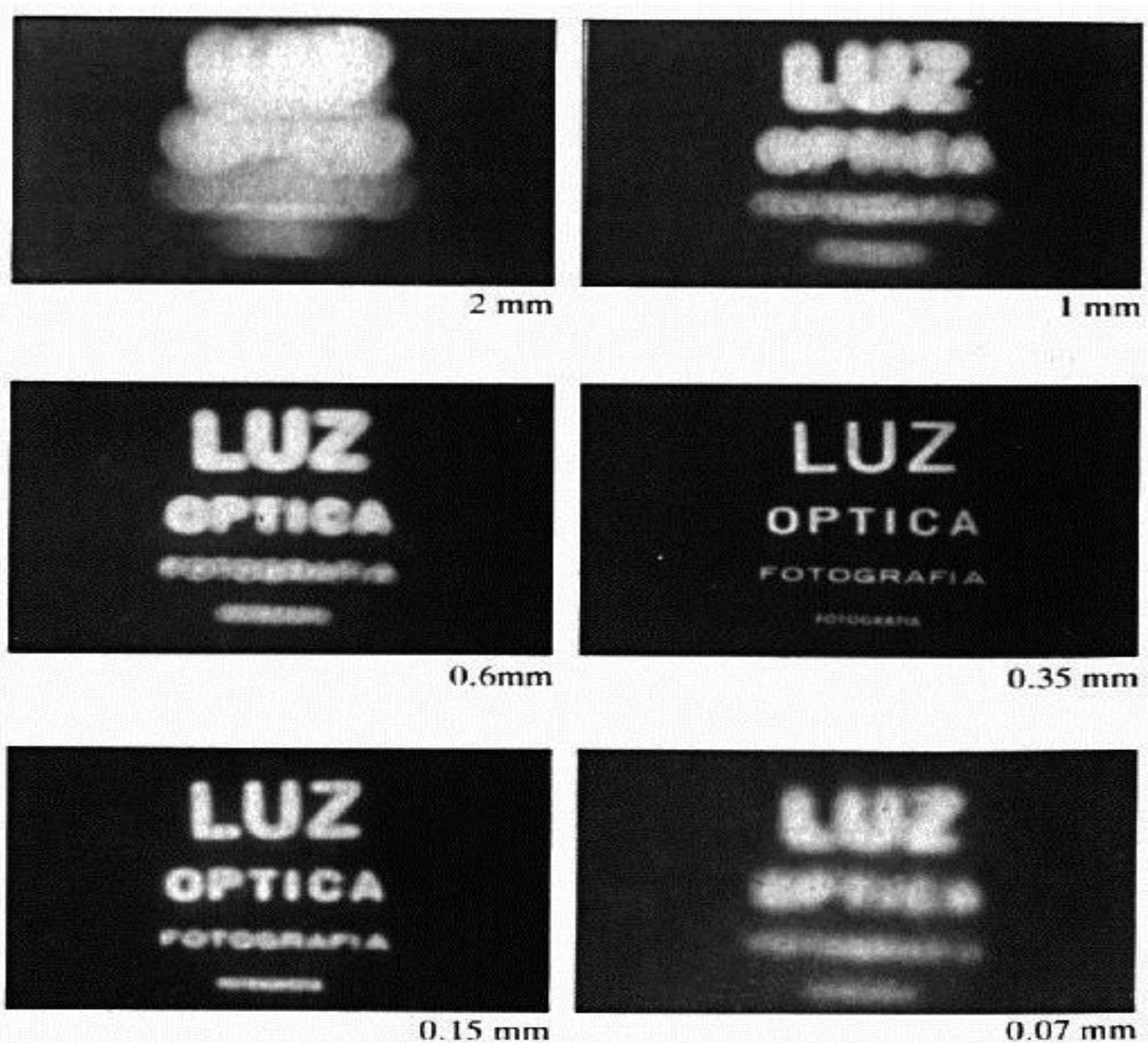


Shrinking the aperture



Why not make the aperture as small as possible?

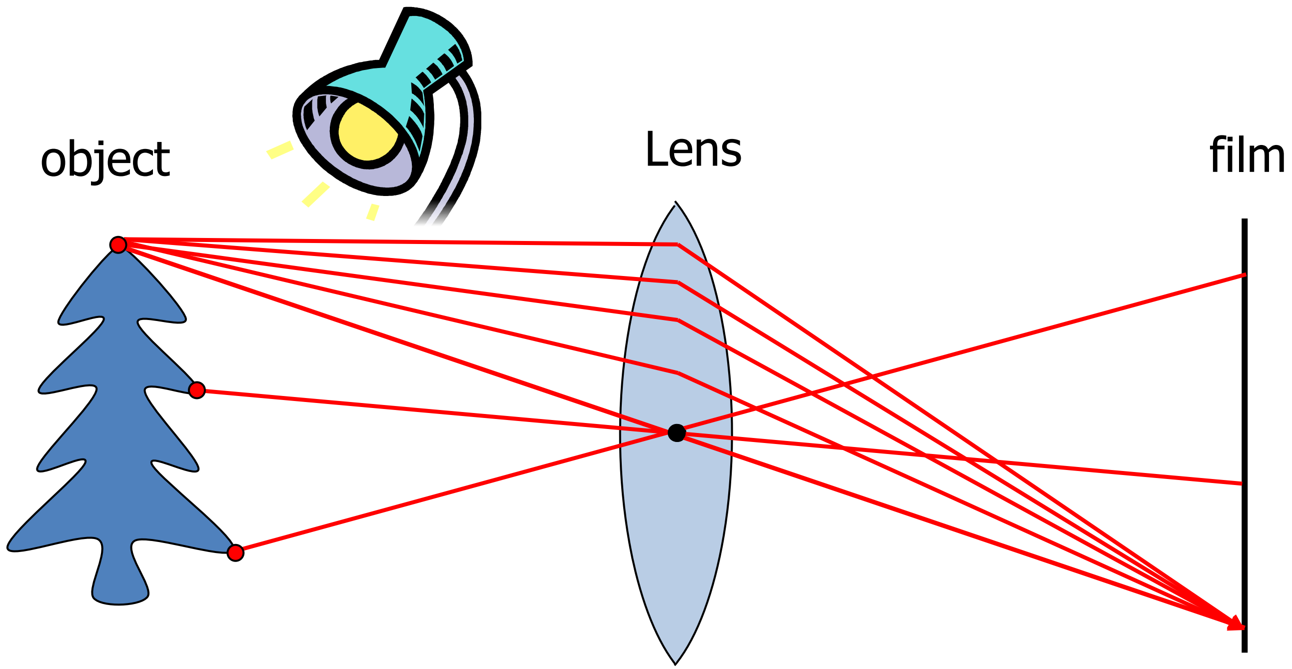
Shrinking the aperture



Why not make the aperture as small as possible?

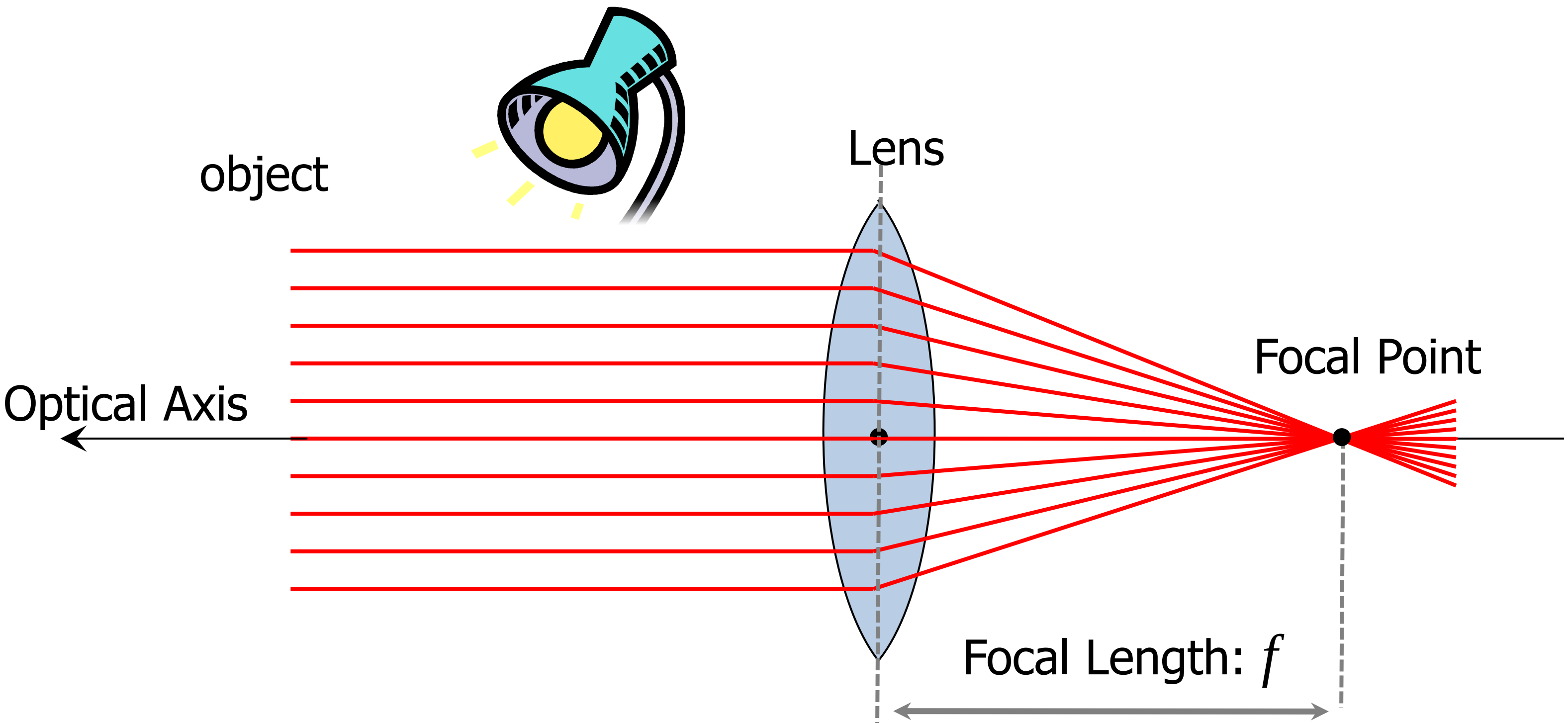
- Less light gets through (must increase the exposure)
- Diffraction effects…

Image formation using a converging lens



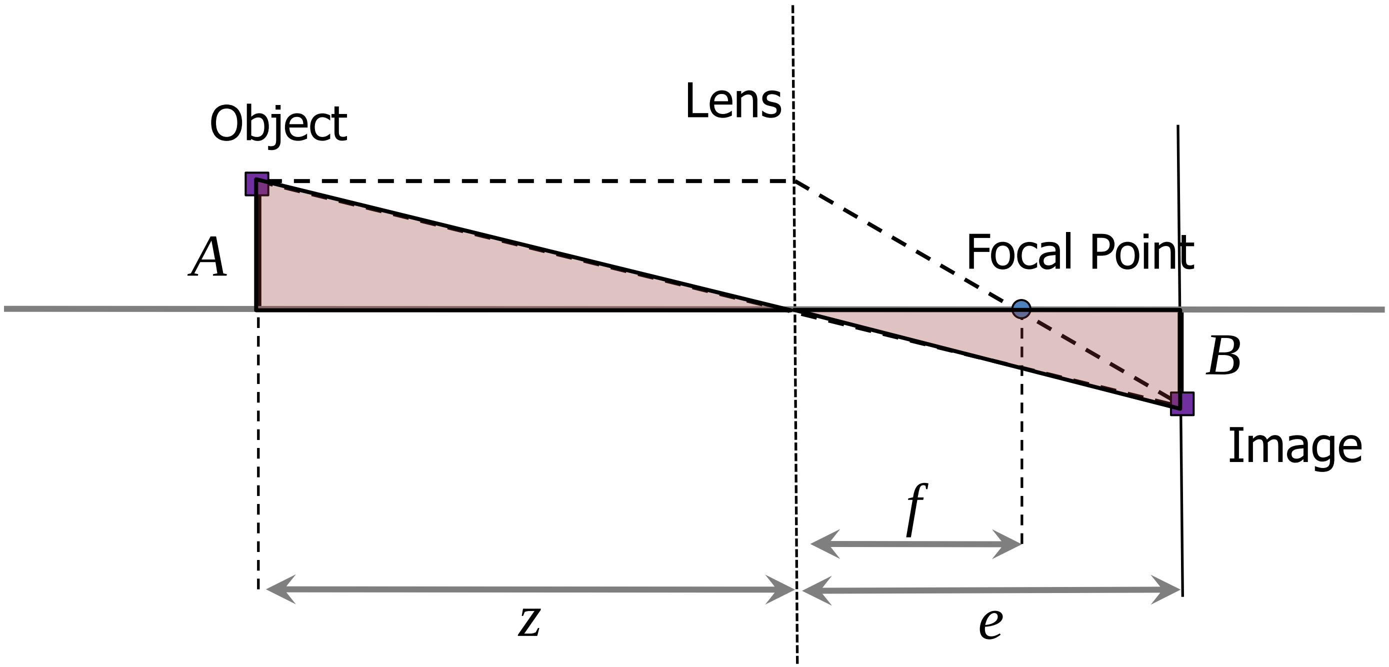
- A lens focuses light onto the film
- Rays passing through the **Optical Center** are not deviated

Image formation using a converging lens



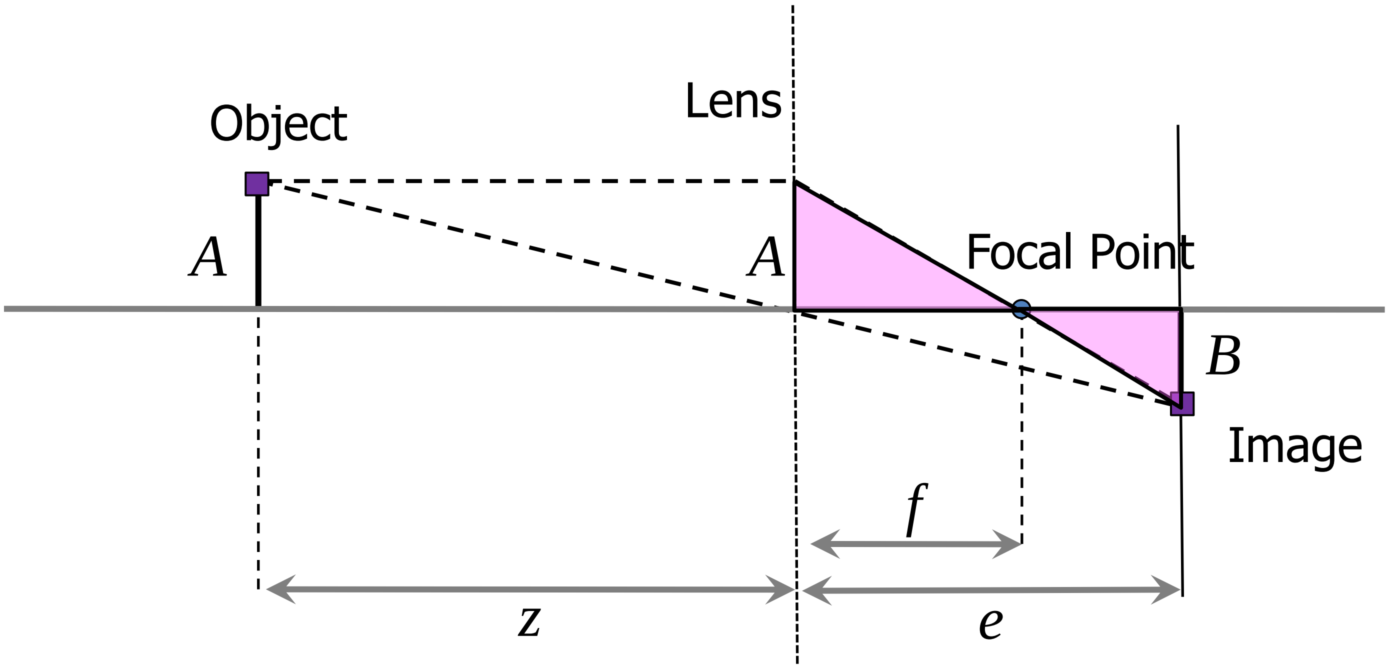
- All rays parallel to the **Optical Axis** converge at the **Focal Point**

Thin lens equation



Find a relationship between *f*, *z*, and *e*

Thin lens equation



- Similar Triangles:

$$\frac{B}{A} = \frac{e}{z}$$

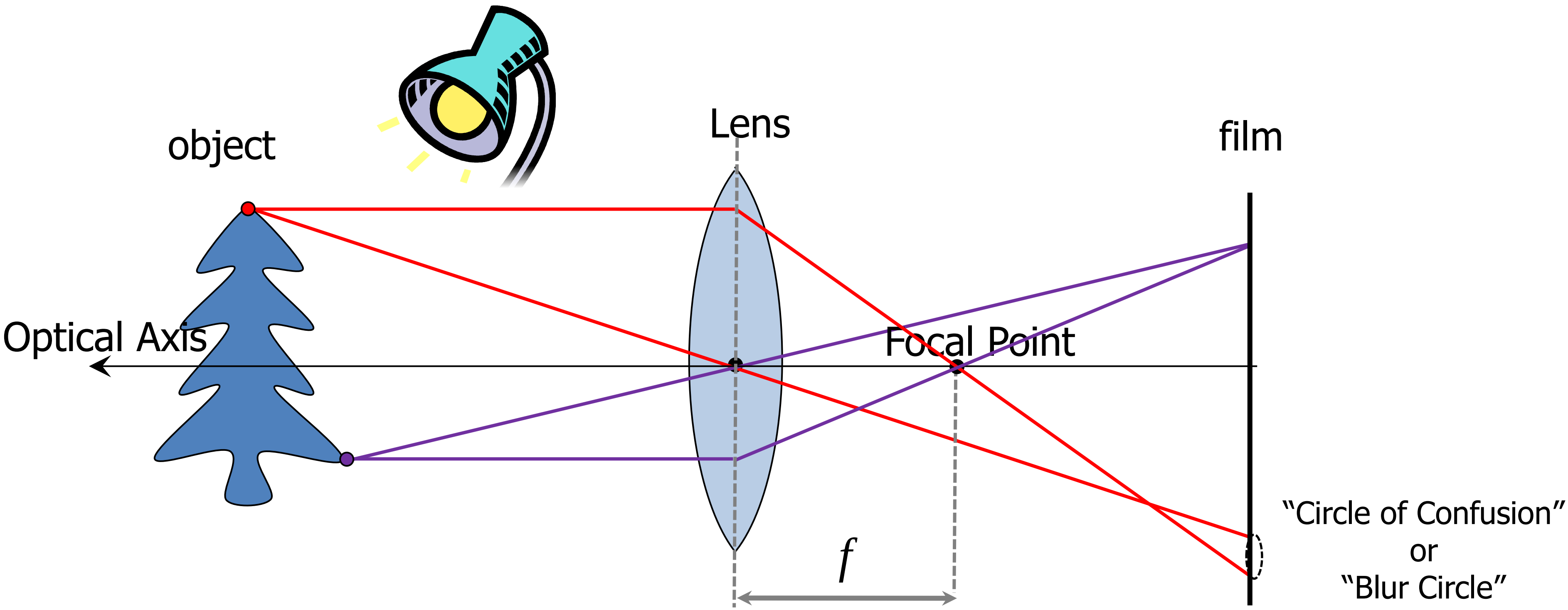
$$\frac{B}{A} = \frac{e-f}{f} = \frac{e}{f} - 1$$

$$\frac{e}{f} - 1 = \frac{e}{z}$$

"Thin lens equation"

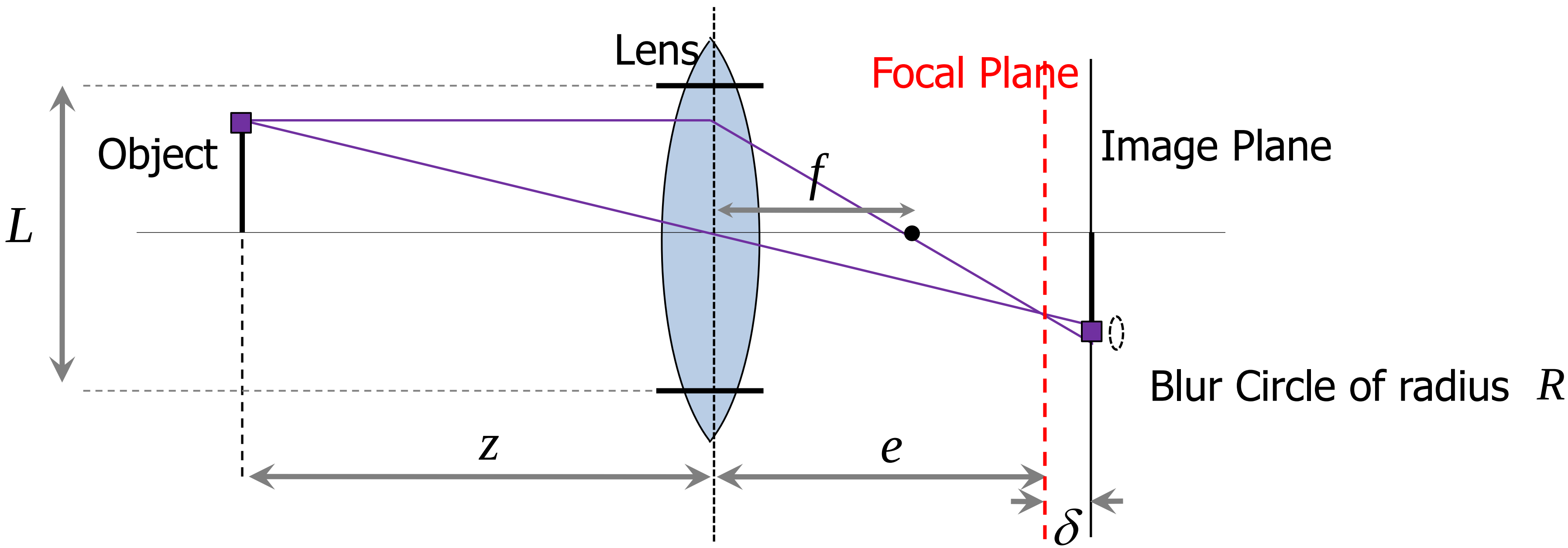
- Any object point satisfying this equation is in focus

“In focus”



- There is a specific distance from the lens, at which world points are “in focus” in the image
- Other points project to a “blur circle” in the image

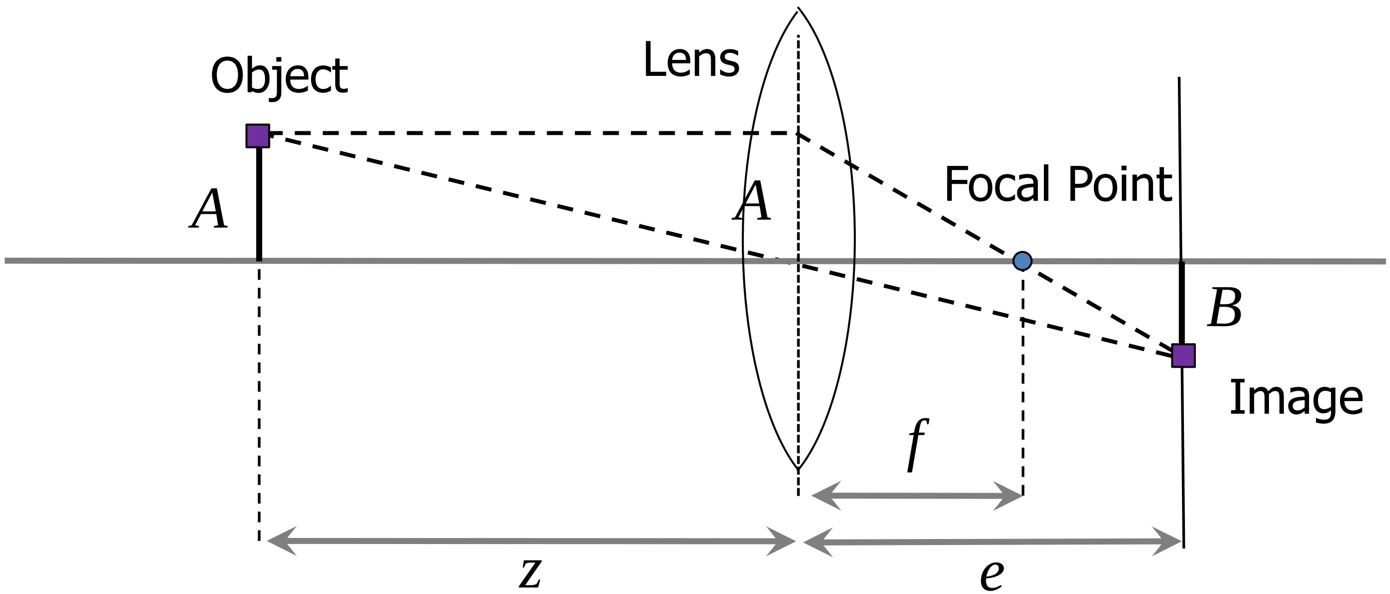
Blur Circle



- Object is out of focus ⇨ Blur Circle has radius: $R = \frac{L\delta}{2e}$
  - A minimal L (pinhole) gives minimal R
  - To capture a ‘good’ image:adjust camera settings, such that R remains smaller than the image resolution

The Pin-hole approximation

- What happens if $z \gg f$?

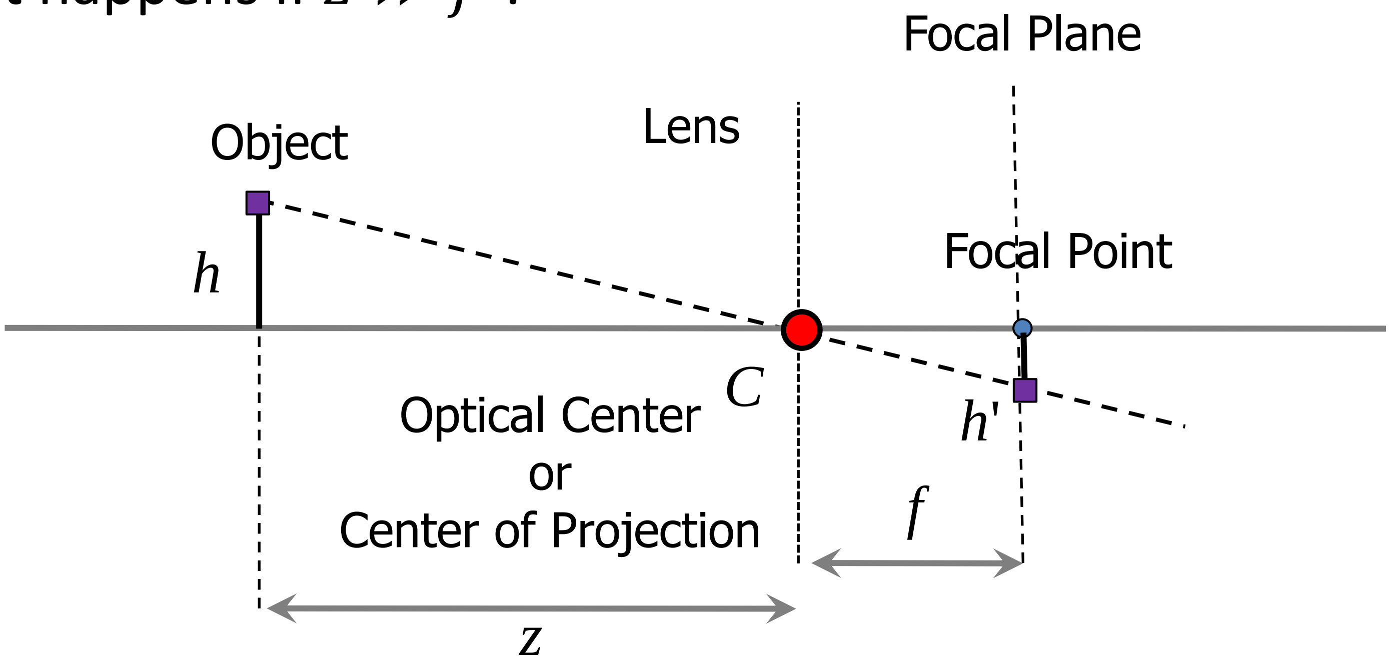


- We need to adjust the image plane such that objects at infinity are in focus

$$\frac{1}{f} = \underbrace{\boxed{\frac{1}{z}}}_{\cong 0} + \frac{1}{e} \Rightarrow \frac{1}{f} \approx \frac{1}{e} \Rightarrow f \approx e$$

The Pin-hole approximation

- What happens if $z \gg f$?



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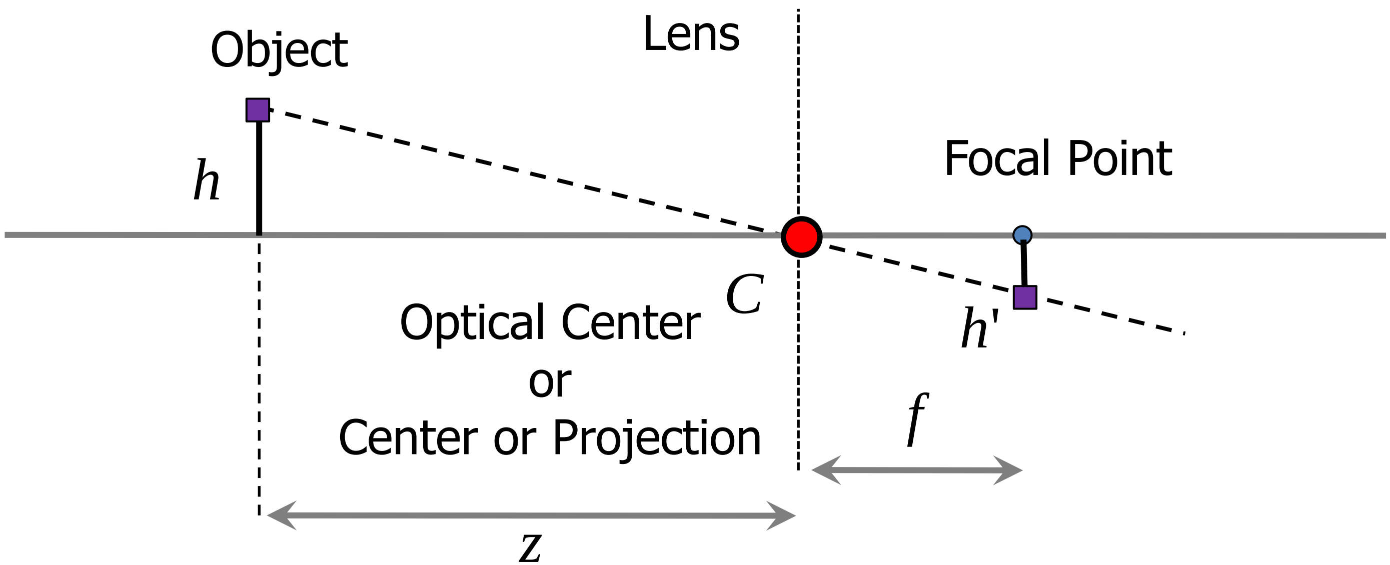
The Pin-hole approximation

- What happens if $z \gg f$?



- We need to adjust the image plane such that objects at infinity are in focus

$$\frac{h'}{h} = \frac{f}{z} \Rightarrow h' = \frac{f}{z} h$$

- The dependence of the apparent size of an object on its depth (i.e. distance from the camera) is known as **perspective**

Perspective effects

- Far away objects appear smaller

Perspective effects

Perspective and art

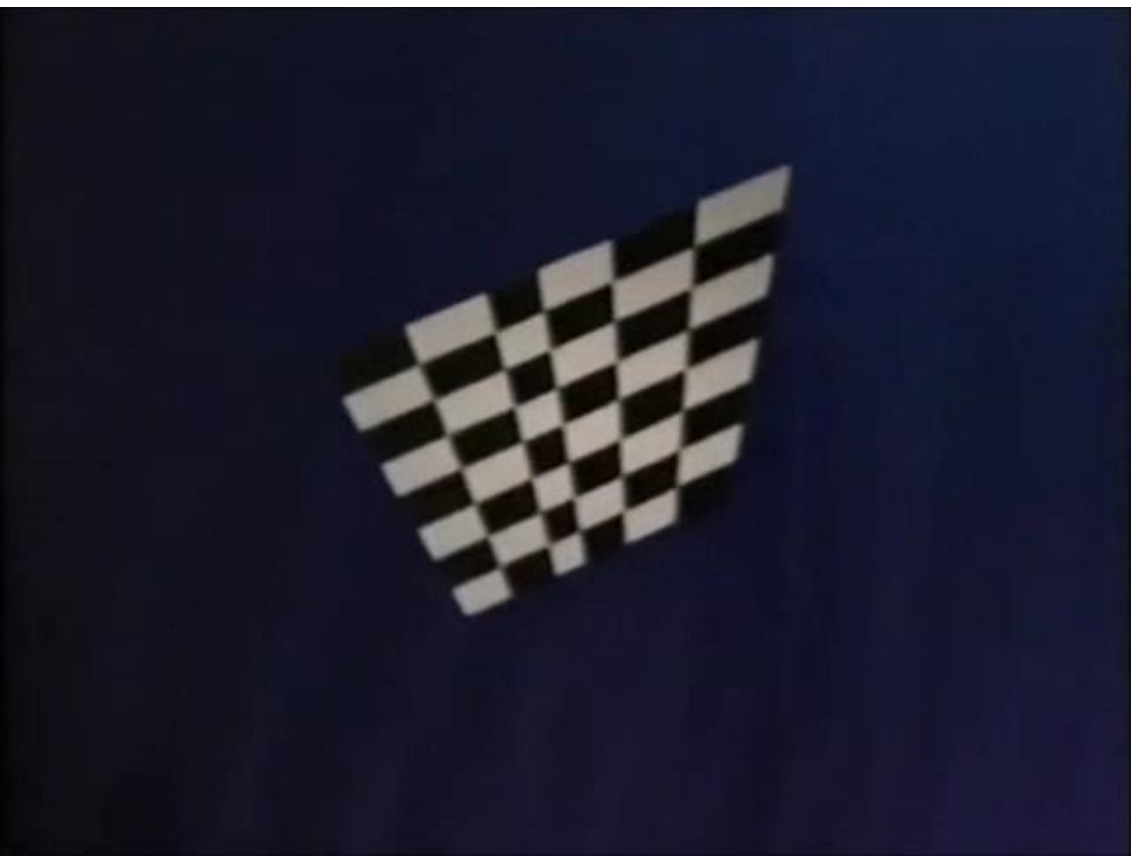
- Use of correct perspective projection indicated in 1st century BCE frescoes
- During Renaissance time, artists developped systematic methods to determine perspective projection (around 1480-1515)

Raphael

Durer

Playing with Perspective

- Perspective gives us very strong depth cues
  ⇨ hence we can perceive a 3D scene by viewing its 2D representation (i.e. image)
- An example where perception of 3D scenes is misleading is the Ames room



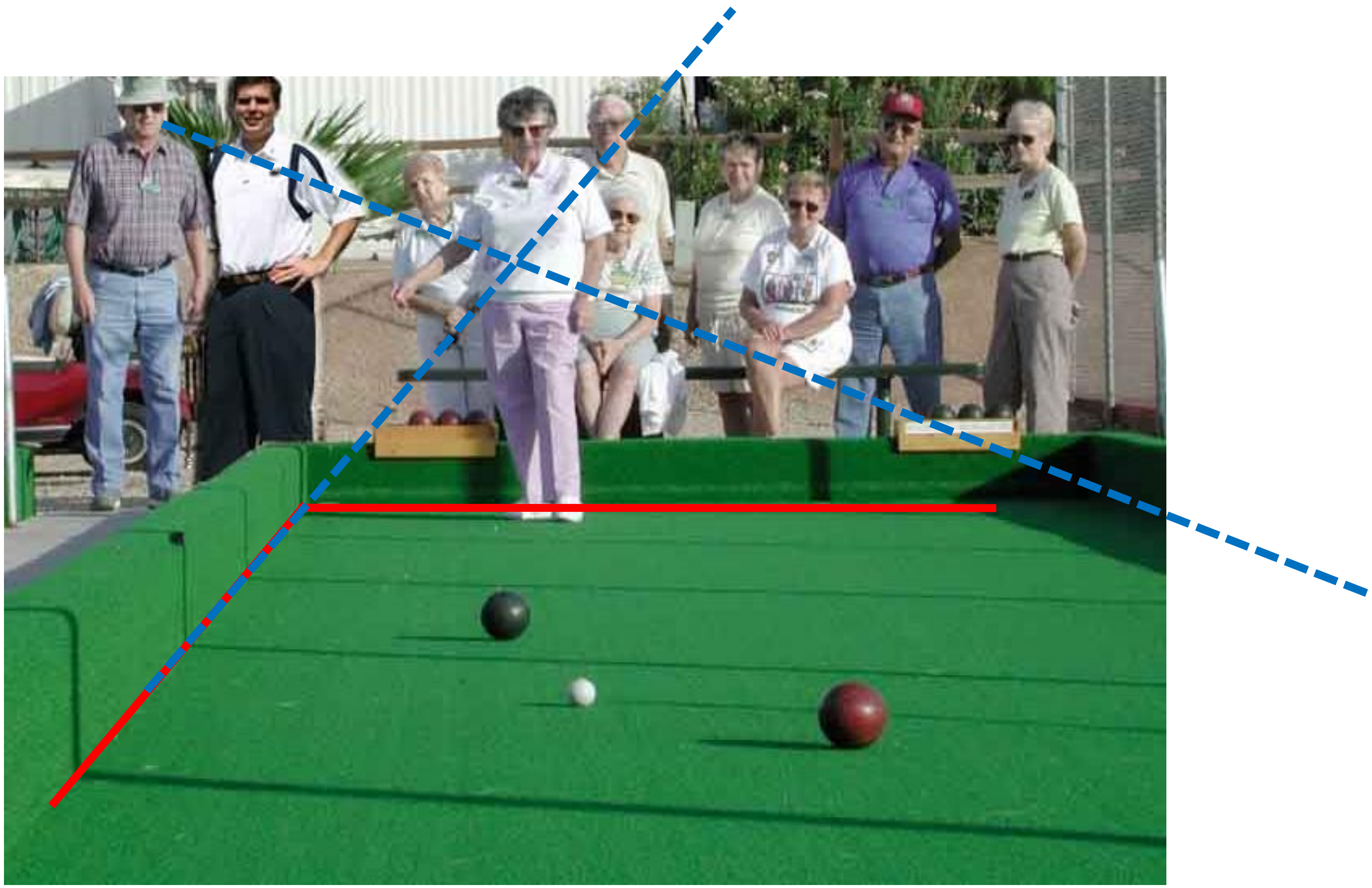
"Ames room"

A clip from "The computer that ate Hollywood" documentary. Dr. Vilayanur S. Ramachandran.

Projective Geometry

What is preserved?

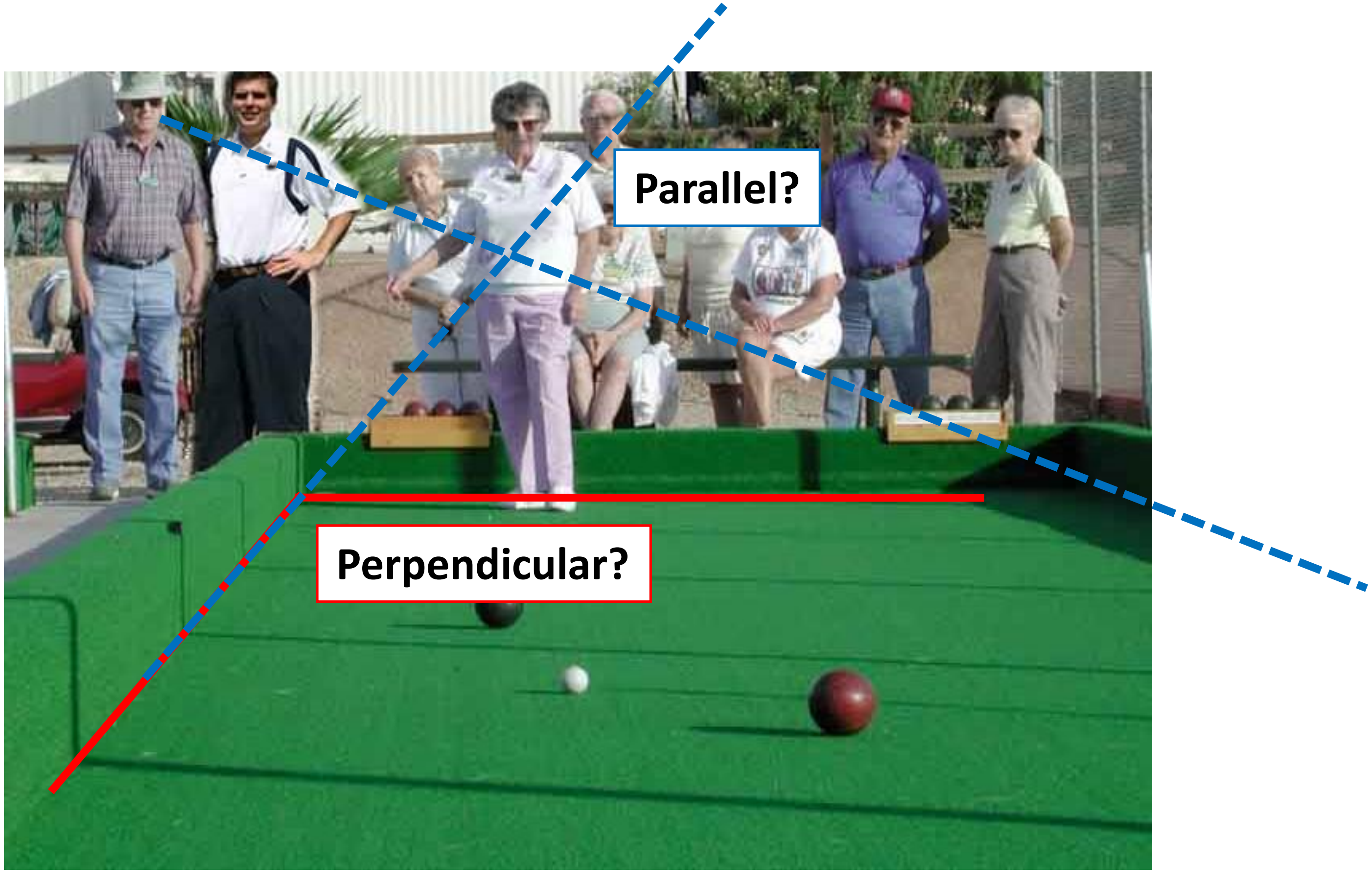
- Straight lines are still straight



Projective Geometry

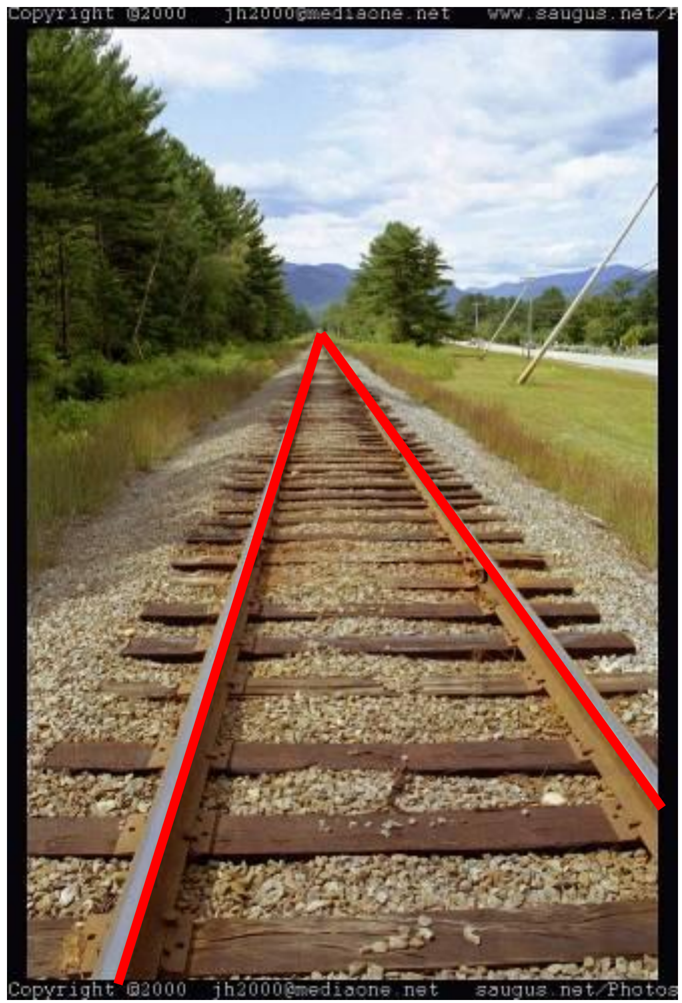
What is lost?

- Length
- Angles



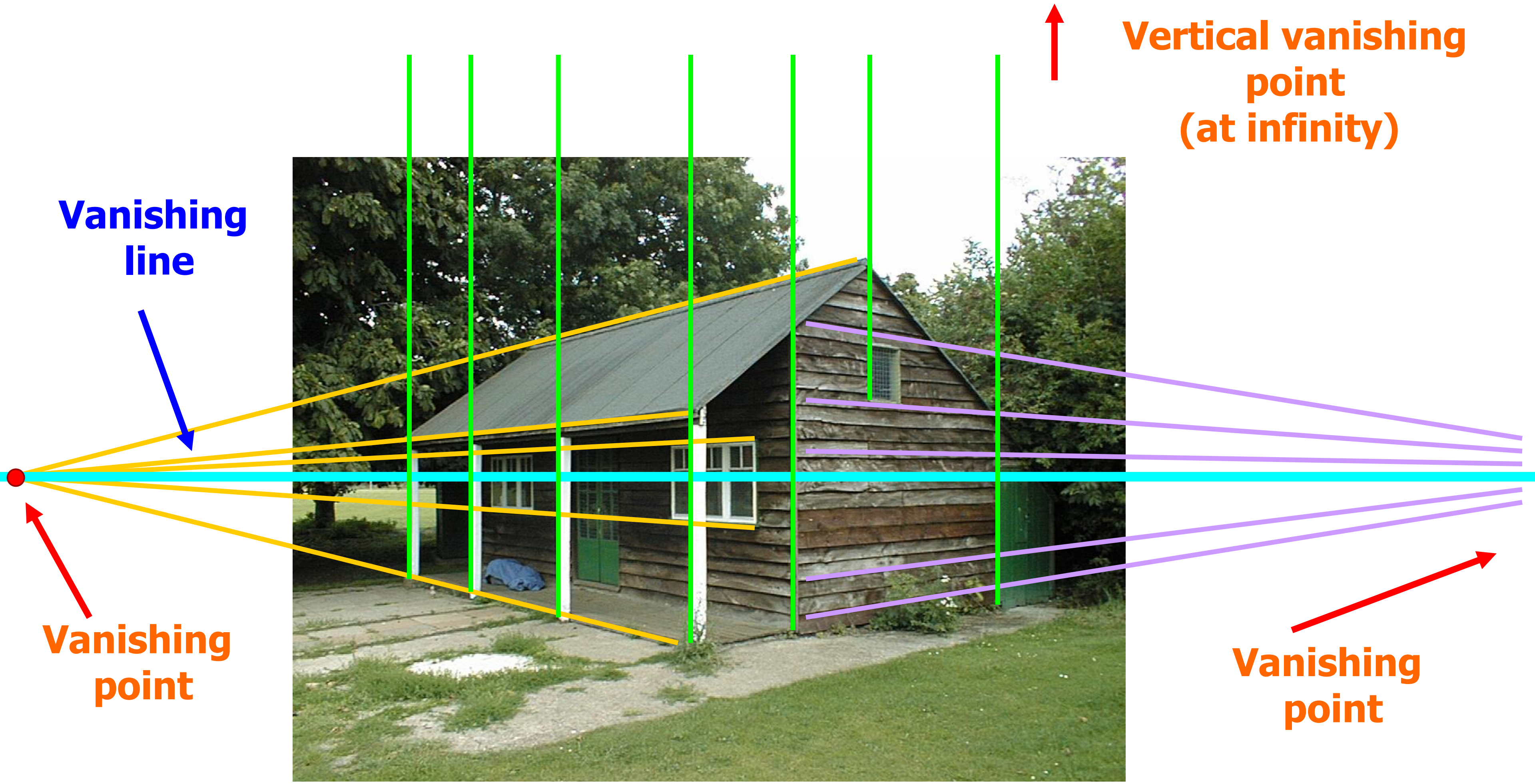
Vanishing points and lines

Parallel lines in the world intersect in the image at a “vanishing point”



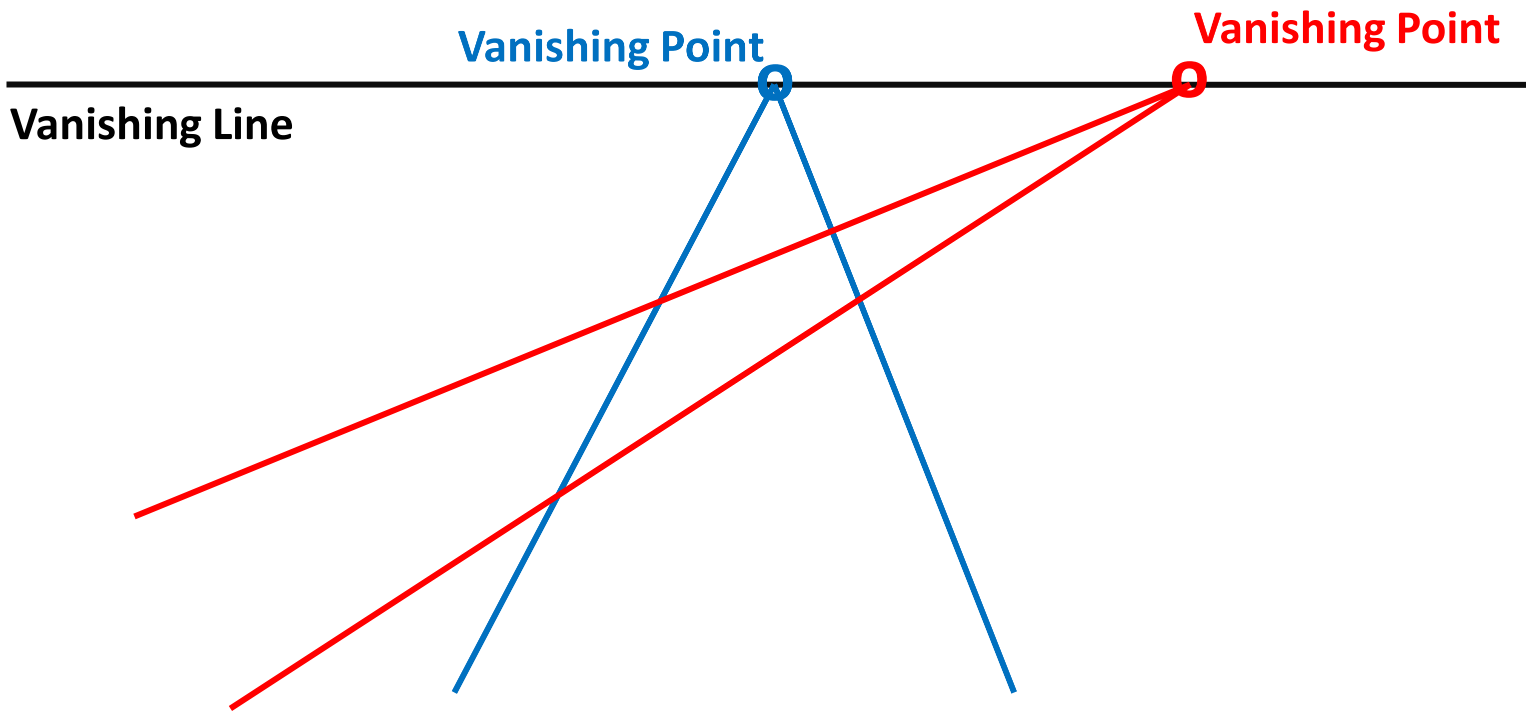
Vanishing points and lines

Parallel lines in the world intersect in the image at a “vanishing point”



Vanishing points and lines

Parallel **planes** in the world intersect in the image at a "vanishing line"



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- Digital camera
- Perspective camera model
- Lens distortion
- Camera calibration
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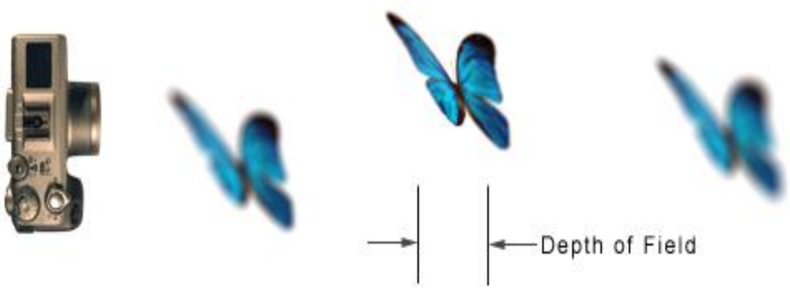
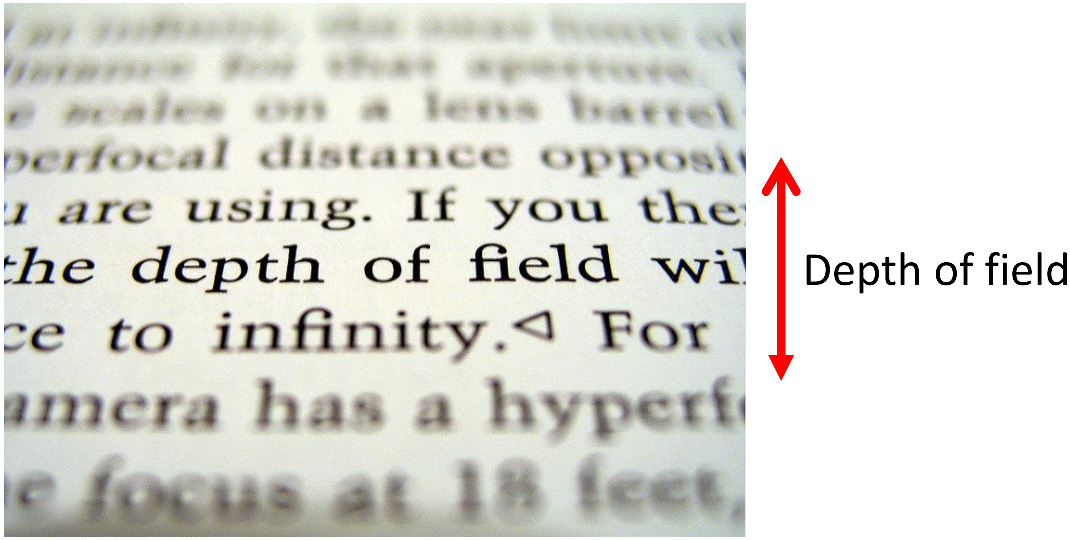
Focus and depth of field

- Depth of field (DOF) is the distance between the nearest and farthest objects in a scene that appear acceptably sharp in an image.

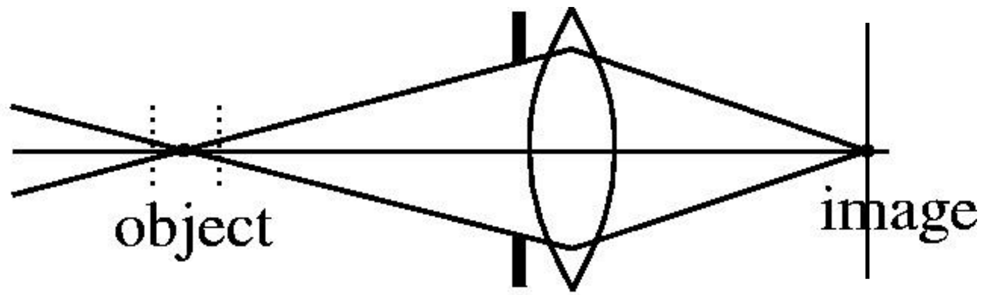
- Although a lens can precisely focus at only one distance at a time, the decrease in sharpness is gradual on each side of the focused distance, so that within the DOF, the unsharpness is imperceptible under normal viewing conditions

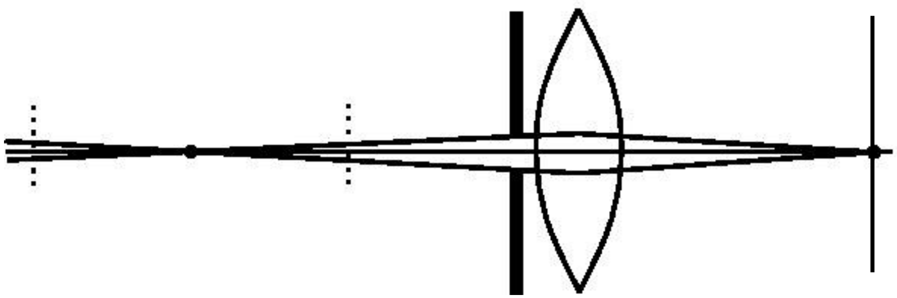




Focus and depth of field

- How does the aperture affect the depth of field?

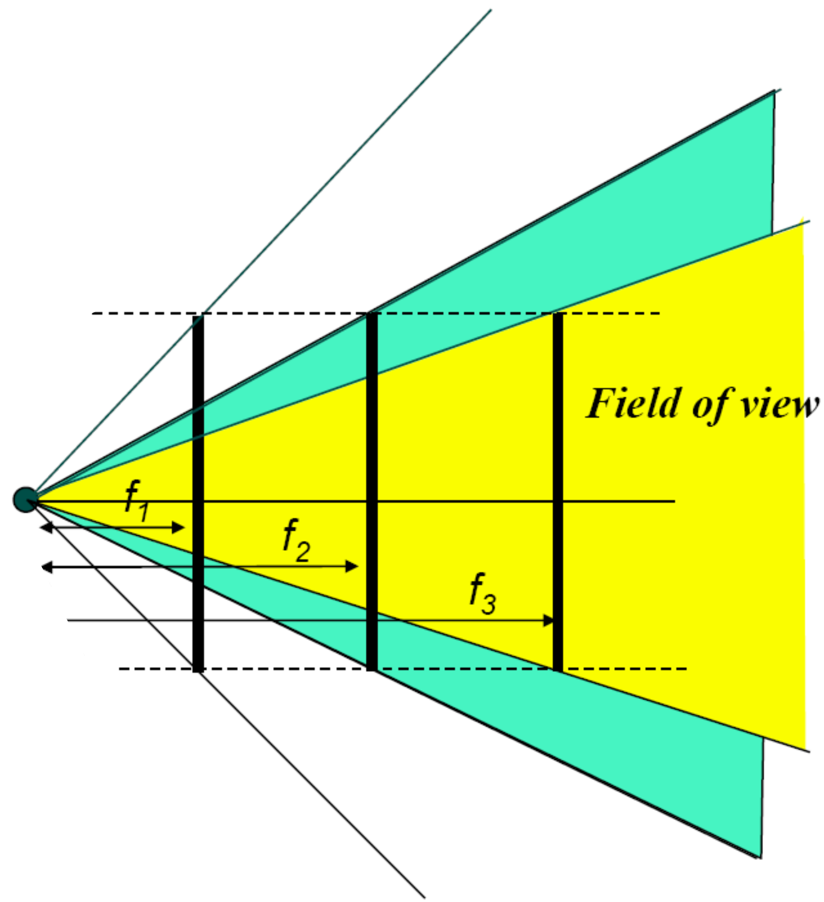




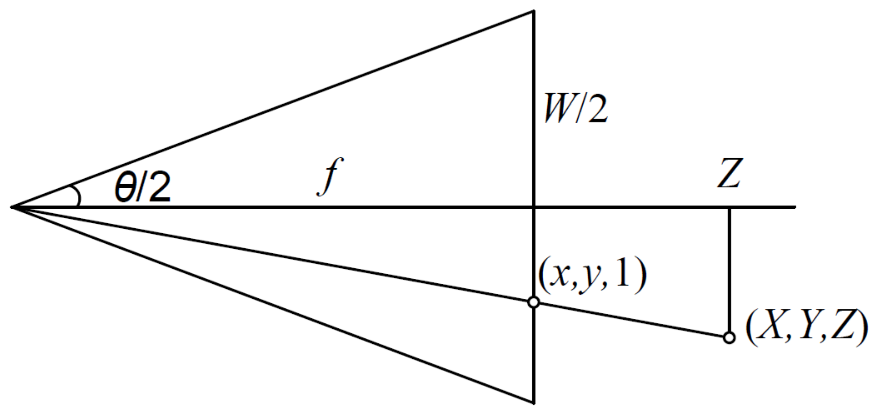
- A smaller aperture increases the range in which the object appears approximately in focus but reduces the amount of light into the camera

Field of view depends on focal length

- As ***f*** gets smaller, image becomes more *wide angle*
  - more world points project onto the finite image plane
- As ***f*** gets larger, image becomes more *narrow angle*
  - smaller part of the world projects onto the finite image plane



Field of view

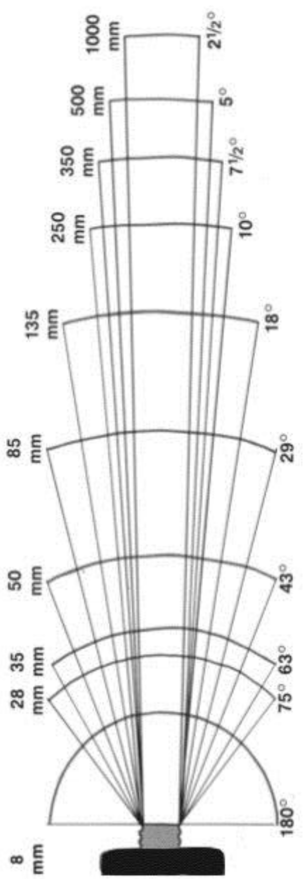


$$\tan\frac{\theta}{2} = \frac{W}{2f} \quad \text{or} \quad f = \frac{W}{2}\left[\tan\frac{\theta}{2}\right]^{-1}$$

Smaller FOV = larger Focal Length

Field of view

Angular measure of portion of 3d space seen by the camera



28 mm lens, 65.5° × 46.4°

50 mm lens, 39.6° × 27.0°

70 mm lens, 28.9° × 19.5°

210 mm lens, 9.8° × 6.5°

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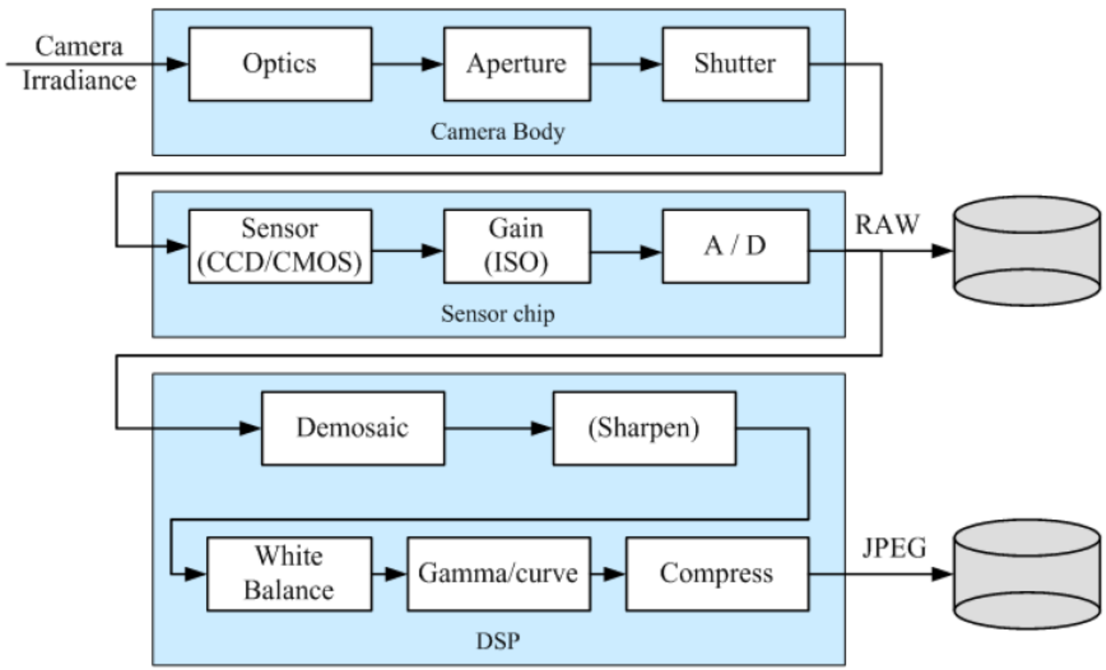
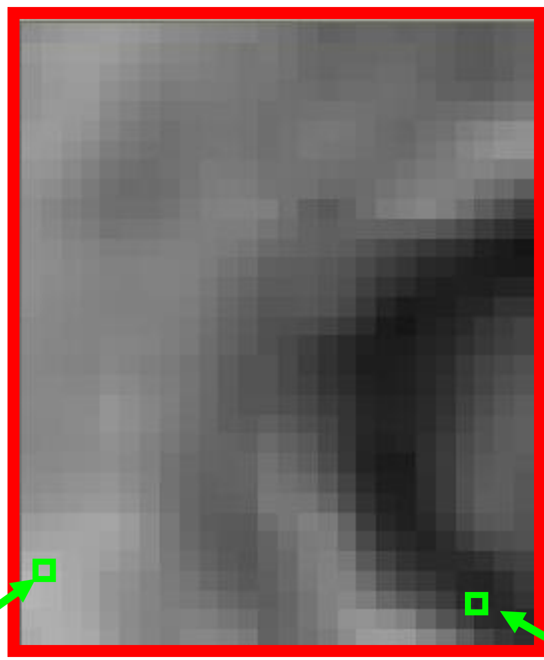
Digital cameras

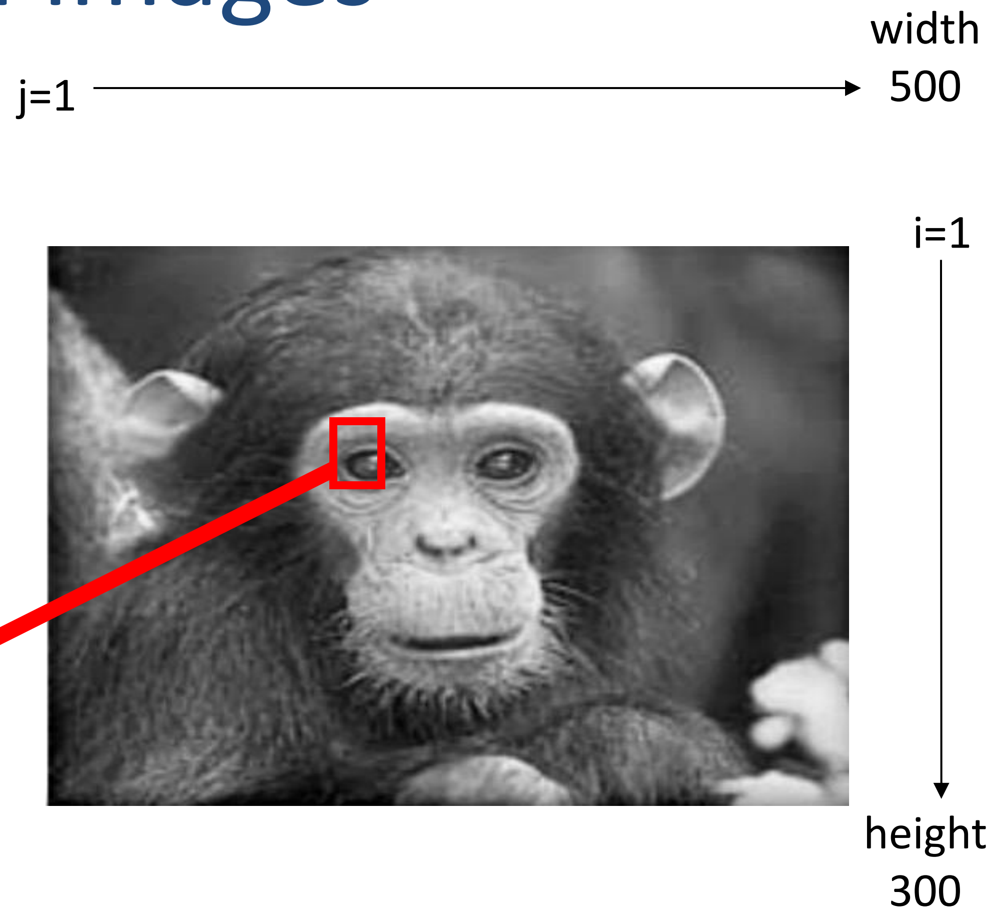
- Film → sensor array
- Often an array of charge coupled devices
- Each CCD/CMOS is light sensitive diode that converts photons (light energy) to electrons



Digital images

Pixel Intensity with 8 bits ranges between [0,255]





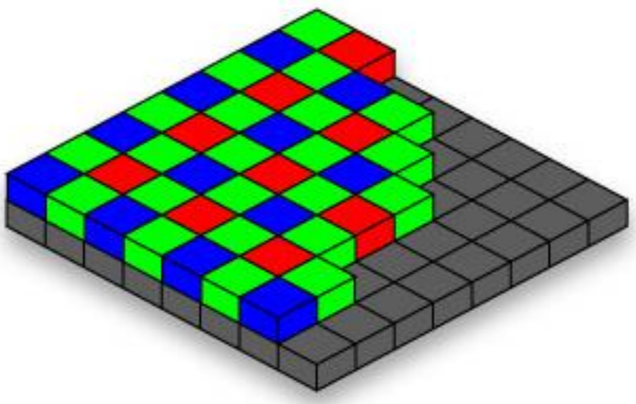
im[176][201] has value 164

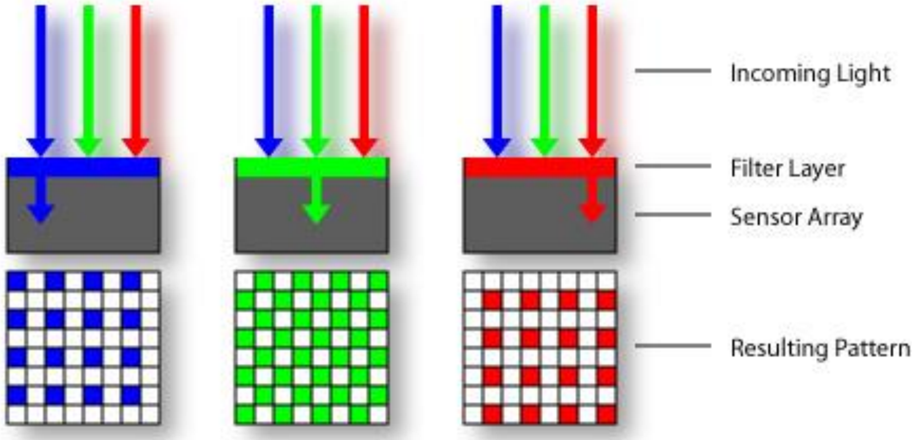
im[194][203] has value 37

NB. Matlab coordinates: [rows, cols]; C/C++ [cols, rows]

Color sensing in digital cameras

Bayer grid

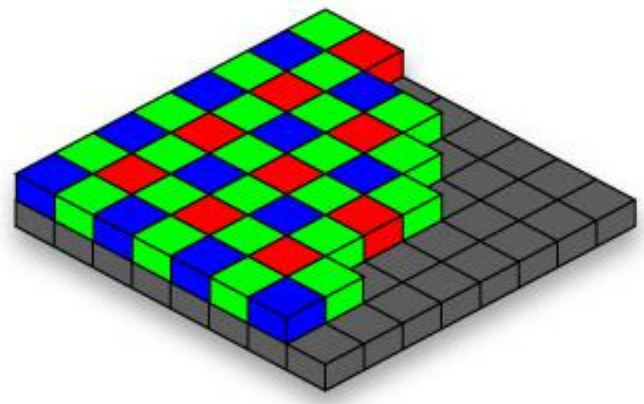




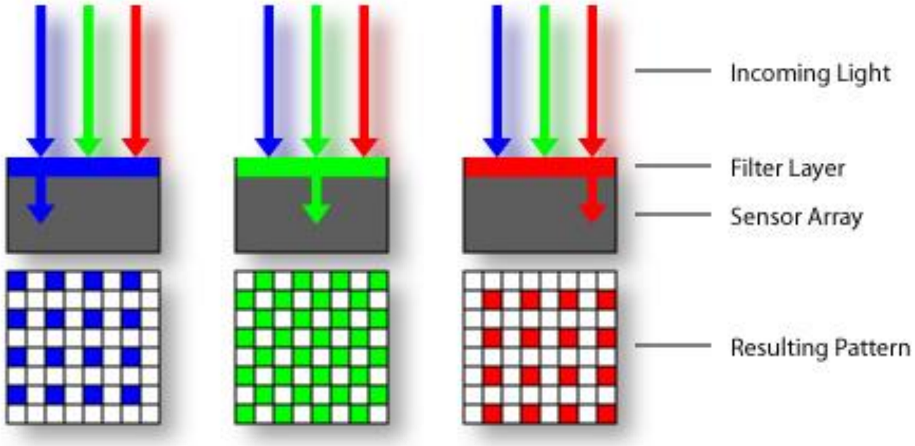
- The Bayer pattern (Bayer 1976) places green filters over half of the sensors (in a checkerboard pattern), and red and blue filters over the remaining ones.
- This is because the luminance signal is mostly determined by green values and the human visual system is much more sensitive to high frequency detail in luminance than in chrominance.

Color sensing in digital cameras

Bayer grid



Estimate missing components from neighboring values (demosaicing)



Foveon chip design (http://www.foveon.com) stacks the red, green, and blue sensors beneath each other but has not gained widespread adoption.

Color images:

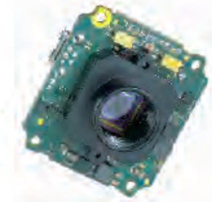
RGB color space

… but there are also many other color spaces… (e.g., YUV)

An example camera datasheet

mvBlueFOX-IGC / -MLC

Technical Details



Sensors

mvBlueFOX-IGC mvBlueFOX-MLC		Resolution (H x V pixels)	Sensor size (optical)	Pixel size (µm)	Frame rate	Sensor technology	Readout type	ADC resolution / output in bits	Sensor
-200w[1,2]	G/C	752 x 480	1/3"	6 x 6	90	CMOS	Global	10 → 10 / 8	Aptina MT9V
-202b	G/C	1280 x 960	1/3"	3.75 x 3.75	24.6	CMOS	Global	10 → 10 / 8	Aptina MT9M
-202d[1]	G/C	1280 x 960	1/3"	3.75 x 3.75	24.6	CMOS	Rolling	10 → 10 / 8	Aptina MT9M
-205[2]	G/C	2592 x 1944	1/2.5"	2.2 x 2.2	5.8	CMOS	Global Reset	10 → 10 / 8	Aptina MT9P

[1] High Dynamic Range (HDR) mode supported

[2] Software trigger supported

Sample: **mvBlueFOX-IGC200wG** means version with housing and 752 x 480 CMOS gray scale sensor.
mvBlueFOX-MLC200wG means single-board version without housing and with 752 x 480 CMOS gray scale sensor.

Hardware Features

Gray scale / Color	Gray scale (G) / Color (C)
Interface	USB 2.0 (up to 480 Mbit/s)
Image formats	Mono8, Mono10, BayerGR8, BayerGR10
Triggers	External hardware based (optional), software based (depending on the sensor) or free run
Size w/o lens (W x H x L) | Weight w/o lens	mvBlueFOX-IGC: 39.8 x 39.8 x 16.5 mm | approx. 10 g mvBlueFOX-MLC: 35 x 33 x 25 mm (without lens mount) | approx. 80 g
Permissible ambient temperature	Operation: 0 .. 45 °C / 30 to 80 % RH Storage: -20 .. 60 °C / 20 to 90 % RH
Lens mounts	Back focus adjustable C/CS-mount lens holder / C-mount, CS-mount or optional S-mount
Digital I/Os	mvBlueFOX-IGC (optional): 1 / 1 opto-isolated mvBlueFOX-MLC: 1 / 1 opto-isolated or 2 / 2 TTL compliant
Conformity	CE, FCC, RoHS
Driver	mvIMPACT Acquire SDK
Operating systems	Windows®, Linux® - 32 bit and 64 bit
Special features	Micro-PLC, automatic gain / exposure control, binning, screw lock connectors

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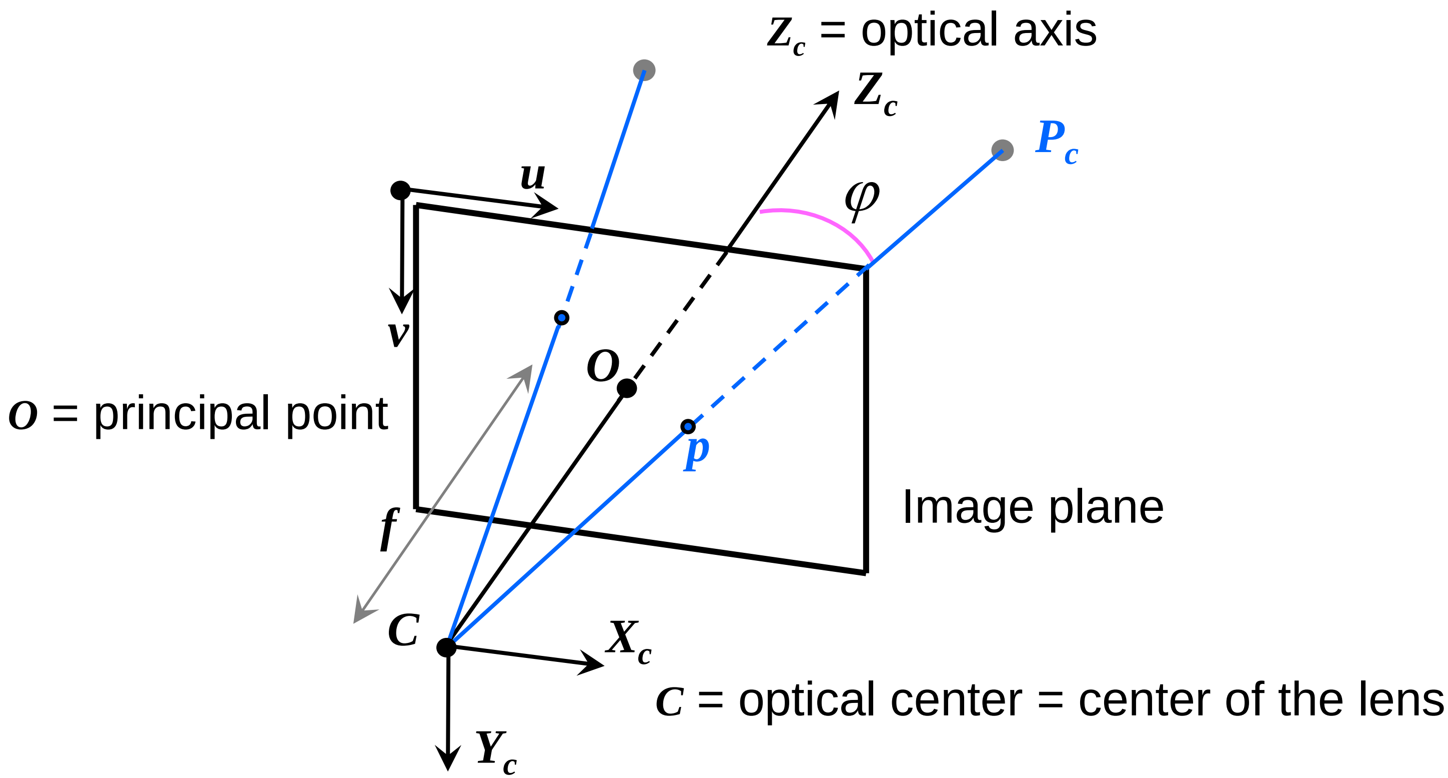
Perspective Camera



- For convenience, the image plane is usually represented in front of C such that the image preserves the same orientation (i.e. not flipped)

- Note: **a camera does not measure distances but angles!**
  ⇨ a camera is a "bearing sensor"

From World to Pixel coordinates

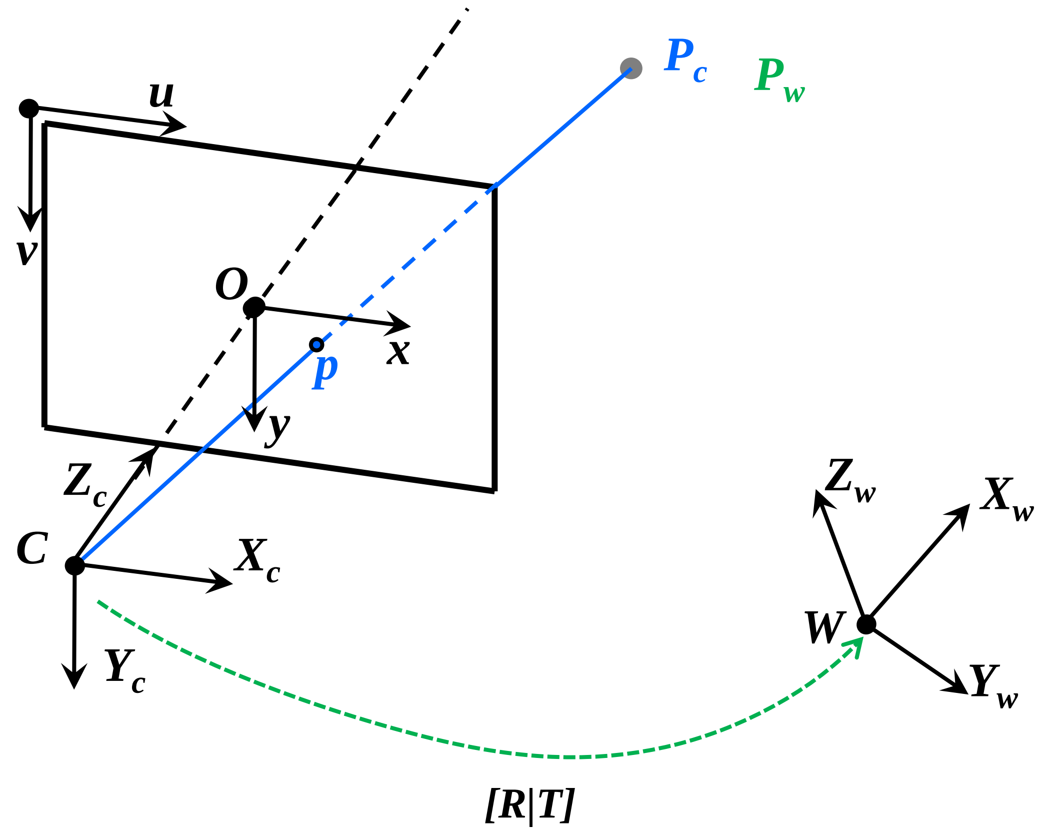
Find pixel coordinates (u,v) of point P_w in the world frame:

0. Convert world point P_w to camera point P_c

Find pixel coordinates (u,v) of point P_c in the camera frame:

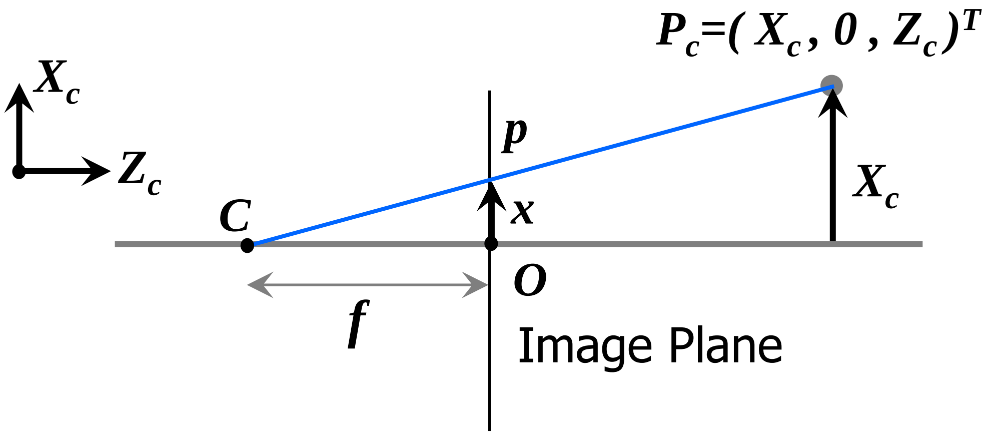
1. Convert P_c to image-plane coordinates (x,y)

2. Convert P_c to (discretised) pixel coordinates (u,v)



Perspective Projection (1)

From the Camera frame to the image plane



- The Camera point $P_c = (X_c, 0, Z_c)^T$ projects to $p = (x, y)$ onto the image plane
- From similar triangles: $$\frac{x}{f} = \frac{X_c}{Z_c} \Rightarrow x = \frac{fX_c}{Z_c}$$
- Similarly, in the general case:

$$\frac{y}{f} = \frac{Y_c}{Z_c} \Rightarrow y = \frac{fY_c}{Z_c}$$

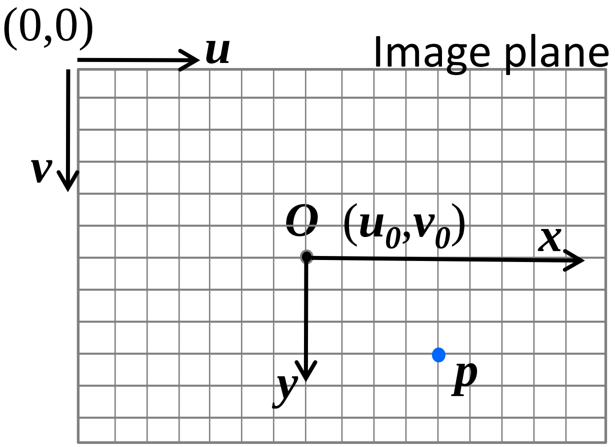
1. Convert P_c to image-plane coordinates (x,y)

2. Convert P_c to (discretised) pixel coordinates (u,v)

Perspective Projection (2)

From the Camera frame to pixel coordinates

- To convert ***p*** from the local image plane coords ***(x,y)*** to the pixel coords ***(u,v)***, we need to account for:
  - the pixel coords of the camera optical center $O = (u_0, v_0)$
  - Scale factors k_u, k_v for the pixel-size in both dimensions



So:

$$u = u_0 + k_u x \Rightarrow u = u_0 + \frac{k_u f X_c}{Z_c}$$

$$v = v_0 + k_v y \Rightarrow v = v_0 + \frac{k_v f Y_c}{Z_c}$$

- Use **Homogeneous Coordinates** for linear mapping from 3D to 2D, by introducing an extra element (scale):

$$p = \begin{pmatrix} u \\ v \end{pmatrix} \quad \Rightarrow \quad \tilde{p} = \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Perspective Projection (3)

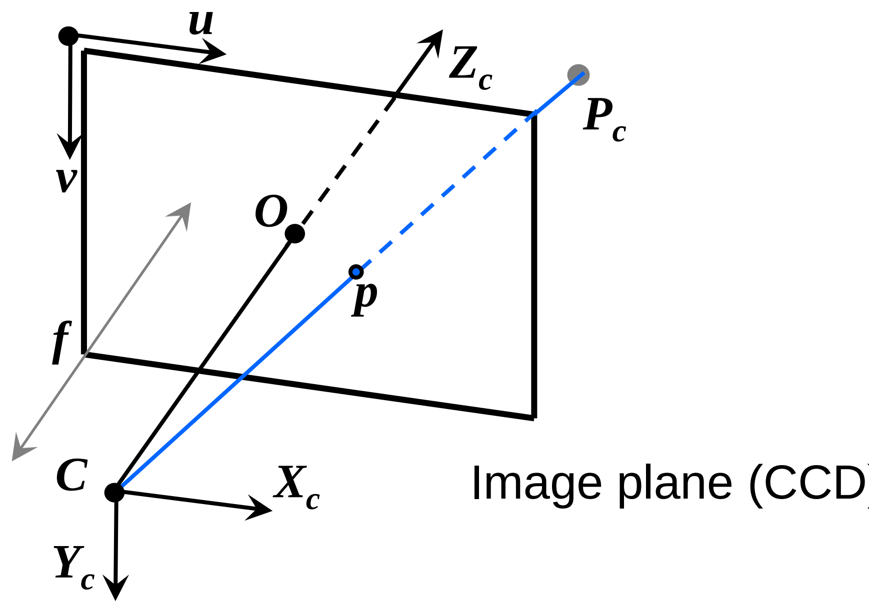
- So:

$$u = u_0 + \frac{k_u f X_c}{Z_c}$$

$$v = v_0 + \frac{k_v f Y_c}{Z_c}$$

Expressed in matrix form and homogenerous coordinates:

$$\begin{bmatrix} \lambda u \\ \lambda v \\ \lambda \end{bmatrix} = \begin{bmatrix} k_u f & 0 & u_0 \\ 0 & k_v f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$



Or alternatively

$$\begin{bmatrix} \lambda u \\ \lambda v \\ \lambda \end{bmatrix} = \begin{bmatrix} \alpha_u & 0 & u_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = K \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

Focal length in pixels

K is called “Calibration matrix” or “Matrix of Intrinsic Parameters”

Sometimes, it is common to assume a skew factor ($K_{12} \neq 0$) to account for possible misalignments between CCD and lens. However, the camera manufacturing process today is so good that we can safely assume $K_{12} = 0$ and $\alpha_u = \alpha_v$.

Exercise 1

- Determine the Intrinsic Parameter Matrix (K) for a digital camera with image size 640×480 pixels and horizontal field of view equal to $90°$
- Assume the principal point in the center of the image and squared pixels
- What is the vertical field of view?

Exercise 1

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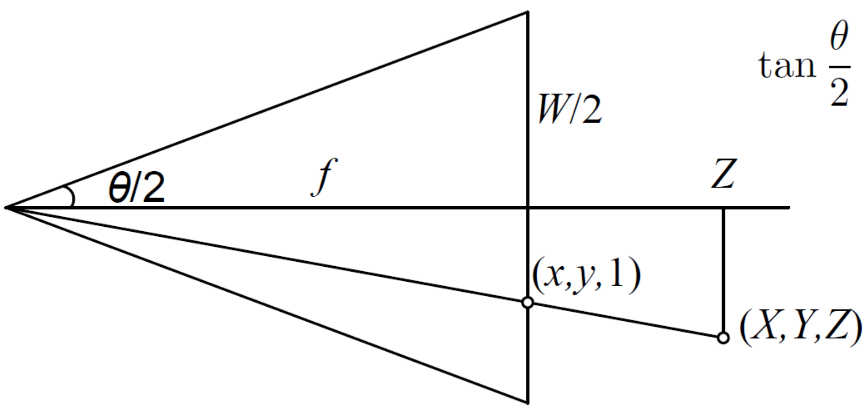


$$\tan\frac{\theta}{2} = \frac{W}{2f} \quad \text{or} \quad f = \frac{W}{2}\left[\tan\frac{\theta}{2}\right]^{-1}$$

$$f = \frac{640}{2\tan\frac{\theta}{2}} = 320\ \boldsymbol{pixels}$$

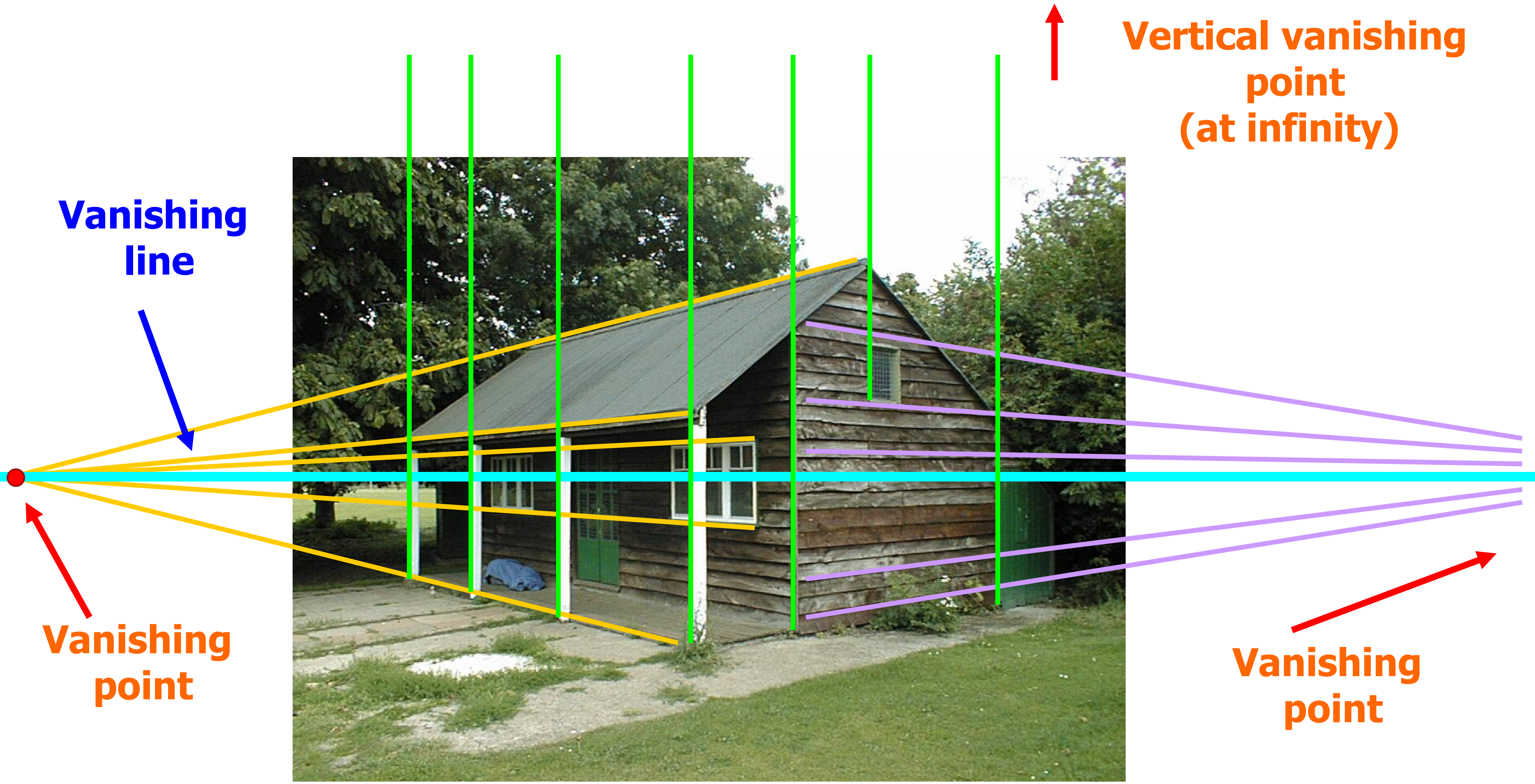
$$K = \begin{bmatrix} 320 & 0 & 320 \\ 0 & 320 & 240 \\ 0 & 0 & 1 \end{bmatrix}$$

- What is the vertical field of view?

$$\theta_V = 2\tan^{-1}\frac{H}{2f} = 2\tan^{-1}\frac{480}{2 \cdot 320} = 73.74°$$

Exercise 2

- Prove that world's parallel lines intersect at a vanishing point in the camera image



Slide from Efros, Photo from Criminisi

Exercise 2

- Prove that world's parallel lines intersect at a vanishing point in the camera image
- Let's consider the perspective projection equation in standard coordinates:

$$u = u_0 + \alpha \frac{X}{Z}$$

$$v = v_0 + \alpha \frac{Y}{Z}$$

- Let's parameterize a 3D line with:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + s \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$

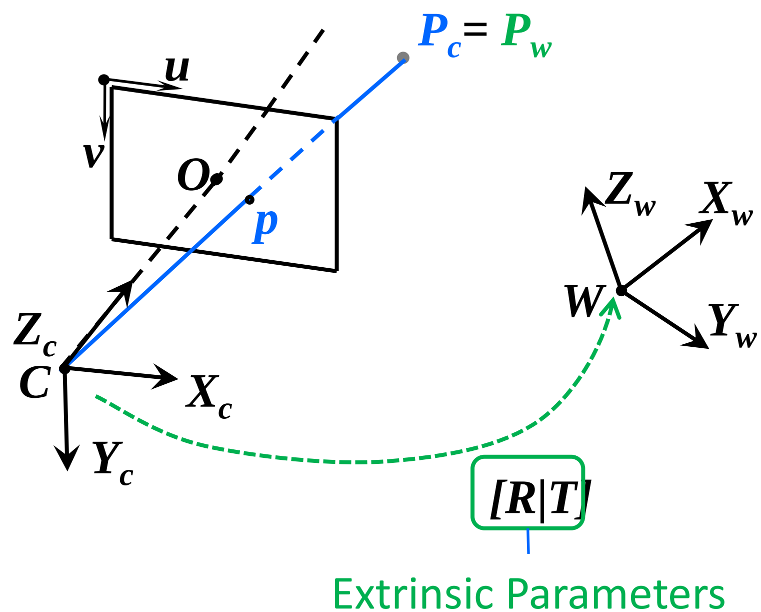
- Now substitute this into the camera perspective projection equation and compute the limit for $s \to \infty$
- What is the intuitive interpretation of this?

Perspective Projection (4)

From the Camera frame to the World frame

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$$

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \left[\begin{array}{ccc|c} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{array}\right] \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} R & T \end{array}\right] \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$



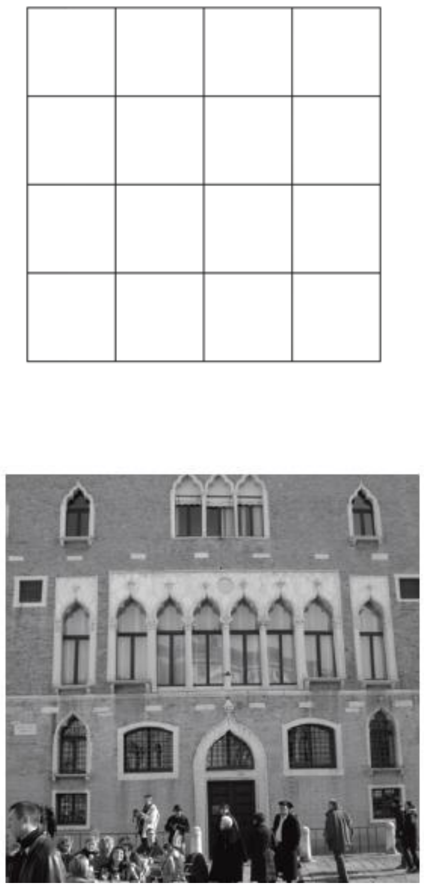
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} \qquad \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \overbrace{\boxed{K\left[R|T\right]}}^{\text{Projection Matrix (M)}} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Perspective Projection Equation

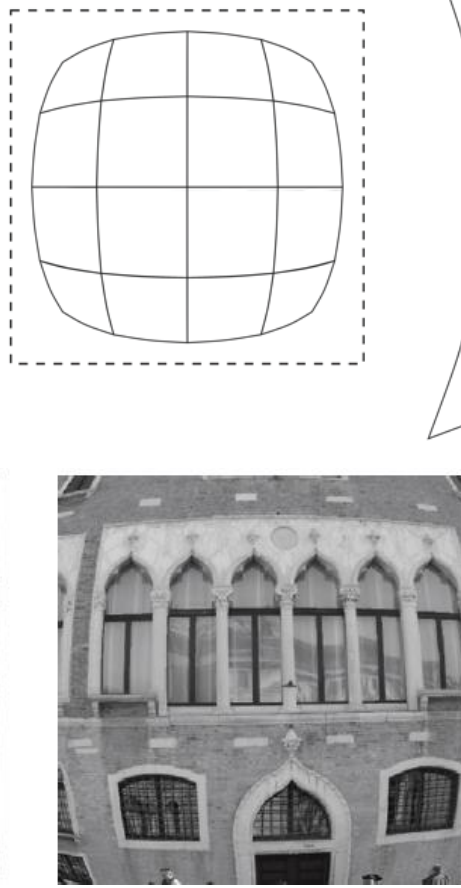
Outline of this lecture

- Perspective camera model
- Lens distortion
- Camera calibration
  - DLT algorithm

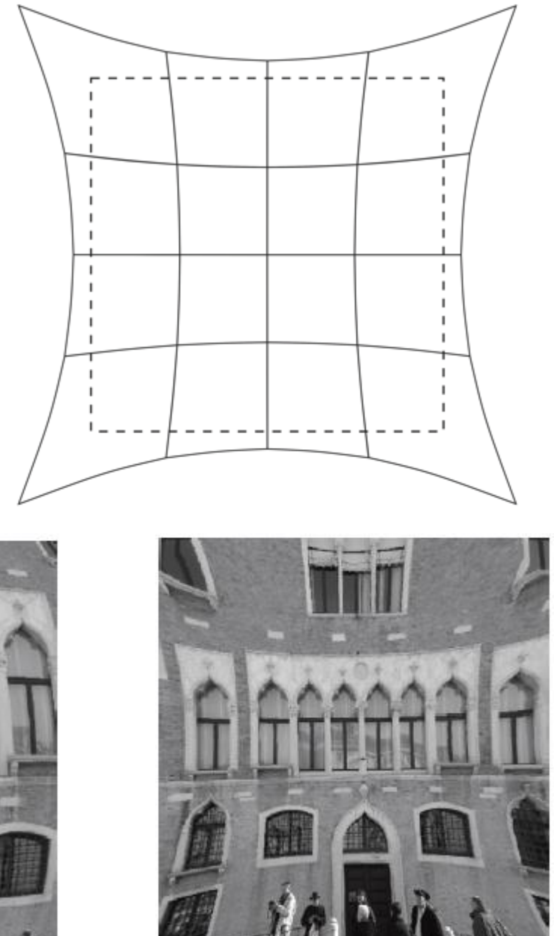
Radial Distortion



No distortion



Barrel distortion



Pincushion

Radial Distortion

- The standard model of radial distortion is a transformation from the ideal coordinates (u, v) (i.e., undistorted) to the real observable coordinates (distorted) (u_d, v_d)

- The amount of distortion of the coordinates of the observed image is a nonlinear function of their radial distance . For most lenses, a simple quadratic model of distortion produces good results

$$\begin{bmatrix} u_d \\ v_d \end{bmatrix} = (1 + k_1 r^2) \begin{bmatrix} u - u_0 \\ v - v_0 \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$$

where $r^2 = (u - u_0)^2 + (v - v_0)^2$

- Depending on the amount of distortion (an thus on the camera field of fiew), higher order terms can be introduced:

$$\begin{bmatrix} u_d \\ v_d \end{bmatrix} = (1 + k_1 r^2 + k_2 r^4 + k_3 r^6) \begin{bmatrix} u - u_0 \\ v - v_0 \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$$

$$r^2 = (u - u_0)^2 + (v - v_0)^2$$

Summary: Perspective projection equations

- To recap, a 3D world point $P = (X_w, Y_w, Z_w)$ projects into the image point $p = (u, v)$

$$\tilde{p} = \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K\left[R \middle| T\right] \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \quad \text{where} \quad K = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \alpha & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

and λ is the depth ($\lambda = Z_C$) of the scene point

- If we want to take into account the radial distortion, then the distorted coordinates (u_d, v_d) (in pixels) can be obtained as

$$\begin{bmatrix} u_d \\ v_d \end{bmatrix} = (1 + k_1 r^2) \begin{bmatrix} u - u_0 \\ v - v_0 \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$$

where

$$r^2 = (u - u_0)^2 + (v - v_0)^2$$

Summary (things to remember)

- Perspective Projection Equation
- Intrinsic and extrinsic parameters (K, R, t)
- Homogeneous coordinates
- Normalized image coordinates
- Image formation equations (including radial distortion)