ROBOTICS & PERCEPTION GROUP

Universität Zürich[UZH]

Lecture 06
Feature Extraction 2

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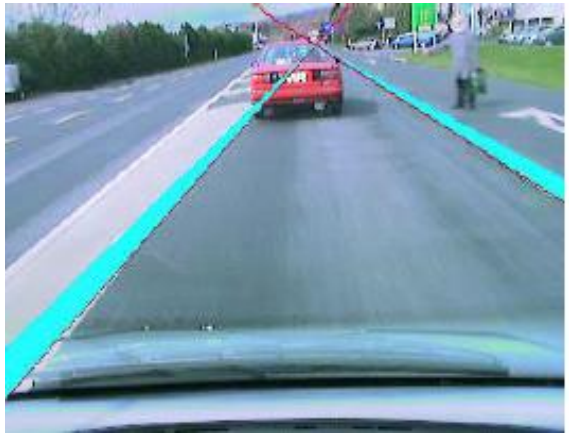
Outline

- Filters for feature extraction
- Point-feature extraction
- Line extraction algorithms

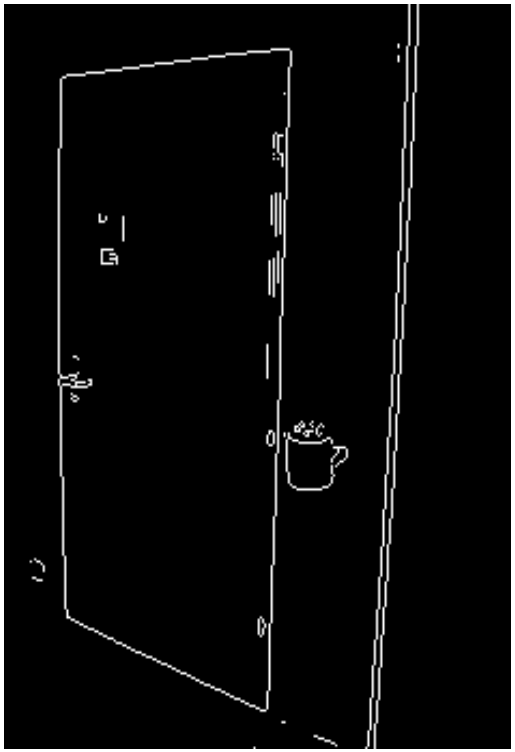
Line extraction

- Supose that you have been commissioned to implement a lane detection for a car driving-assistance system
- How would you proceed?



Line extraction

- How do we extract lines from edges?

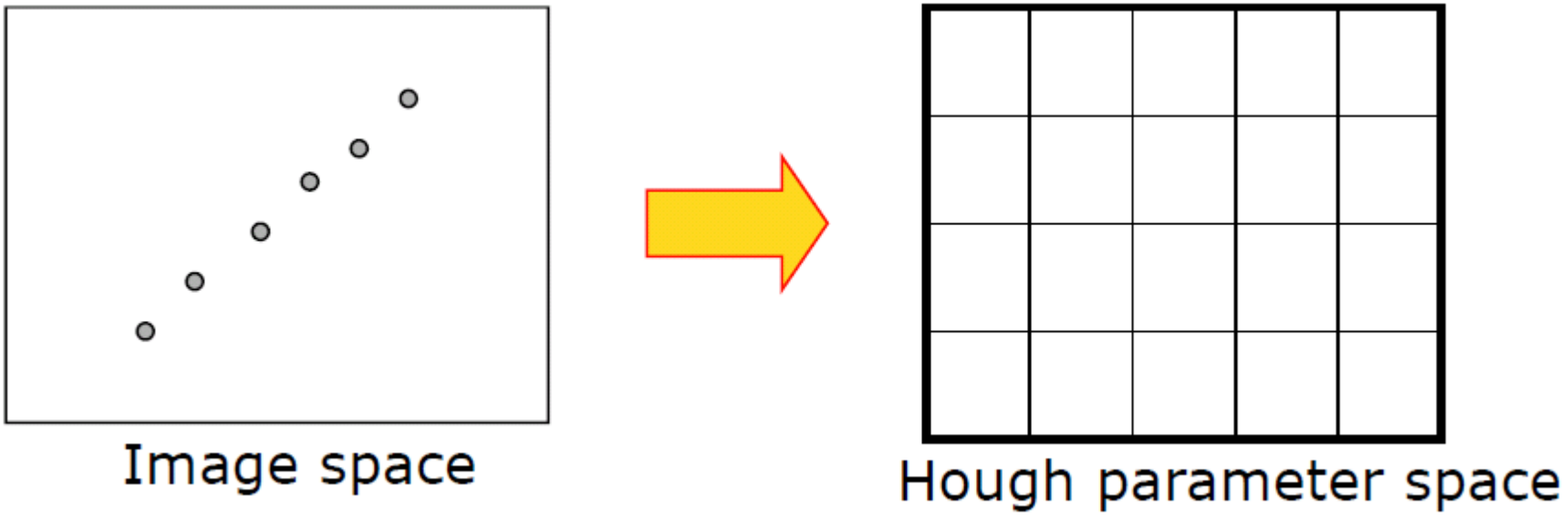


Two popular line extraction algorithms

- Hough transform (used also to detect circles, ellipses, and any sort of shape)
- RANSAC (Random Sample Consensus)

Hough-Transform

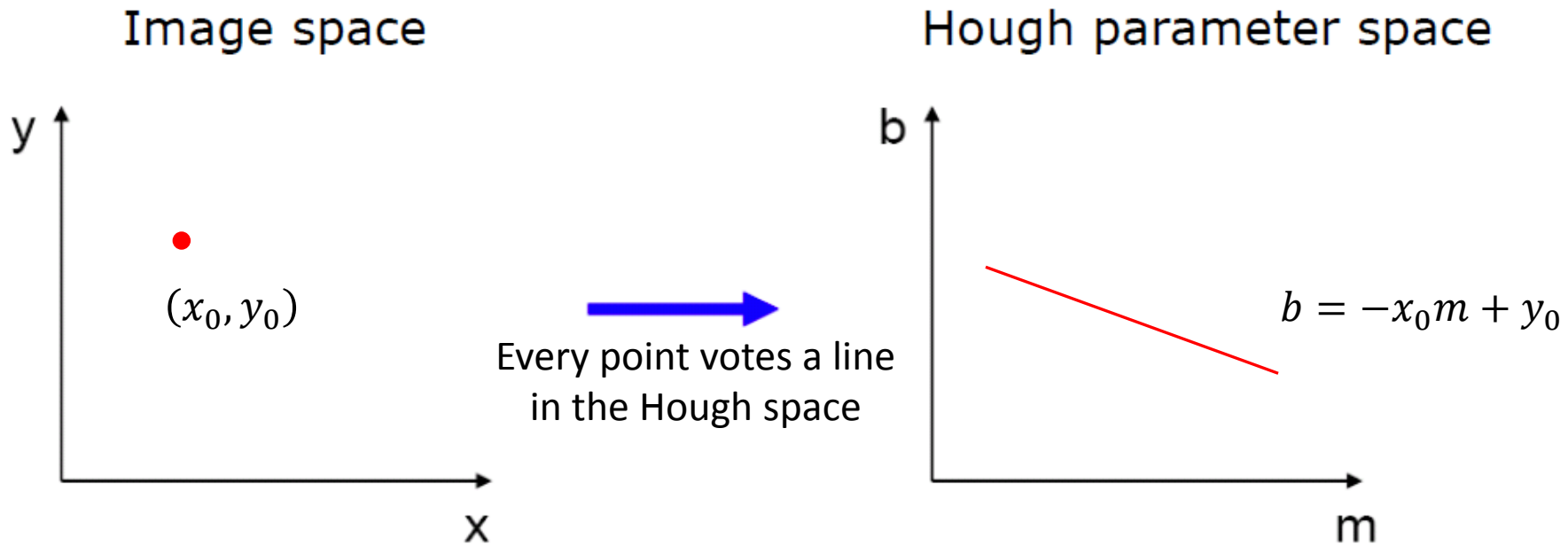
- Finds lines from a binary edge image using a voting procedure
- The voting space (or accumulator) is called **Hough space**



1. P. Hough, Machine Analysis of Bubble Chamber Pictures, Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959
2. J. Richard, O. Duda, P.E. Hart (April 1971). "Use of the Hough Transformation to Detect Lines and Curves in Pictures". Artificial Intelligence Center (SRI International)

Hough-Transform

- Let (x_0, y_0) be an image point
- We can represent all the lines passing through it by $y_0 = mx_0 + b$
- The Hough transform works by parameterizing this expression in terms of m and b: $b = -x_0 m + y_0$
- This is represented by a line in the Hough space



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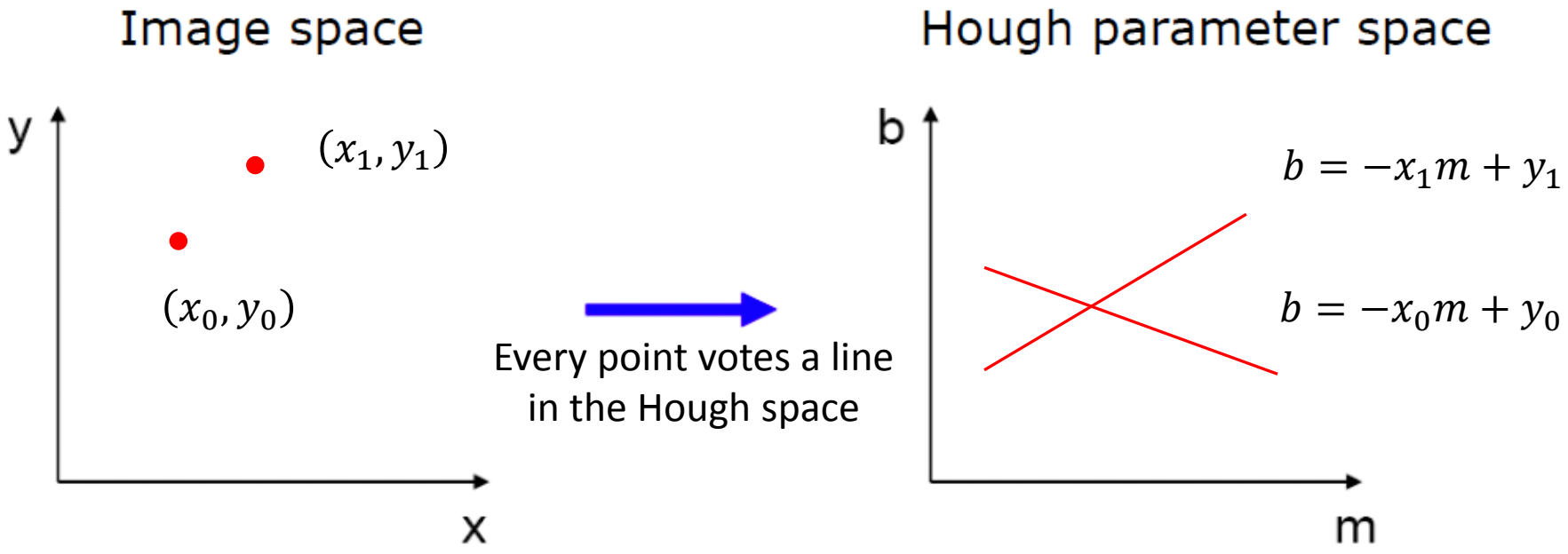
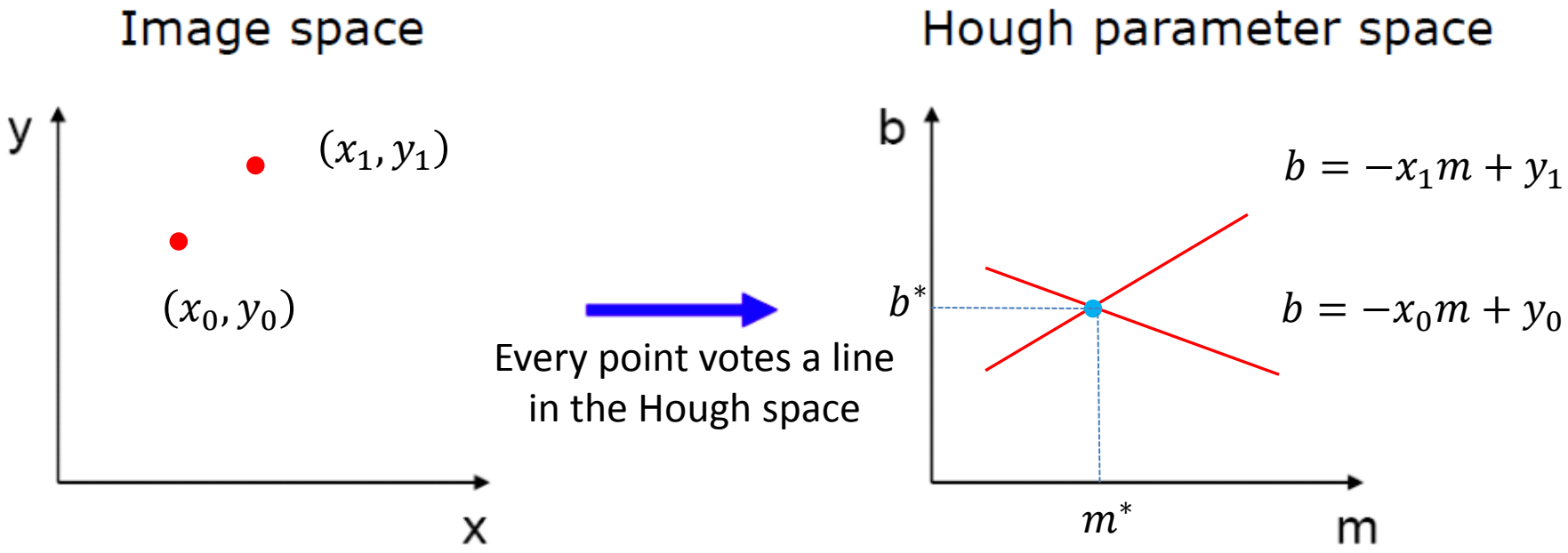
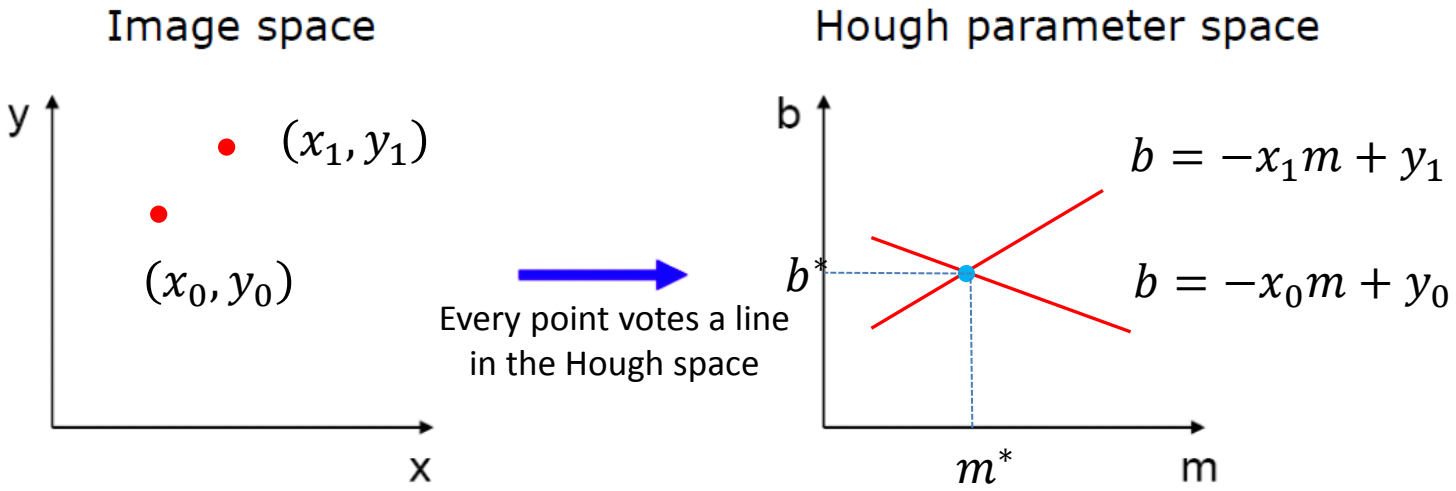


Hough-Transform

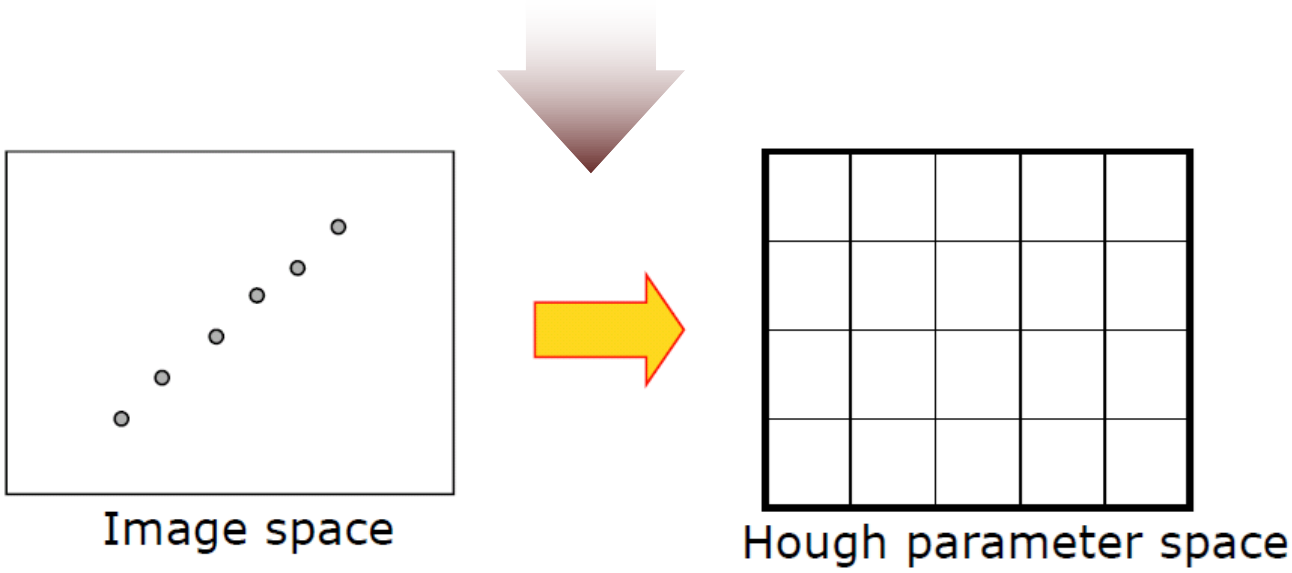
- How do we compute the line (b^*, m^*) that contains both (x_0, y_0) and (x_1, y_1)?
  - It is the intersection of the lines $b = -x_0 m + y_0$ and $b = -x_1 m + y_1$



Hough-Transform



- Each point in image space, votes for line-parameters in Hough parameter space



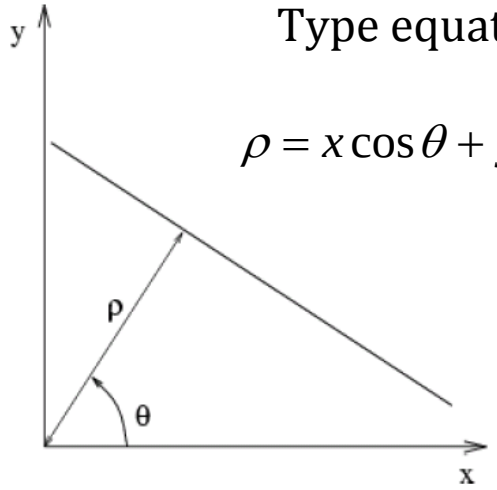
Hough-Transform

- Problems with the (m, b) space:
  - Unbounded parameter domain
    - m, b can assume any value in $[-\infty, +\infty]$

Hough-Transform

- Problems with the (m, b) space:
  - Unbounded parameter domain
    - m, b can assume any value in $[-\infty, +\infty]$

- Alternative line representation: polar representation -> Bounded parameter domain! Can you tell what the bounds are?



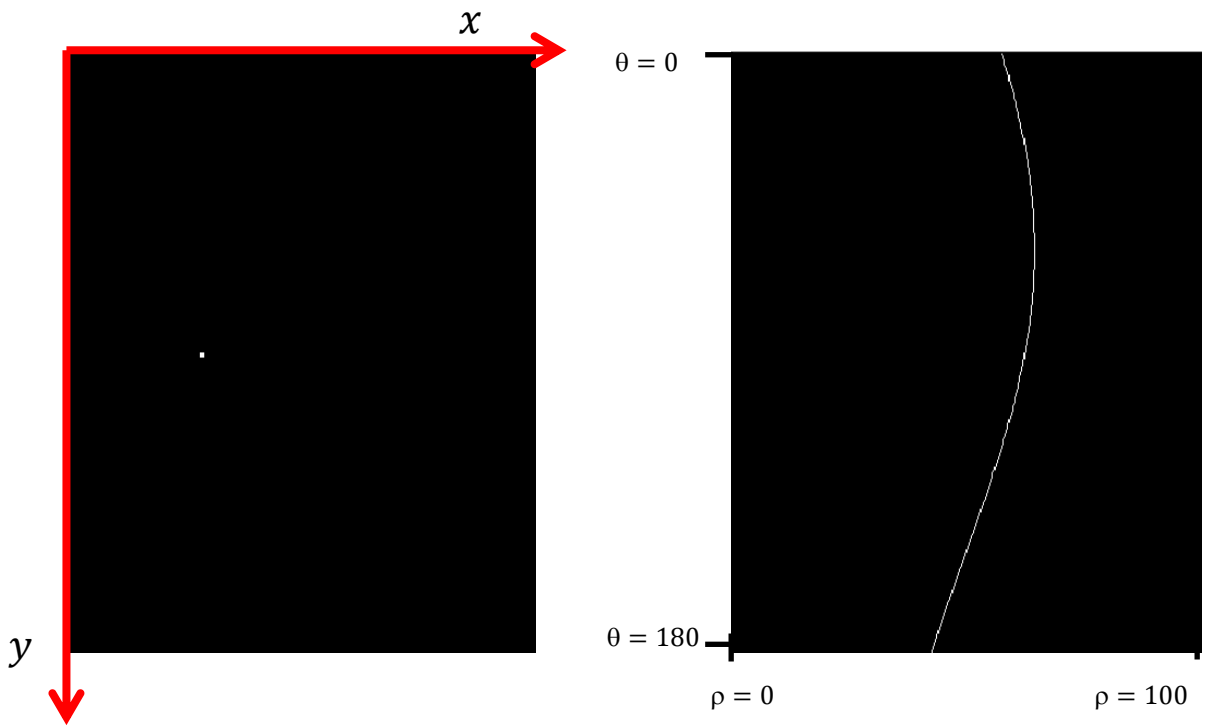
Type equation here.

$$\rho = x\cos\theta + y\sin\theta$$

Hough-Transform

- Each point in image space will map to a sinusoid in the (ρ, θ) parameter space

$$\rho = x\cos\theta + y\sin\theta$$



Hough-Transform

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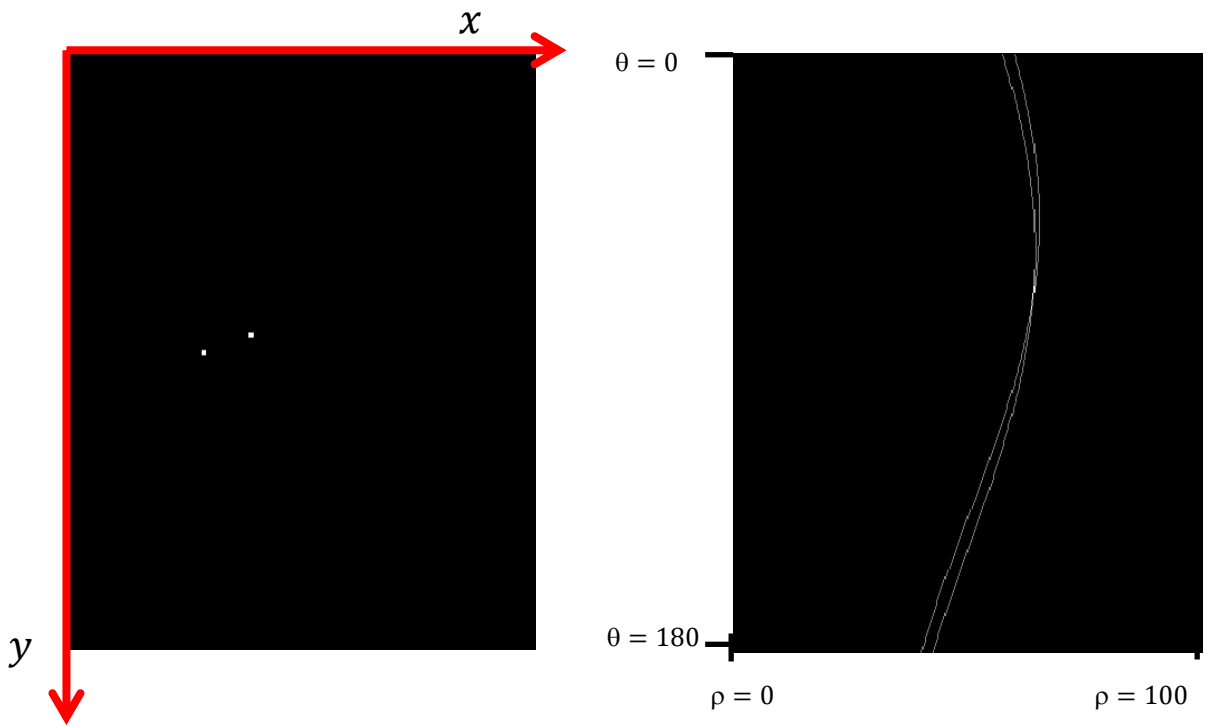


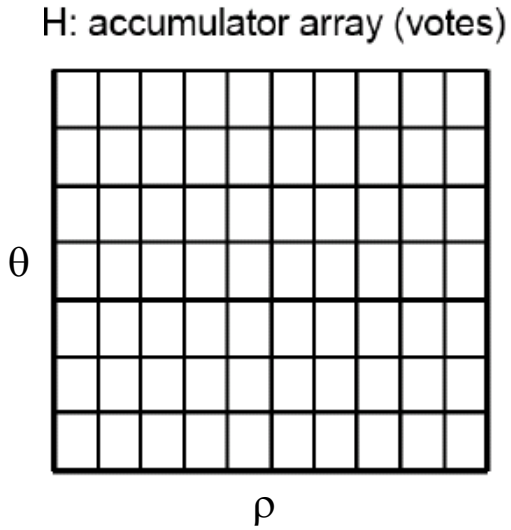
Hough-Transform Algorithm

1. Initialize accumulator H to all zeros

2. for each edge point (x,y) in the image

- **for** all θ in [0,180]
  - Compute $\rho = x \cos\theta + y \sin\theta$
  - $H(\theta, \rho) = H(\theta, \rho) + 1$
- **end**

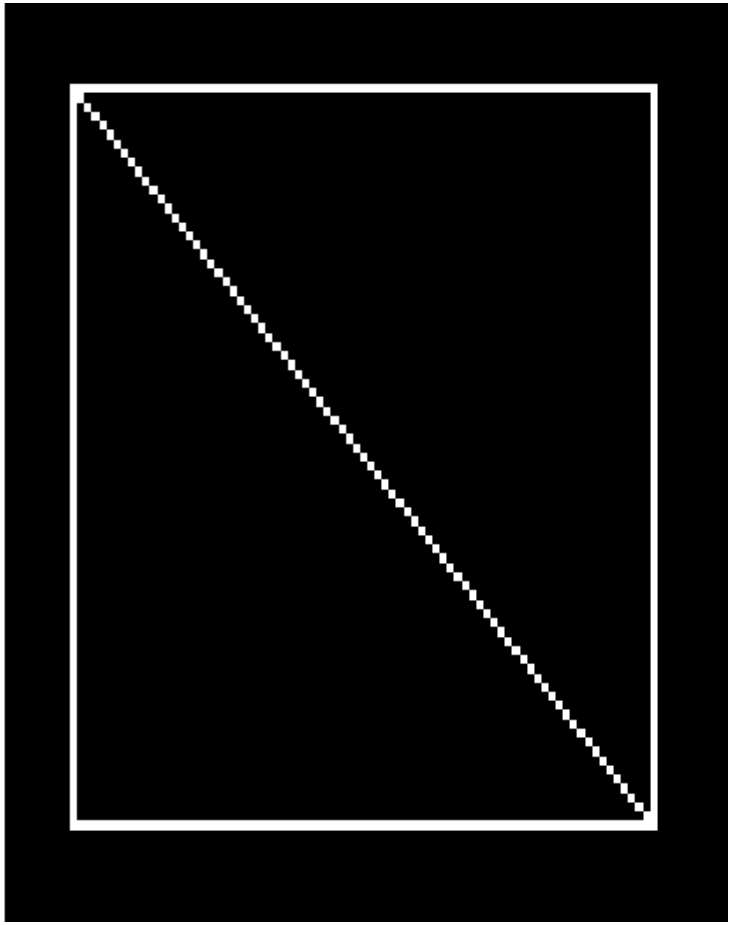
end

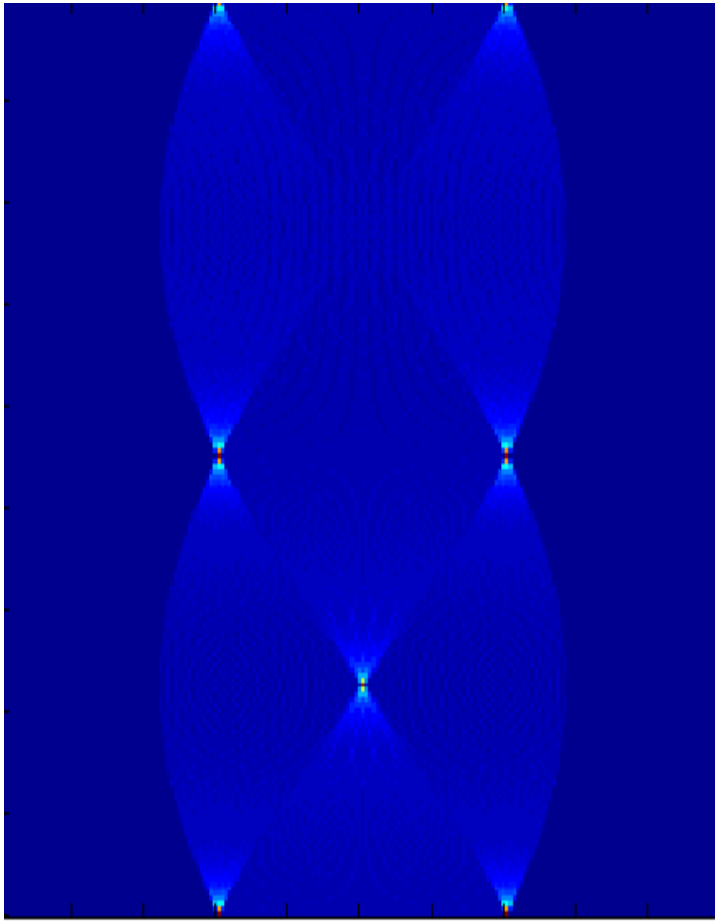


3. Find the values of (θ, ρ) where $H(\theta, \rho)$ is a local maximum

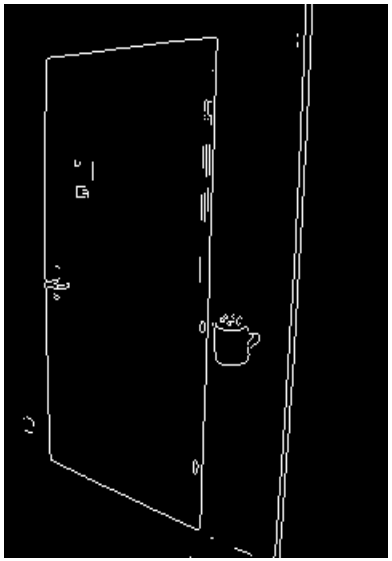
4. The detected line in the image is given by: $\rho = x \cos\theta + y \sin\theta$

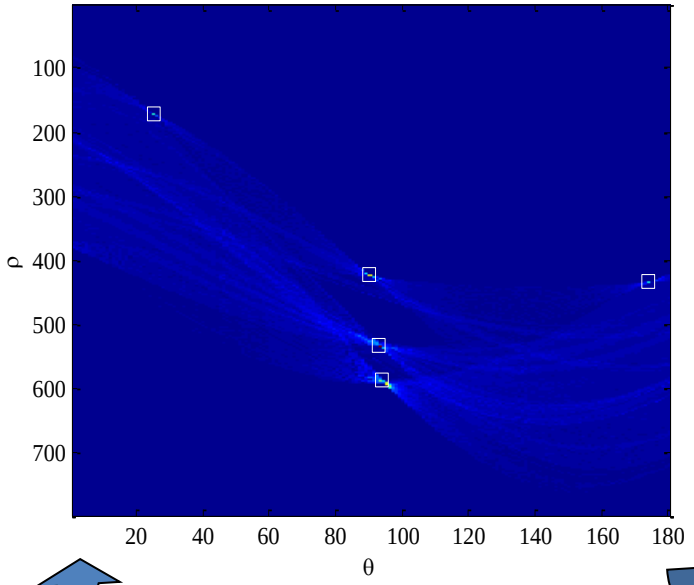
Examples

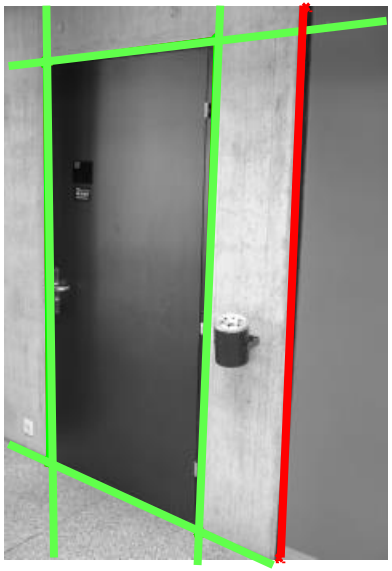




Examples

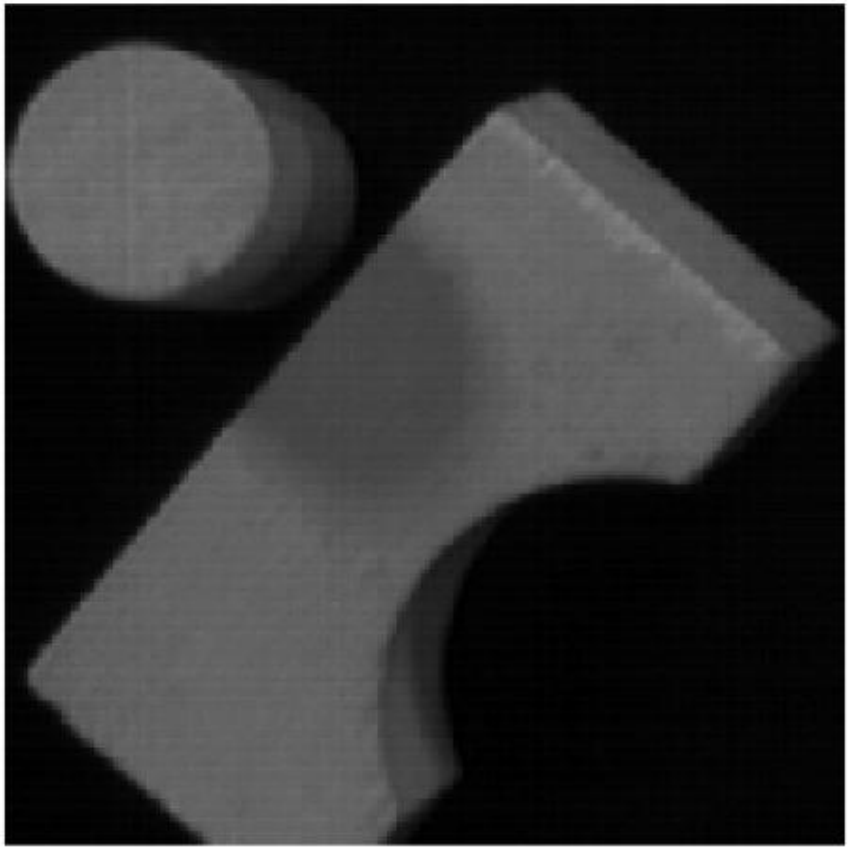


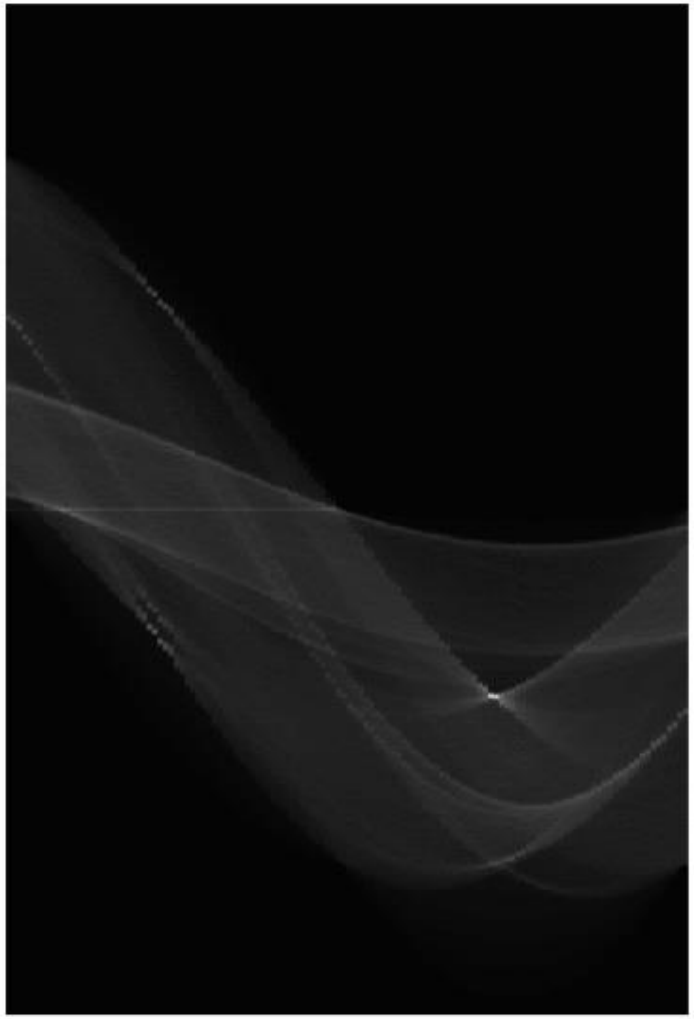




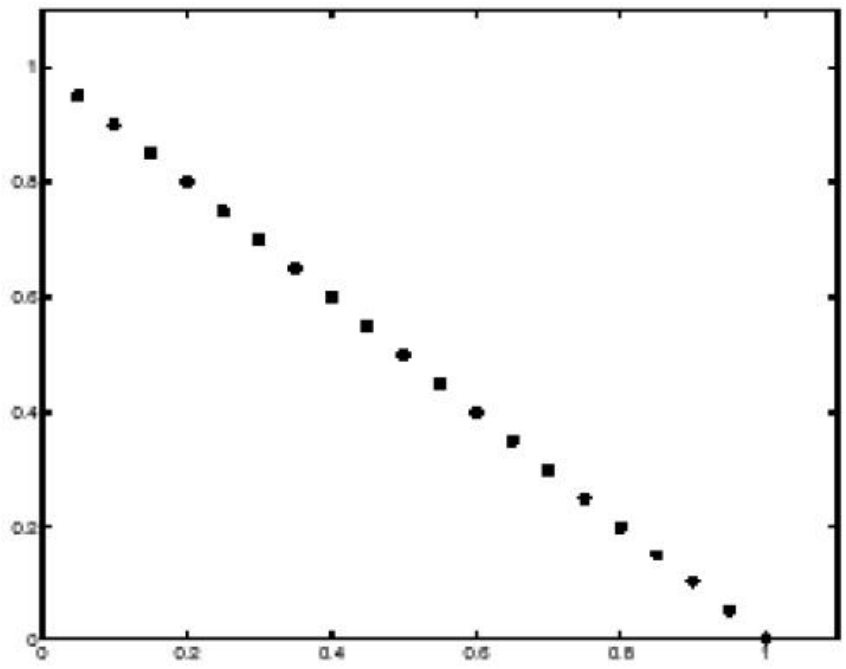
Hough Transform

Examples

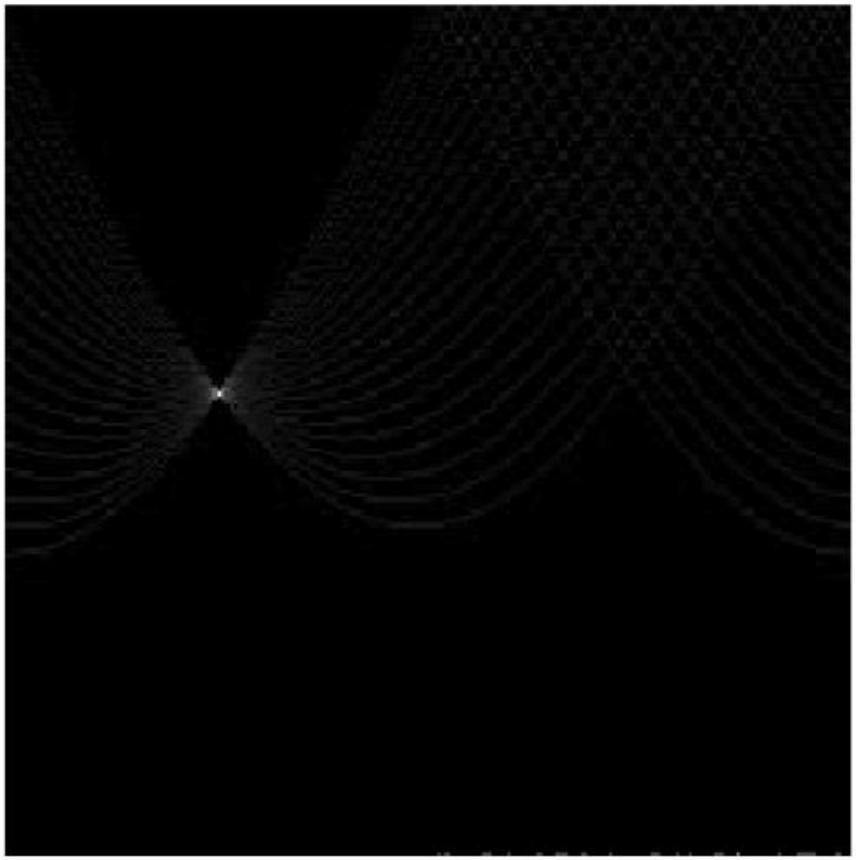




Examples



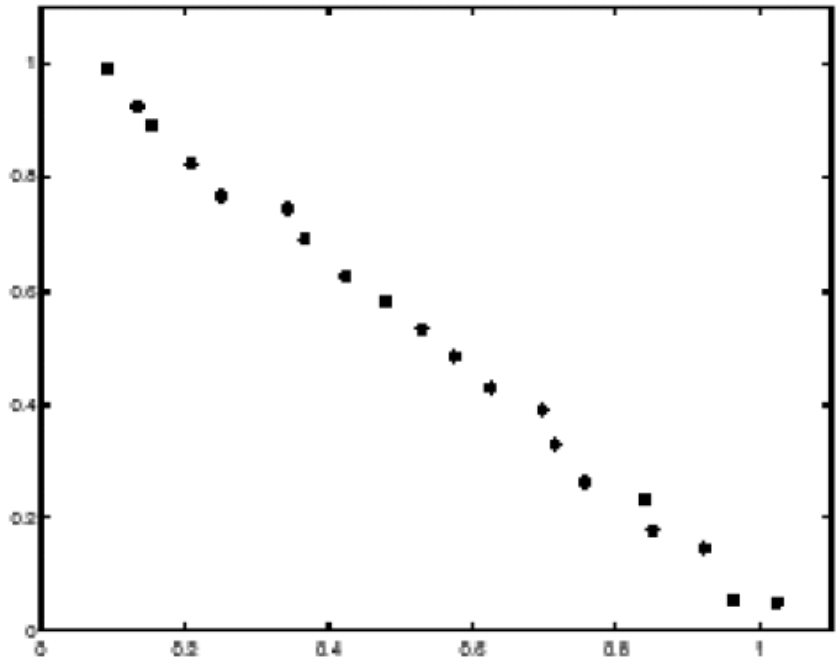
features



votes

Problems: Noise

- Effects of noise: peaks get fuzzy and hard to locate
- How to overcome this?
  - Increase bin size (decrease resolution of the Hough space); however, this reduces the accuracy of the line parameters
  - Smooth the Hough image with a box or Gaussian filter; why?



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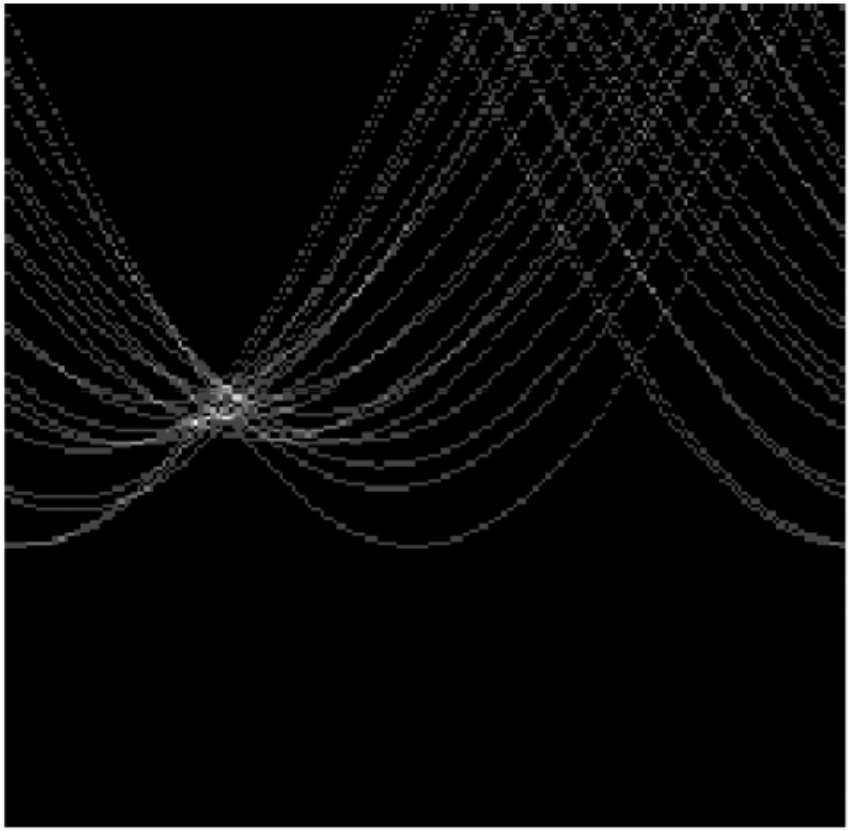

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Problems: Outliers

- How would Hough work with this type of data?

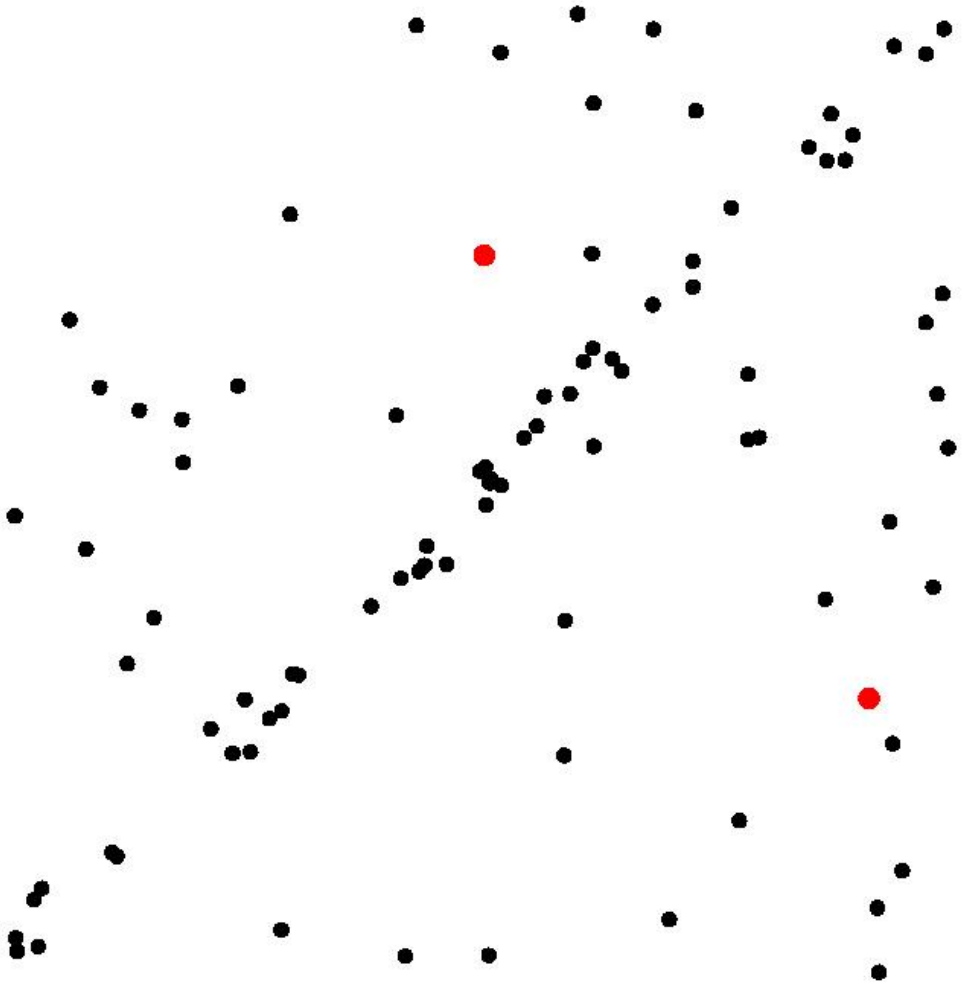
RANSAC (RAndom SAmple Consensus)

- RANSAC has become the standard method for model fitting in the presence of outliers (very noisy points or wrong data)
- It can be applied to line fitting but also to thousands of different problems where the goal is to estimate the parameters of a model from the data (e.g., camera calibration, structure from motion, DLT, homography, etc.)
- We will see many examples of RANSAC applications in the next lecture
- Let's now focus on line extraction

M. A.Fischler and R. C.Bolles. Random sample consensus: A paradigm for model fitting with apphcatlons to image analysis and automated cartography. Graphics and Image Processing, 24(6):381–395, 1981.

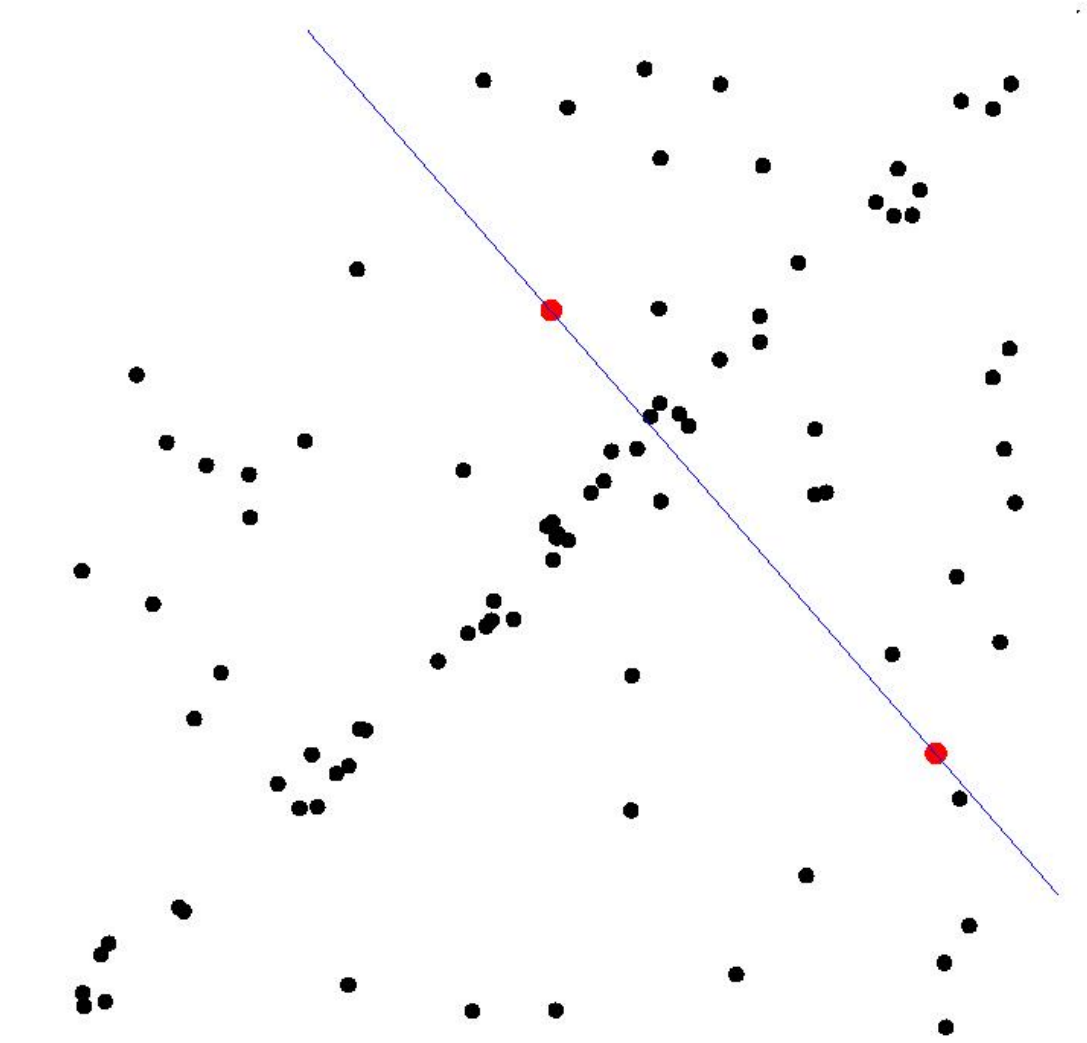
RANSAC

RANSAC



- **Select sample of 2 points at random**

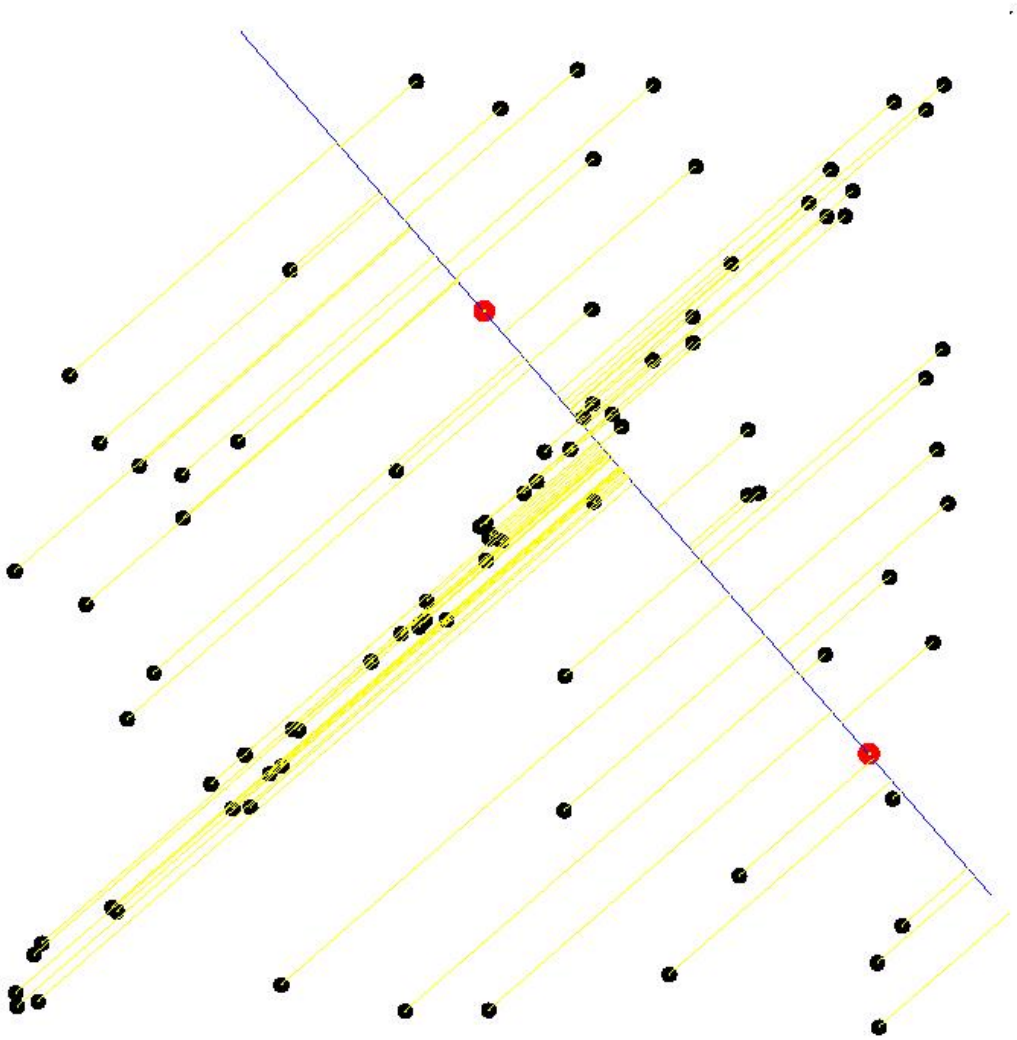
RANSAC



- Select sample of 2 points at random

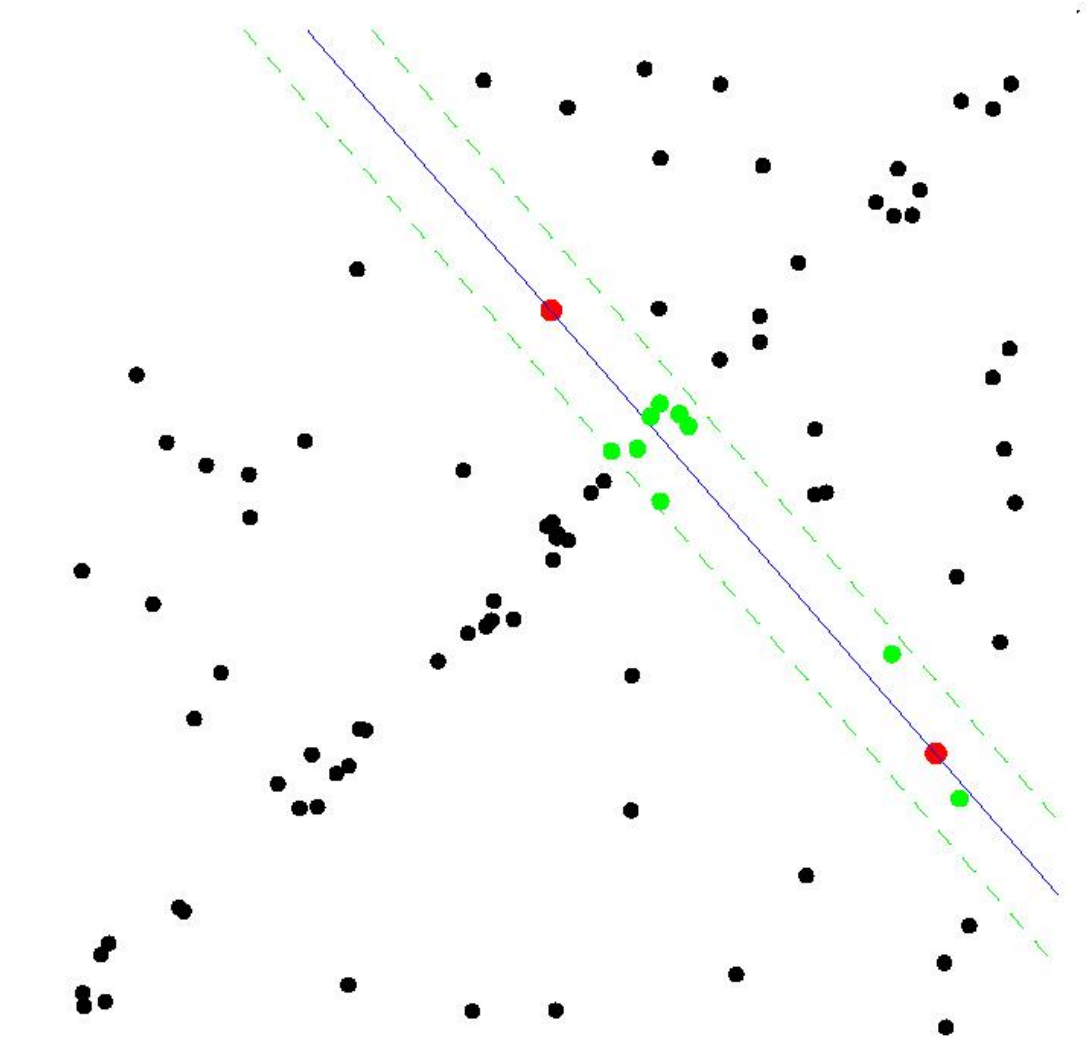
- **Calculate model parameters that fit the data in the sample**

RANSAC



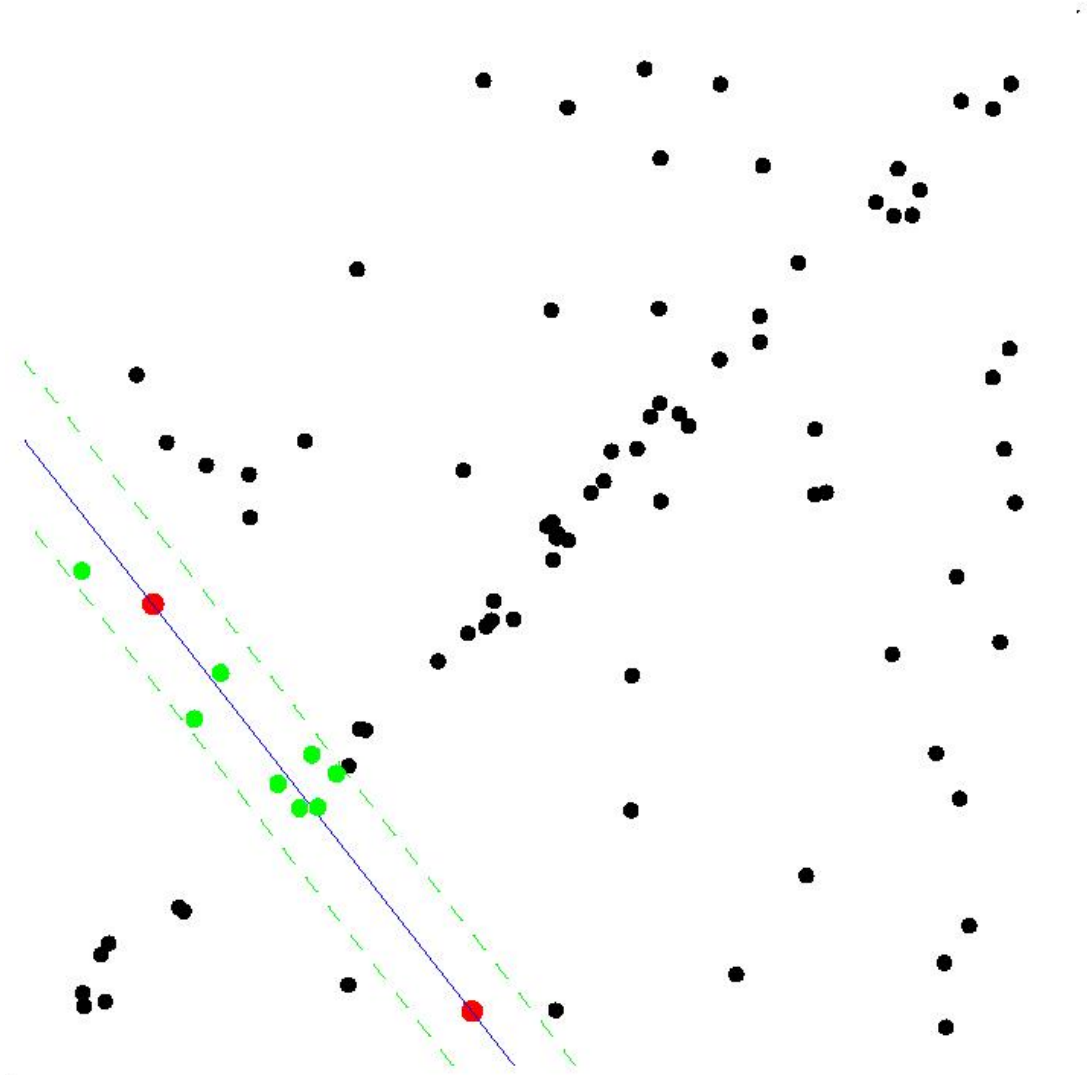
- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- **Calculate error function for each data point**

RANSAC



- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- **Select data that supports current hypothesis**

RANSAC



- Select sample of 2 points at random
- Calculate model parameters that fit the data in the sample
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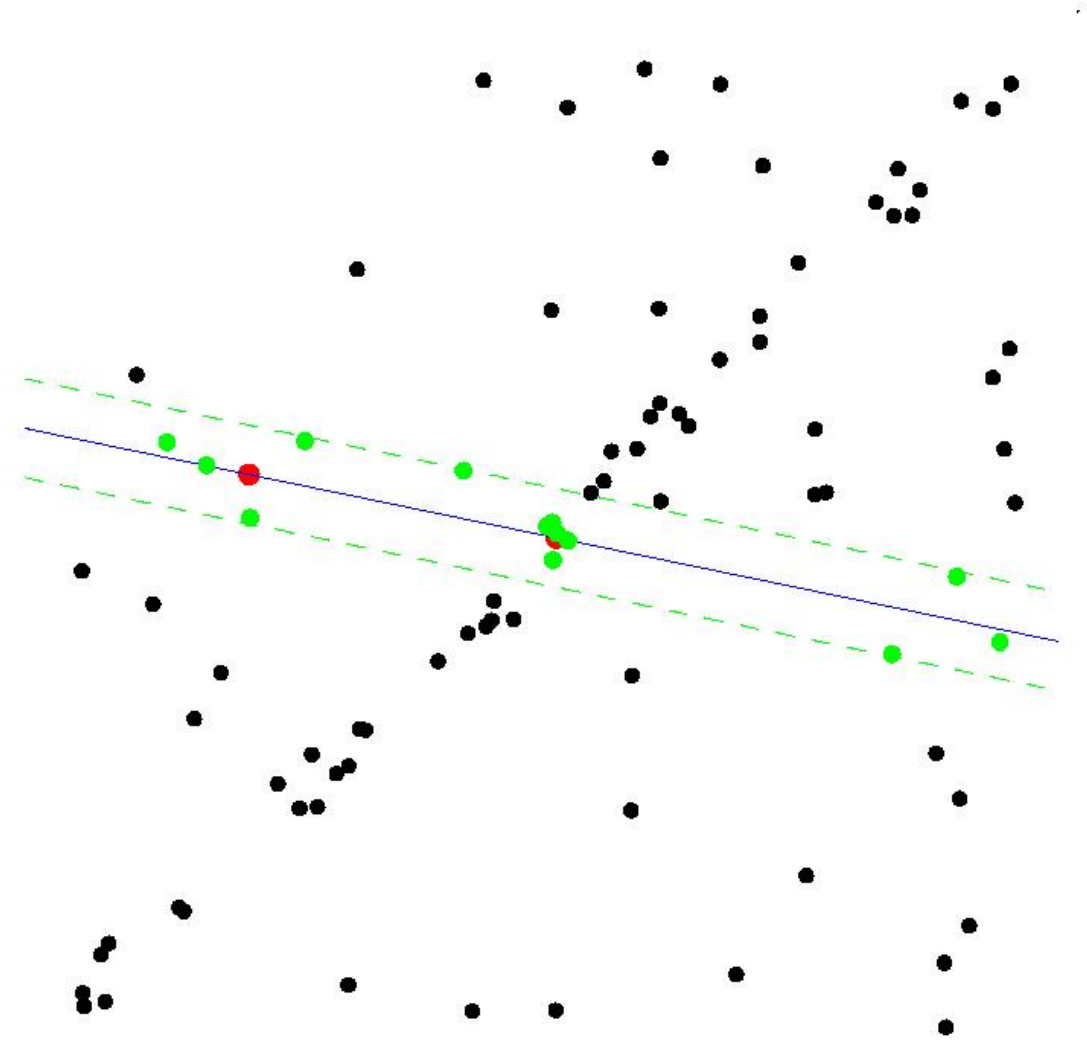
- Select data that supports current hypothesis
- **Repeat sampling**

RANSAC



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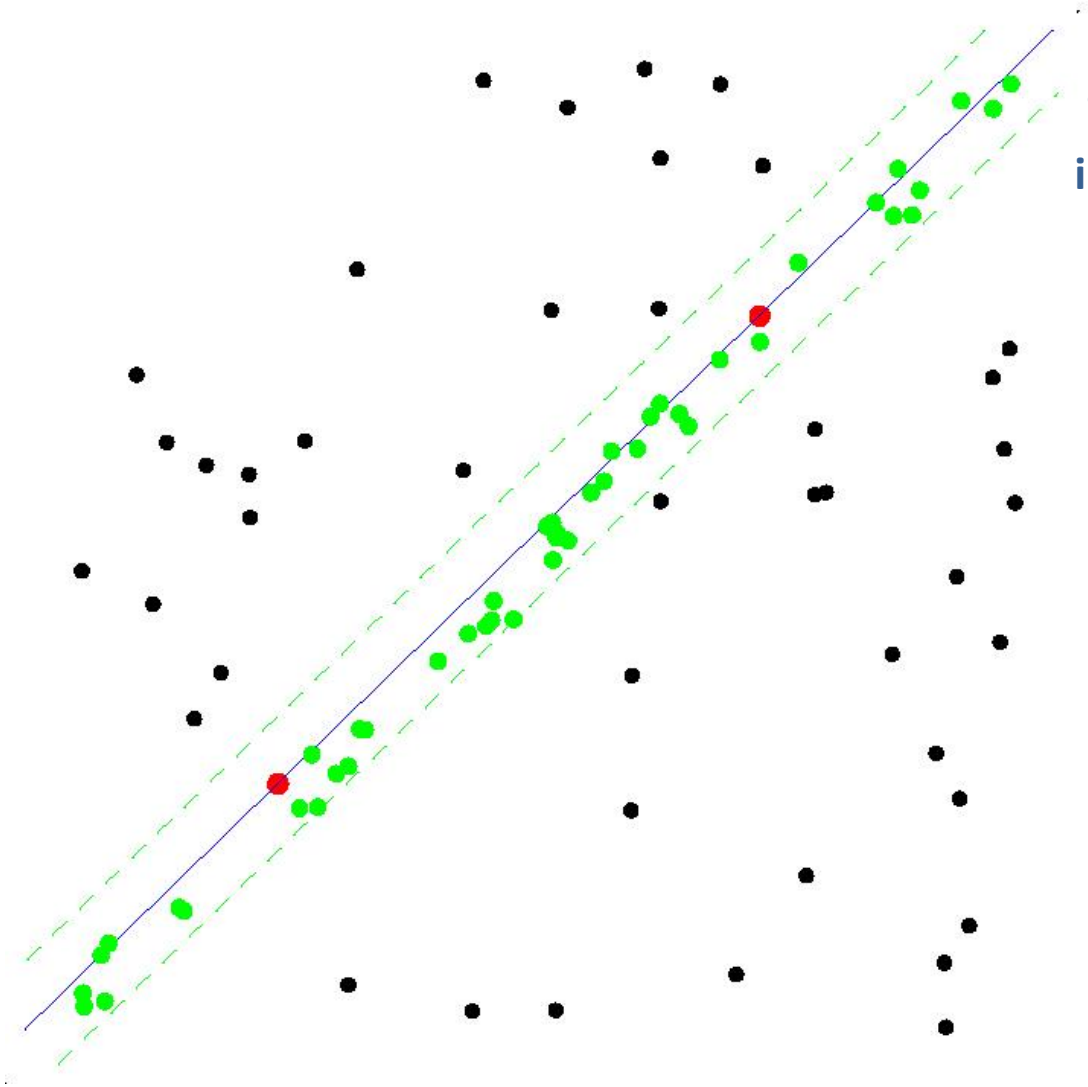
RANSAC



Set with the maximum number of inliers obtained within k iterations

RANSAC

How many iterations does RANSAC need?

- Ideally: check all possible combinations of **2** points in a dataset of **N** points.
- No. all pairwise combinations: **N(N-1)/2**

  ⇨ computationally unfeasible if **N** is too large.

  example: 1000 edge points ⇨ need to check all 1000*999/2= **500'000** possibilities!

- Do we really need to check all possibilities or can we stop RANSAC after some iterations?
  Checking a **subset** of combinations is enough if we have a **rough** estimate of the percentage of inliers in our dataset

- This can be done in a probabilistic way

RANSAC

How many iterations does RANSAC need?

- ***w*** := number of inliers/*N*

  N := total number of data points

  ⇨ ***w*** : fraction of inliers in the dataset ⇨ ***w =*** P(selecting an inlier-point out of the dataset)
- Assumption: the 2 points necessary to estimate a line are selected independently

  ⇨ w^2 = P(both selected points are inliers)

  ⇨ $1-w^2$ = P(at least one of these two points is an outlier)
- Let ***k*** := no. RANSAC iterations executed so far
- ⇨ $(1-w^2)^k$ = P(RANSAC never selected two points that are both inliers)
- Let ***p*** := P(probability of success)
- ⇨ $1-p = (1-w^2)^k$ and therefore :

$$k = \frac{\log(1-p)}{\log(1-w^2)}$$

RANSAC

How many iterations does RANSAC need?

- The number of iterations $\boldsymbol{k}$ is

$$k = \frac{\log(1-p)}{\log(1-w^2)}$$

- ⇨ knowing the fraction of inliers $\boldsymbol{w}$, after $\boldsymbol{k}$ RANSAC iterations we will have a probability $\boldsymbol{p}$ of finding a set of points free of outliers

- Example: if we want a probability of success $\boldsymbol{p}$=99% and we know that $\boldsymbol{w}$=50% ⇨ $\boldsymbol{k}$=16 iterations – these are dramatically fewer than the number of all possible combinations! **As you can see, the number of points does not influence the estimated number of iterations, only *w* does!**

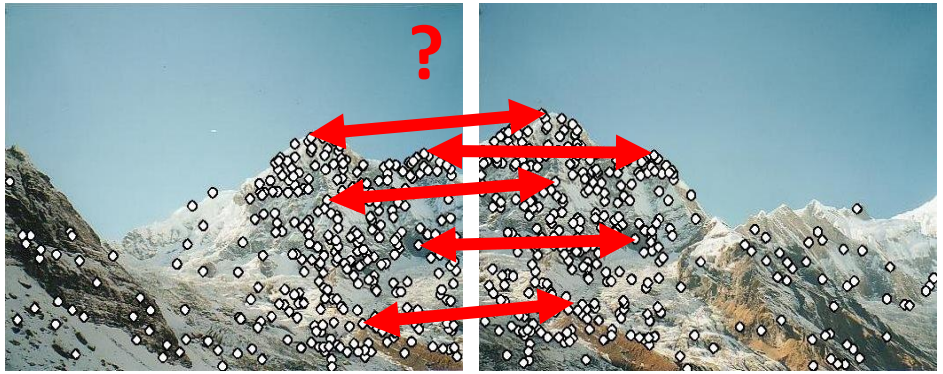
- In practice we only need a rough estimate of $\boldsymbol{w}$.
  More advanced variants of RANSAC estimate the fraction of inliers and adaptively update it at every iteration

RANSAC

1. Initial: let *A* be a set of *N* points
2. **repeat**
3. Randomly select a sample of 2 points from *A*
4. Fit a line through the 2 points
5. Compute the distances of all other points to this line
6. Construct the inlier set (i.e. count the number of points whose distance < *d*)
7. Store these inliers
8. **until** maximum number of iterations ***k*** reached
9. The set with the maximum number of inliers is chosen as a solution to the problem

RANSAC: applications

- **RANSAC** = **RAN**dom **SA**mple **C**onsensus.
- A generic & robust fitting algorithm of models in the presence of outliers (i.e. points which do not satisfy a model)
- Generally applicable algorithm to any problem where the goal is to **identify the inliers which satisfy a predefined model**.



- Typical applications in robotics are:
  line extraction from 2D range data, plane extraction from 3D data, feature matching, structure from motion, camera calibration, homography estimation, etc.
- RANSAC is **iterative** ⇨ the probability to find a set free of outliers increases as more iterations are used
- `RANSAC is **non-deterministic** ⇨ results are different between runs