Tutorial „Modeling Locomotion“

Mass-spring models
to describe human running

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Locomorph Summer School 2012
Odense, Denmark (USD)

Overview

1 **Basics**

2 Horizontal and vertical oscillator

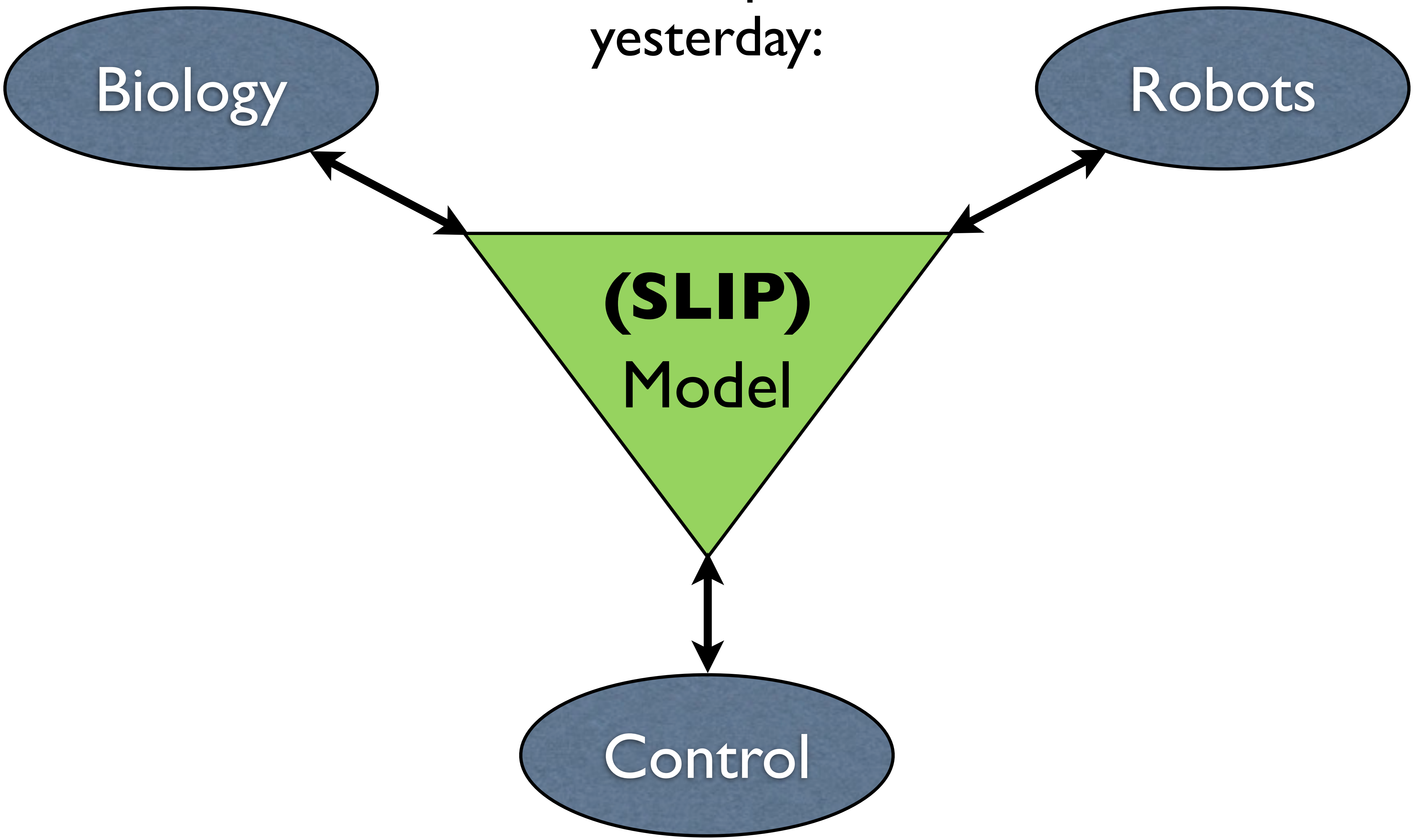
3 Mass-spring bouncing in 1D (*Hopping*)

4 Mass-spring bouncing in 2D (*SLIP model*)

5 Steps-to-fall map: A kind of sensitivity analysis

Basics

From Helmut‘s presentation yesterday:



Basics

Source: AP

Basics

We use **Newtonian physics** to **make predictions** of the object's motion

particular instant of time:

$$\boldsymbol{F} = m \cdot \boldsymbol{a} \qquad \boldsymbol{a} = \ddot{\boldsymbol{r}}$$

force — external cause of motion

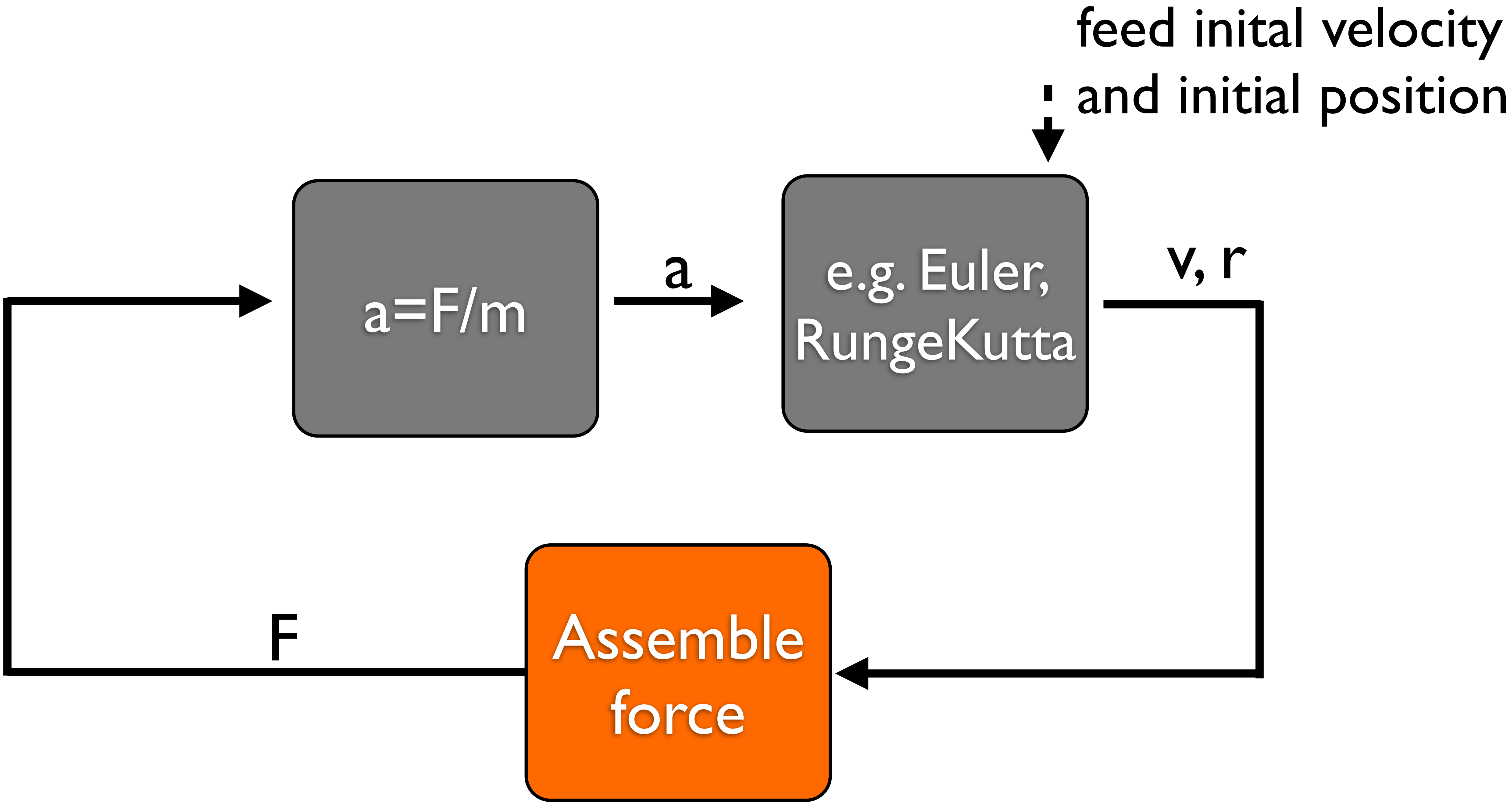
mass

acceleration — effect of force

position

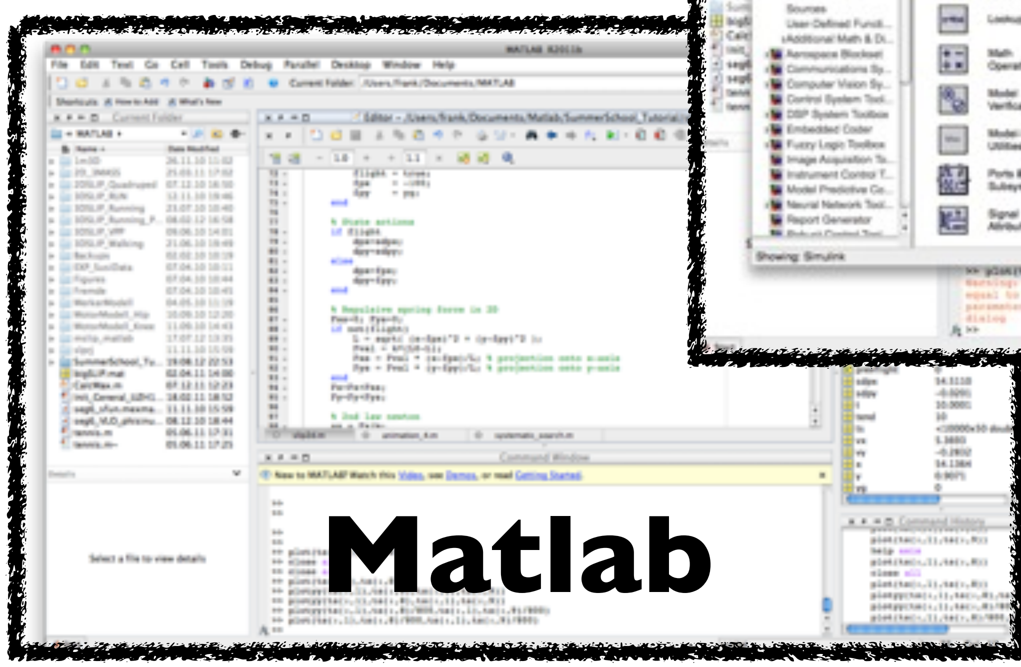
Basics

Initial value problem

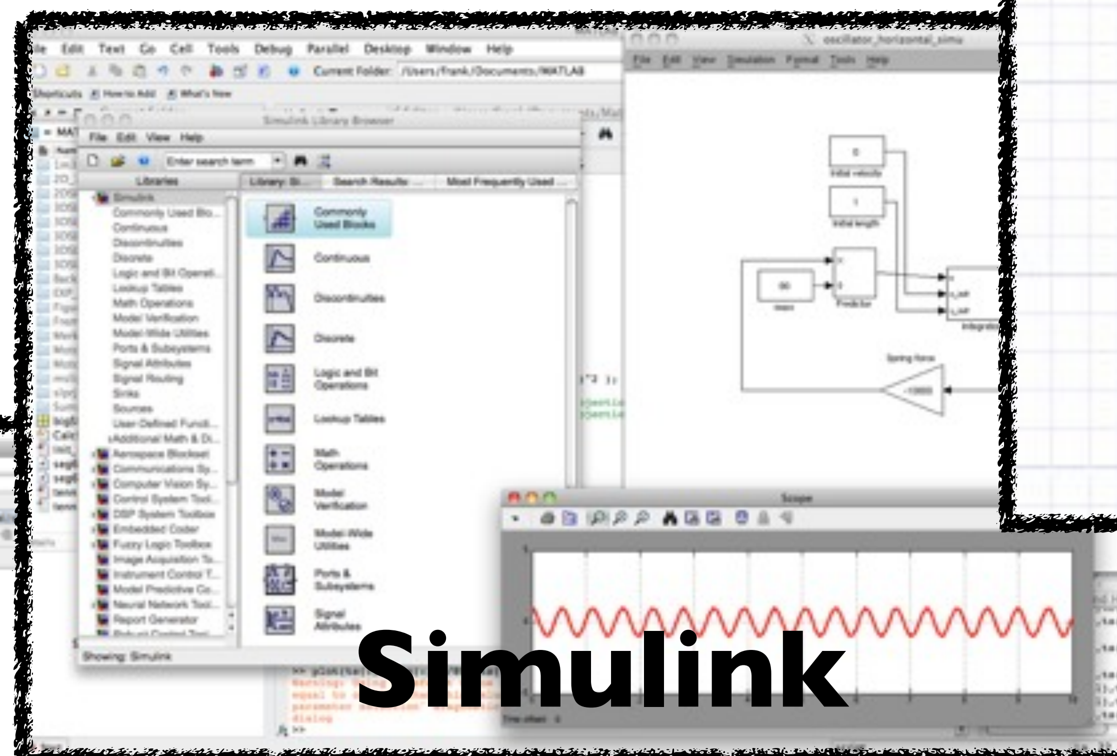


Basics

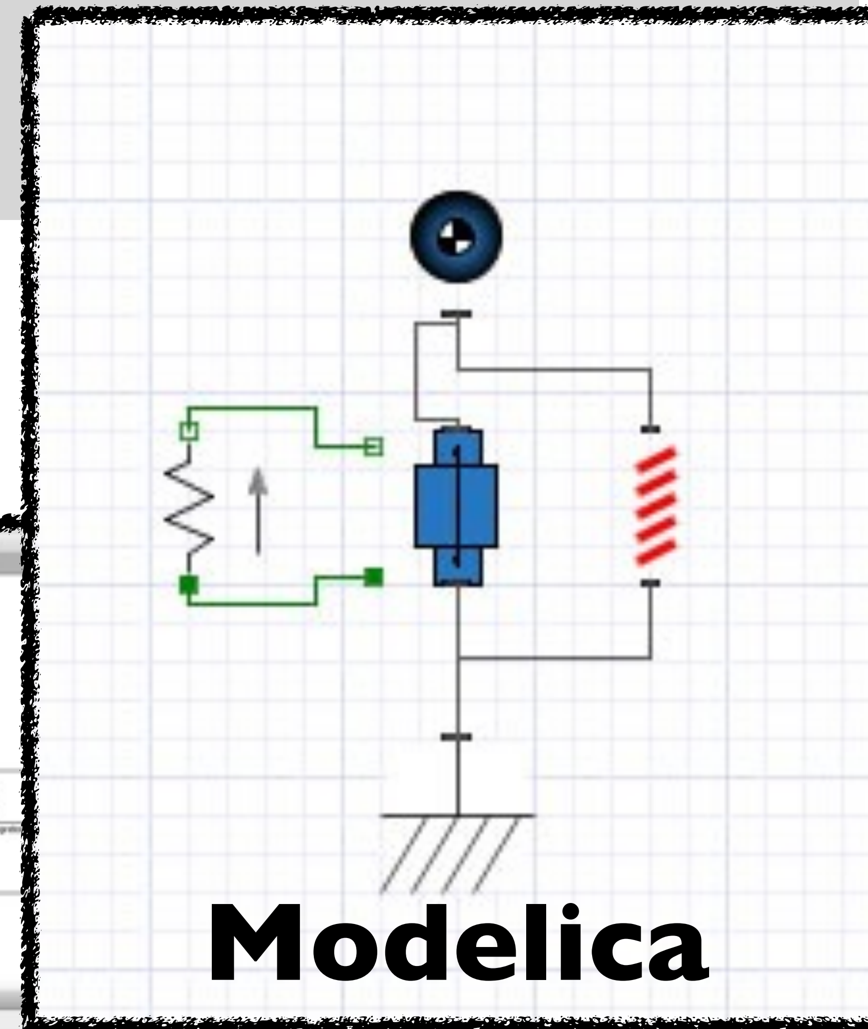
Abstraction level



User-friendly and
C speed if compiled
with Matlab Coder



Causal graphical
programming.
C speed but
large init time



General purpose,
under development,
10x slower than C

Tutorial outline

Horizontal oscillator

↓

Vertical oscillator

Hopper

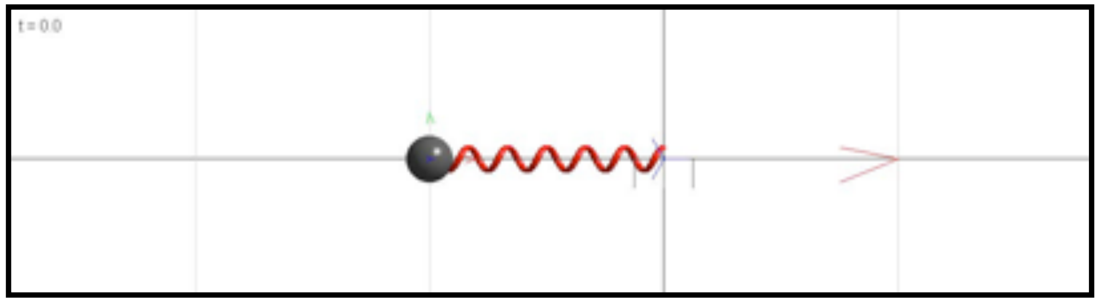
SLIP model

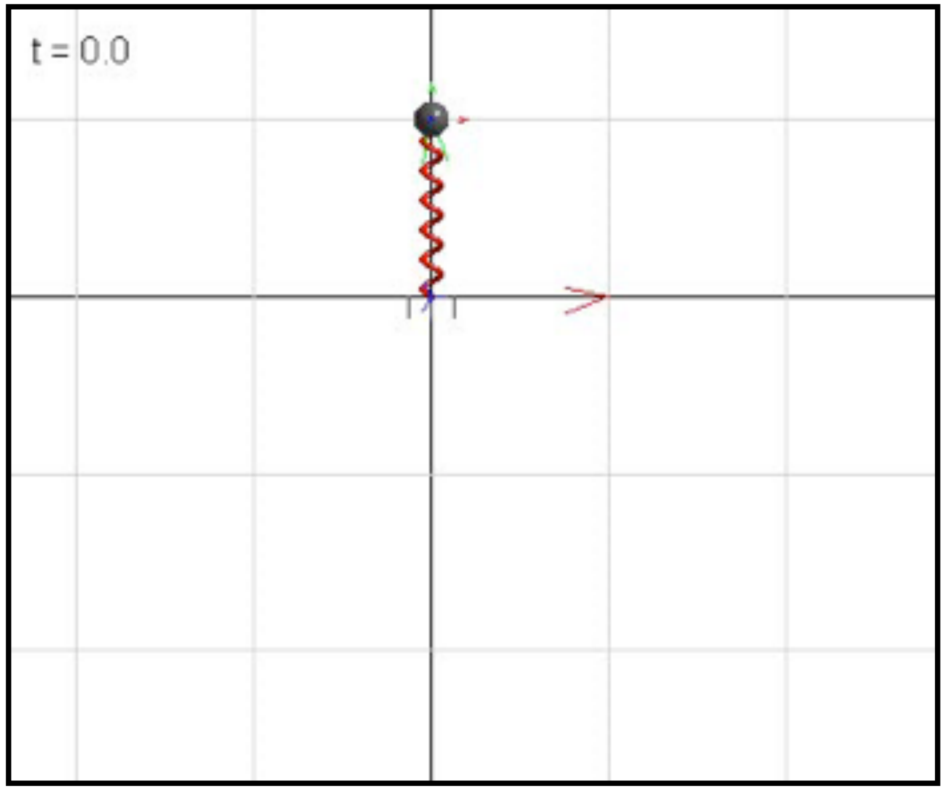
Steps-to-fall map of SLIP model

Overview

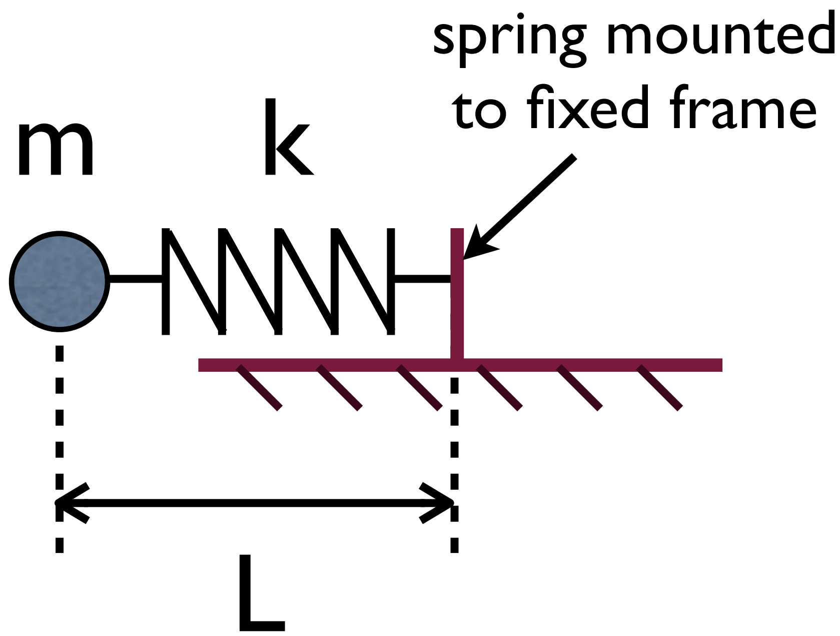
1 Basics

2 **Horizontal and vertical oscillator**





Horizontal oscillator



spring force:

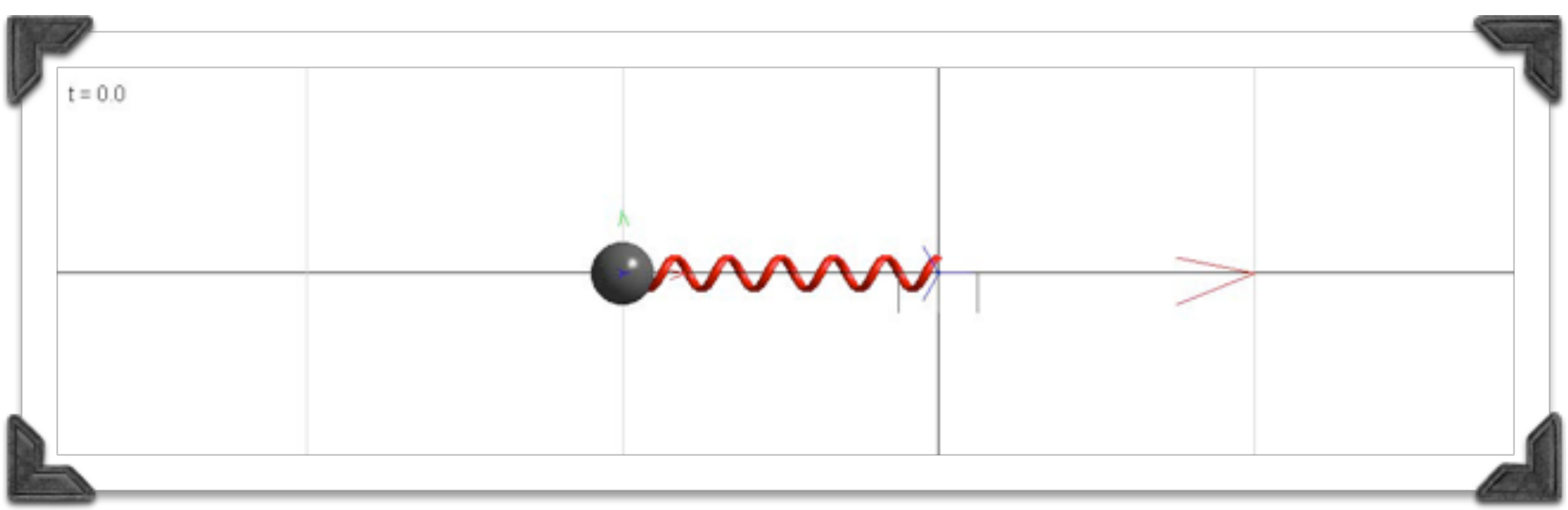
$$\boldsymbol{F} = -k \cdot (L - L_0) \cdot \hat{\boldsymbol{e}}_{\mathrm{spring}}$$

1D and with suitable coord. frame:

$$F = -k \cdot L$$

spring **stiffness** k :

$$k > 0$$



Basic structure of the **M files** (oscillator_horizontal.m)

Part I: Initialization

Part II: Time-stepping

```
clear all

% 1 Integrator parameters
dt=1e-4;
tend=10;
t=0;
i=1;

% 2 Output parameters
dtsample=1e-2;
N=ceil(tend/dtsample);
di=ceil(dtsample/dt);
ts=zeros(N,10);

% 3 Parameters
m=80;
k=20000;

% 4 Initial values
L=1;
v=0;

% 5 Just declarations
a=0;
F=0;

% ---------------------------------------------------- Time-looping -------
while t<tend
    % 6 Assemble force
    F = -k*L;

    % 7 2nd law newton
    a = F/m;

    % 8 Output
    if mod(i,di)==0
        is=i/di;
        ts(is,1:6)=[t L v a F m/2*v^2 + k/2*L^2];
    end

    % 9 Euler
    v = v + dt * a;
    L = L + dt * v;

    % 10 Advance in time
    t=t+dt;
    i=i+1;
end
```

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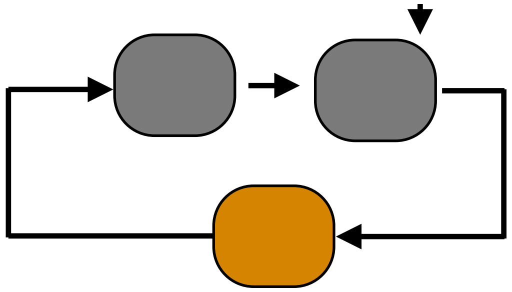
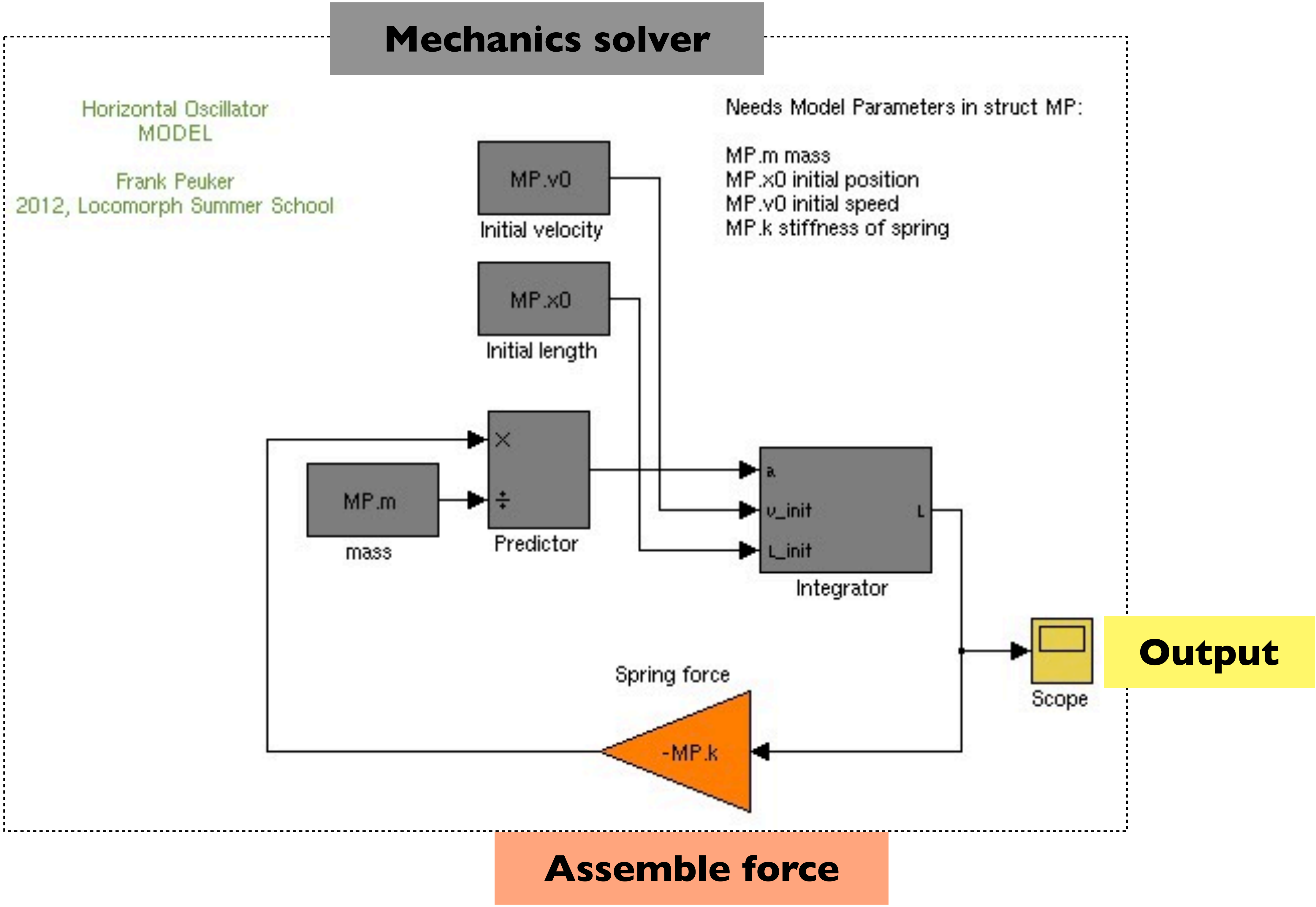
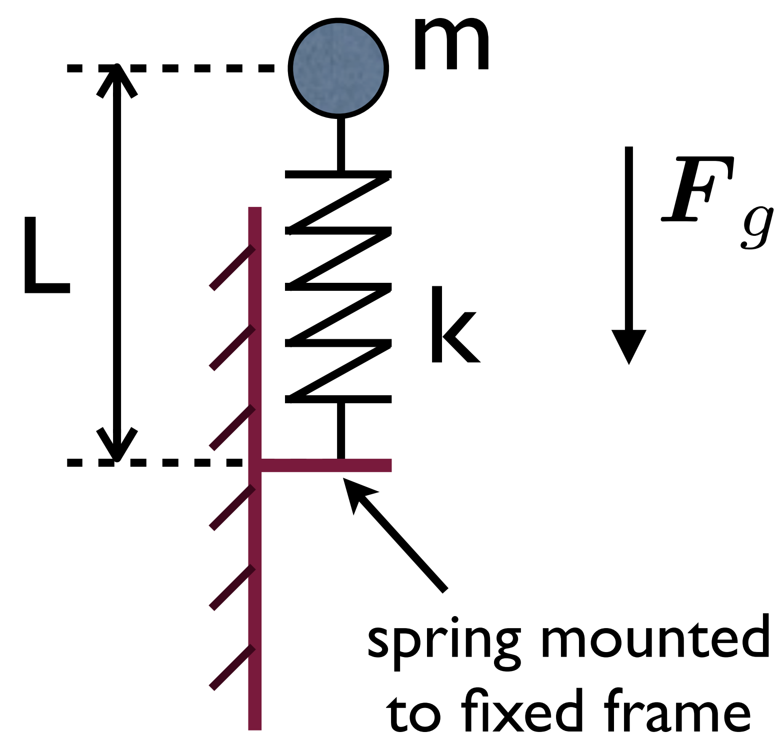
Basic structure of the **Simulink file** (oscillator_horizontal_simu.mdl)

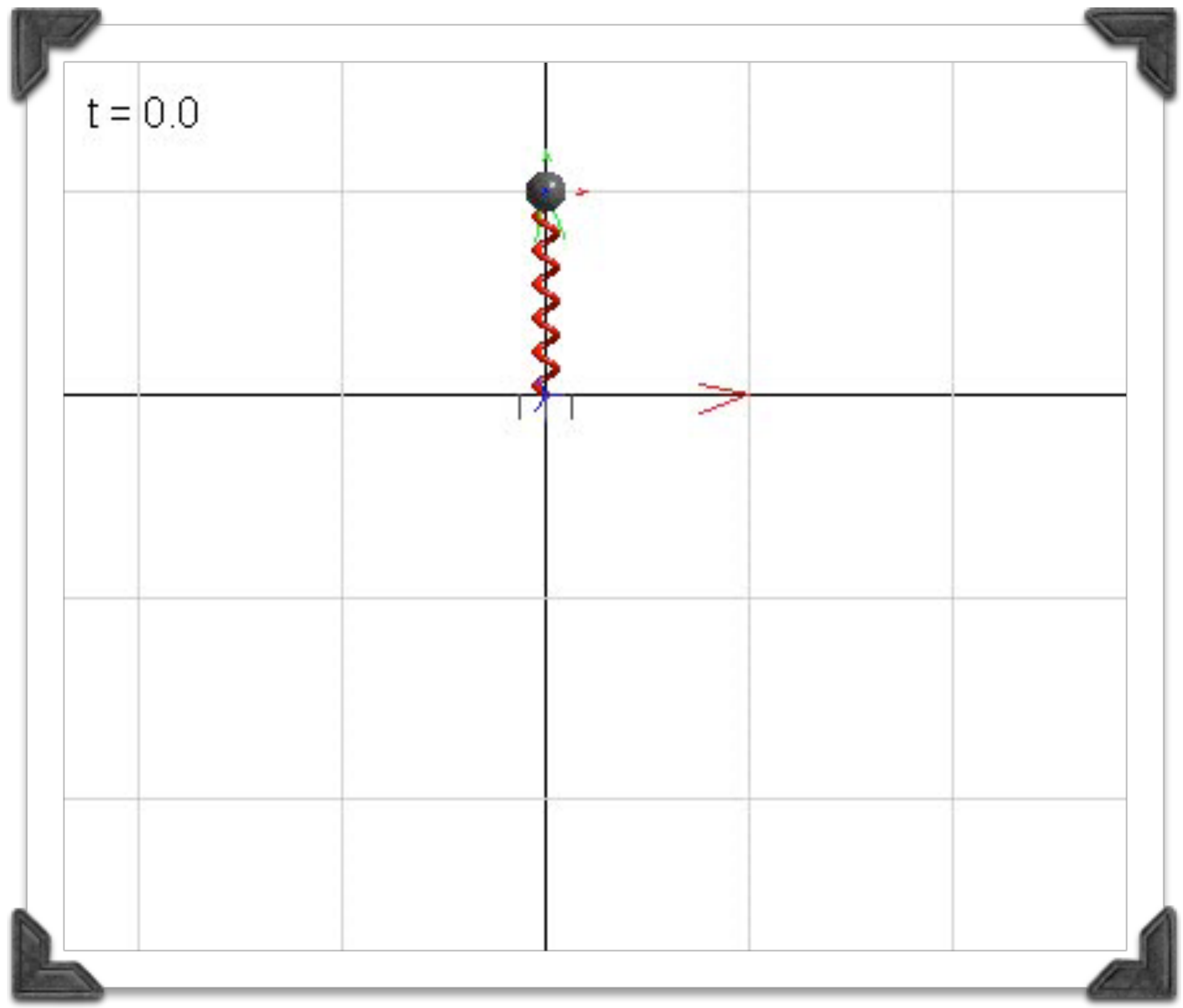




Vertical oscillator

HandsOn





gravitational force:

$$\boldsymbol{F_g} = m \cdot \boldsymbol{g}$$

1D, numerical value:

$$F_g = -9.81 \cdot m$$

Vertical oscillator

Solution

oscillator_vertOsci.m

oscillator_vertical_simu.m

Overview

1 Basics

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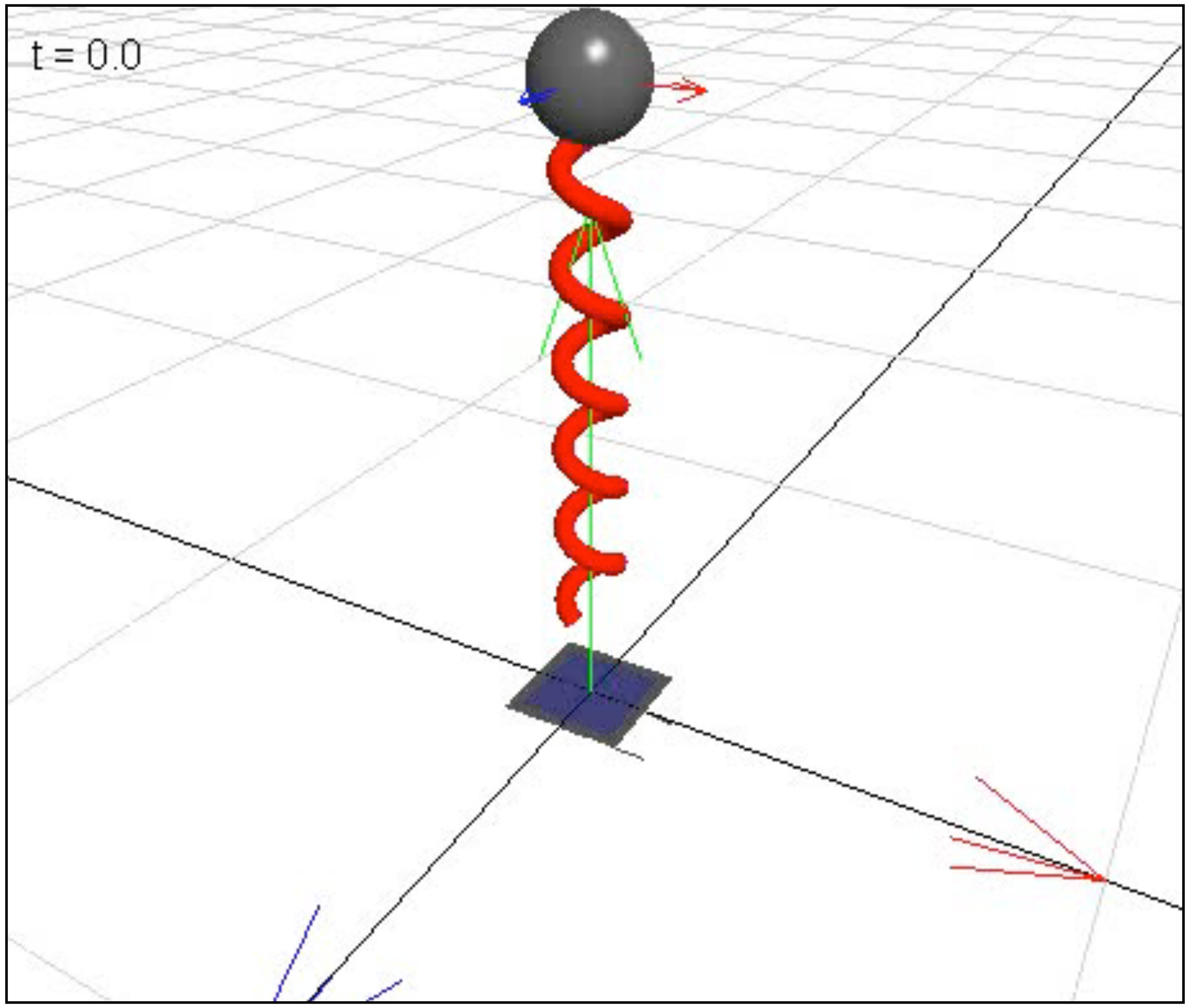
3 **Mass-spring bouncing in 1D (*Hopping*)**

4

5

Ground reaction force (arrow)



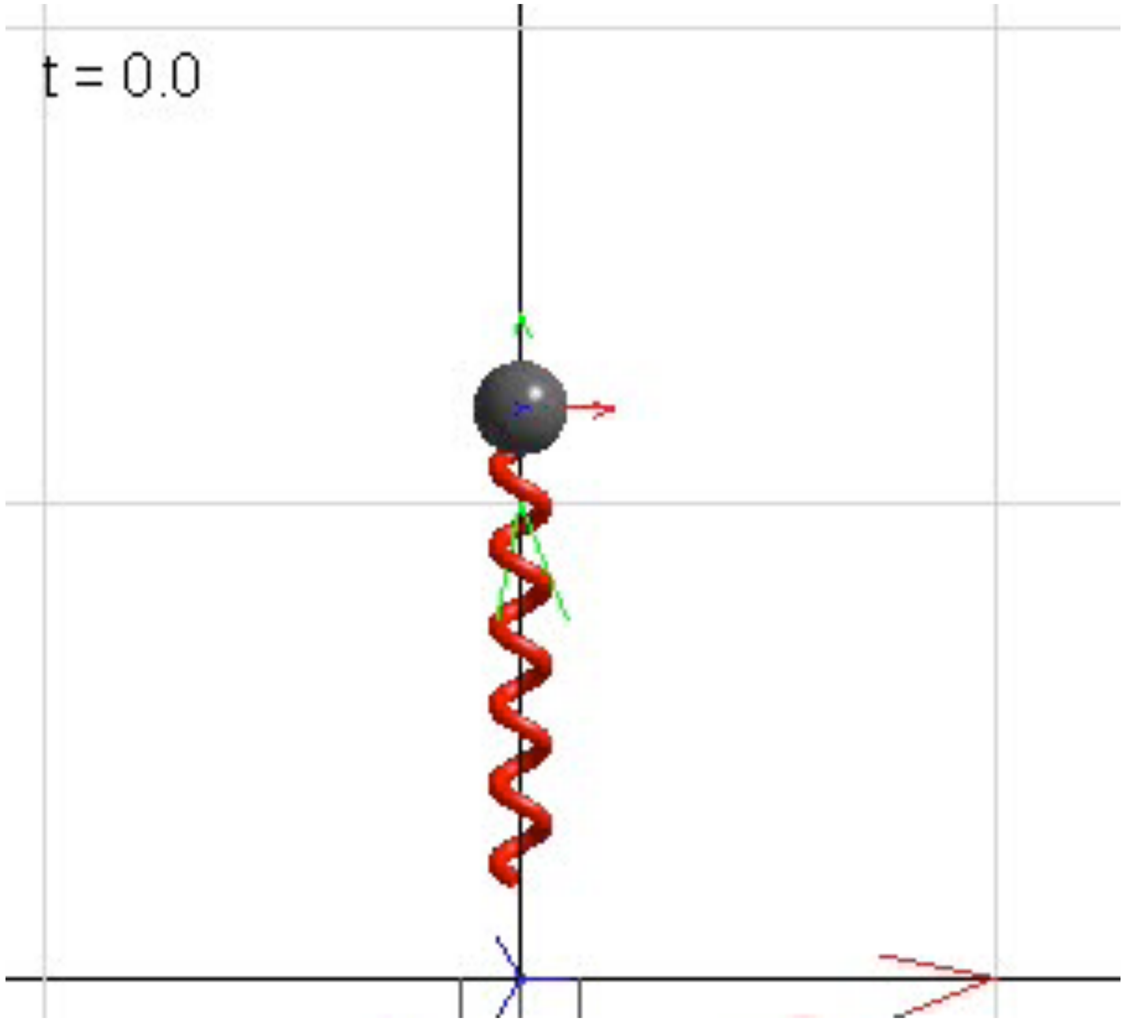
Spring mounted to mass

Events occur (hybrid sytem):

- Touch-down TD
- lift-off LO

Why to model contact this way?

It‘s simple.



New parameters:

- Flight foot point: `flight_fp`
- Stance foot point: `stance_fp`
- ground height: `yg`
- Foot point `fp` (just for visualization)

During ground contact:

- Freeze foot point: `stance_fp=const.`

Hopper: A hybrid system

HandsOn

Goals:

1) graph „Succession of flight and stance“
2) Trivial animation

Typical human hopper:

mass = 80 kg
stiffness = 20 kN/m

init height y0 = 1.2 m

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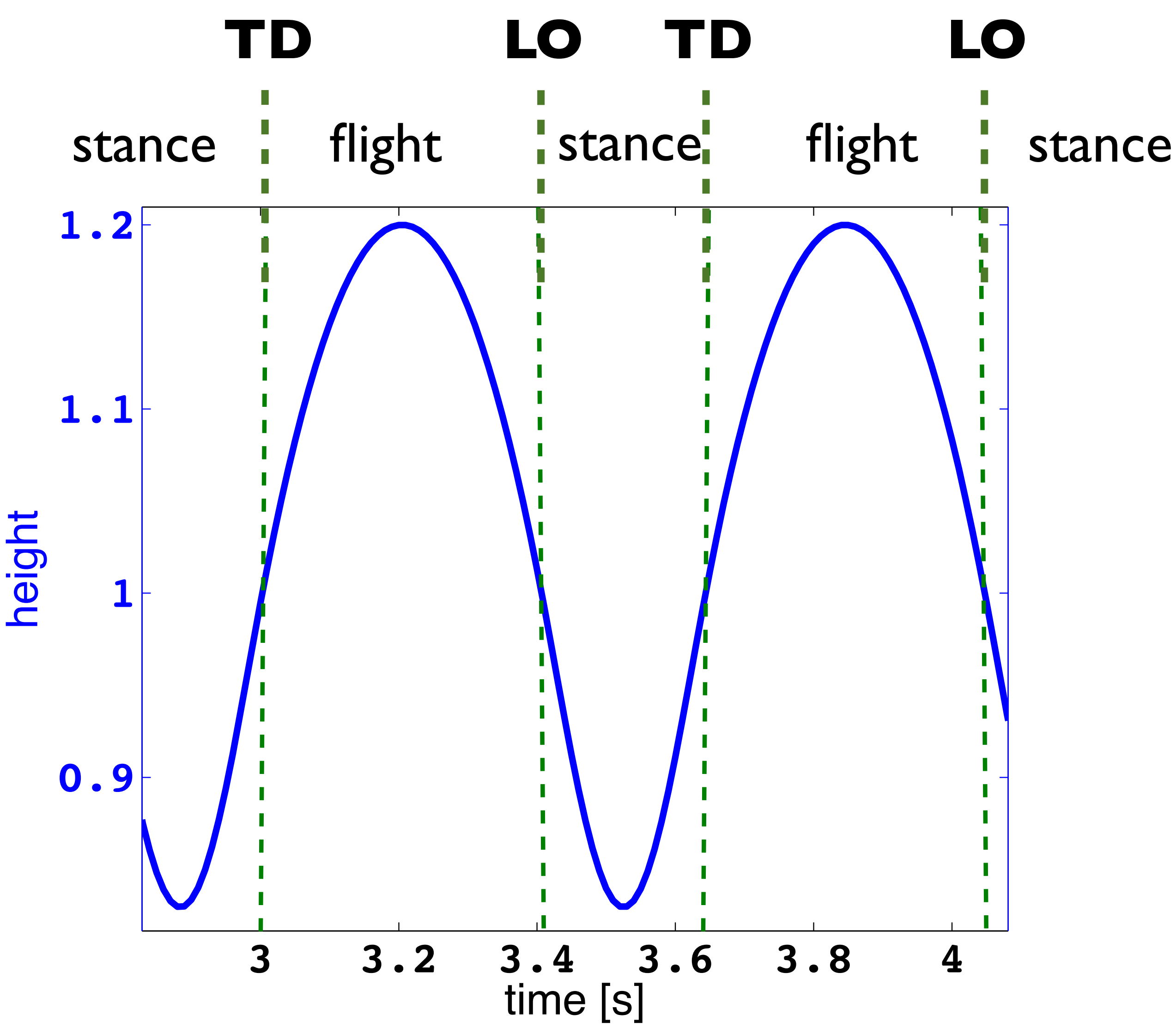

Hopper: A hybrid system

Solution

hopper.m

hopper_simu.mdl

Overview

1. Basics
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3. Mass-spring bouncing in 1D (*Hopping*)
4. **Mass-spring bouncing in 2D (*SLIP model*)**
5.

Running: A hybrid system

<u>states</u>:
stance + flight

<u>events</u>:
touch-down + lift-off

One-legged hopper suffices to describe running

Permit horizontal motion

HandsOn

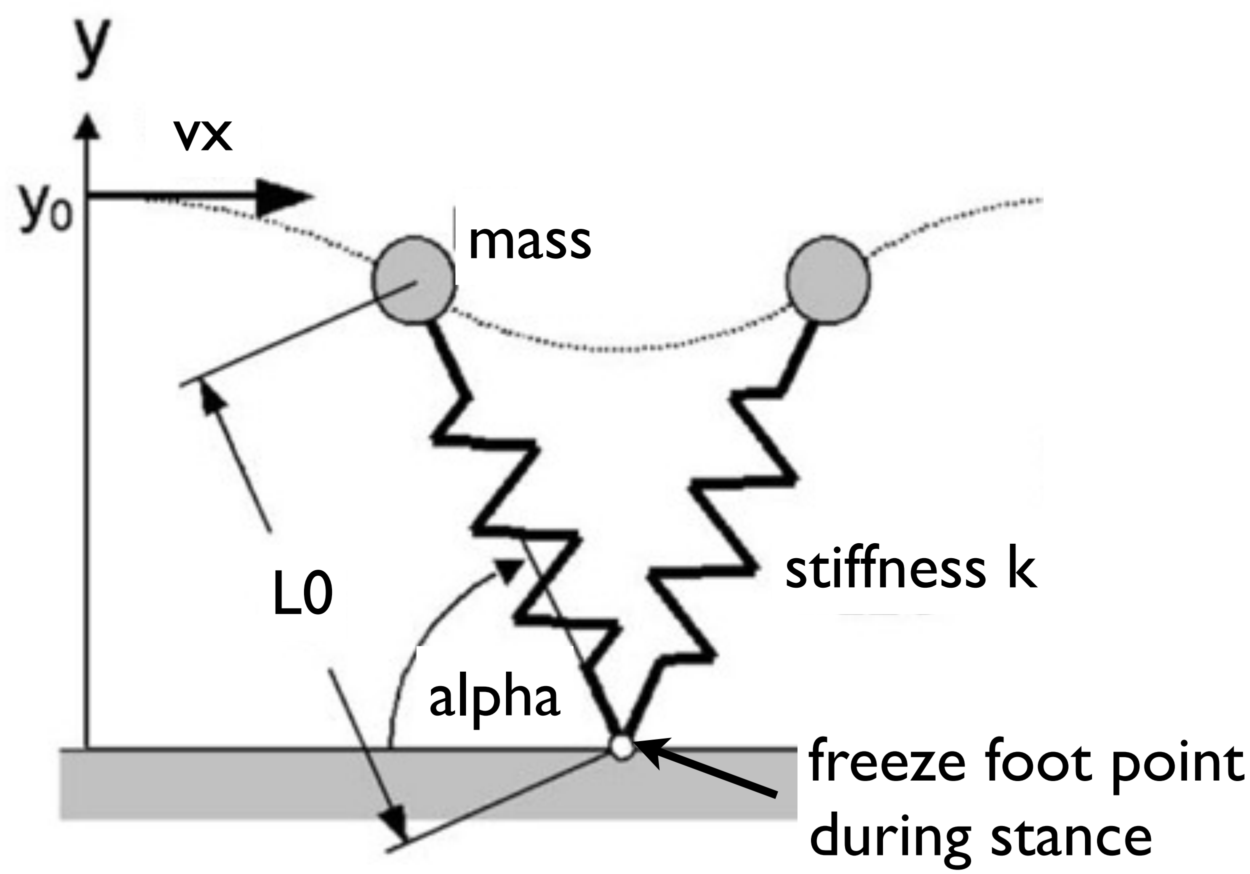
New parameters (wrt 1D hopper):

- Advance variables to 2D
- Angle of attack *alpha*, Initial horizontal velocity *vx*

Typical human runner:

mass = 80 kg
leg stiffness = 20 kN/m
alpha = 68 deg

init height y0 = 1m
init speed v0 = 5 m/s



Seyfarth (2002)

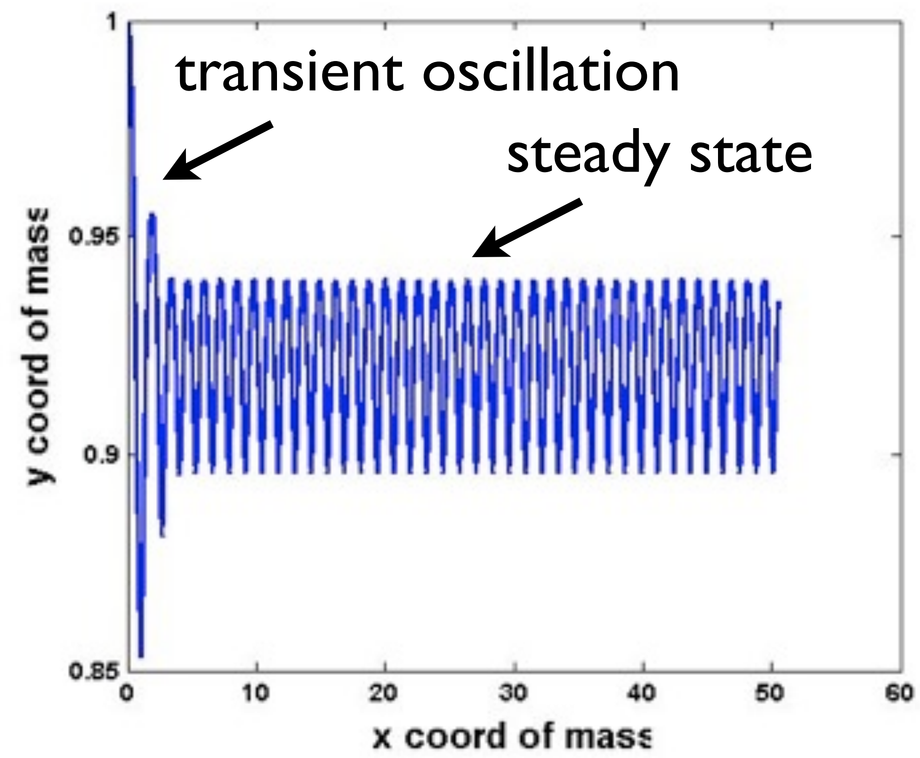
init speed v0 = 5 m/s

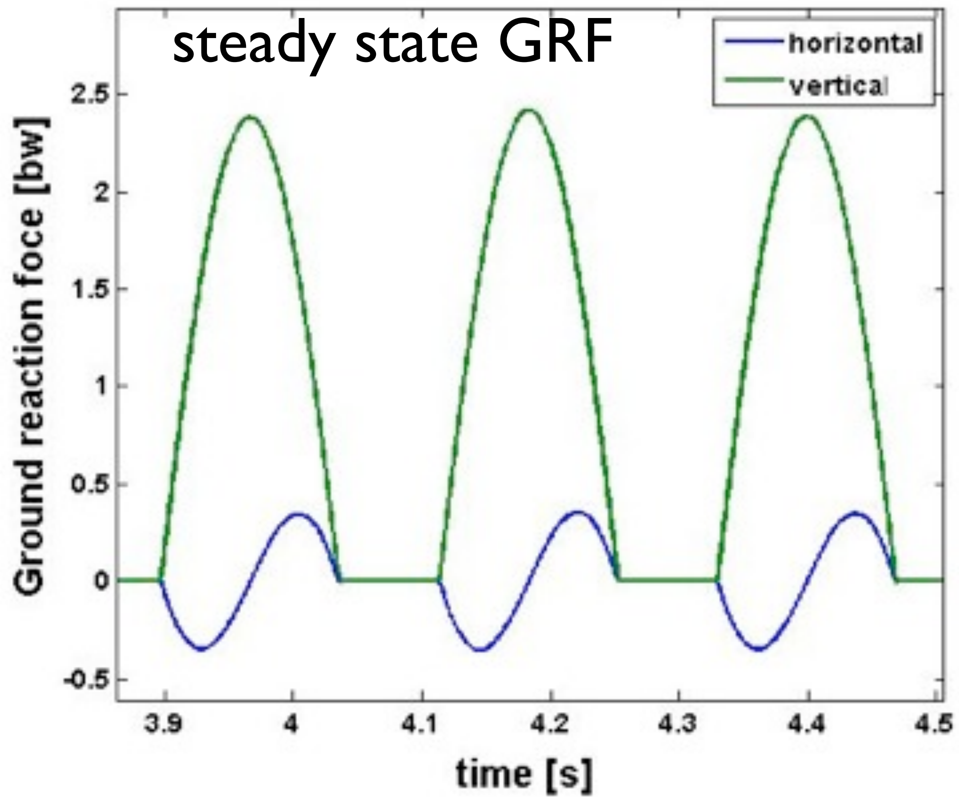
Typical measures

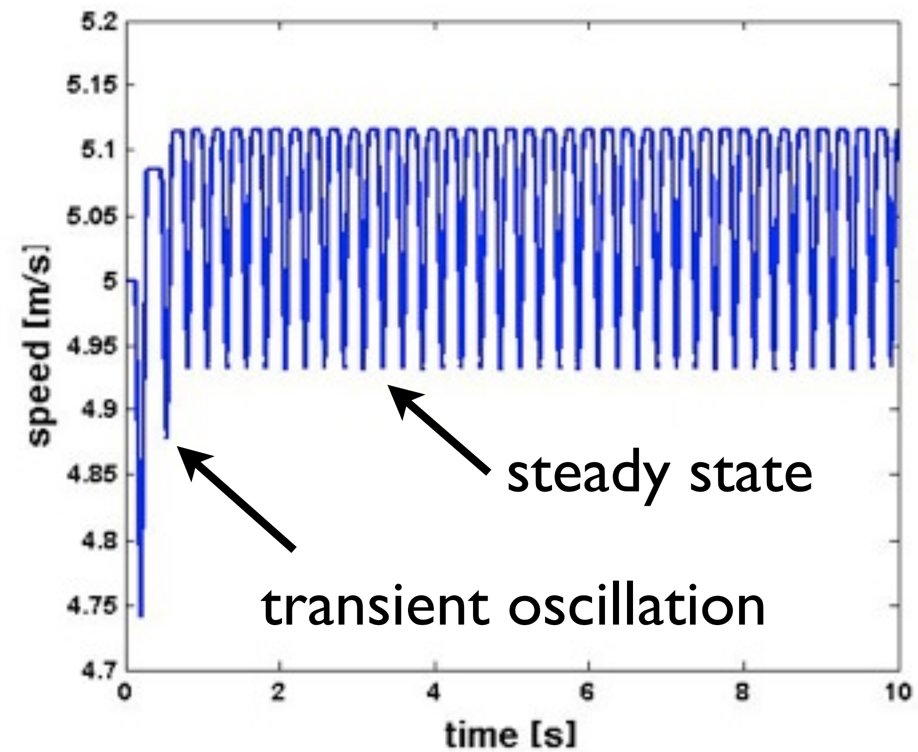
HandsOn

Goals:

1) graph „x vs y“
2) graph „horizontal speed vs time“
3) graph „Ground reaction force“
4) Trivial animation





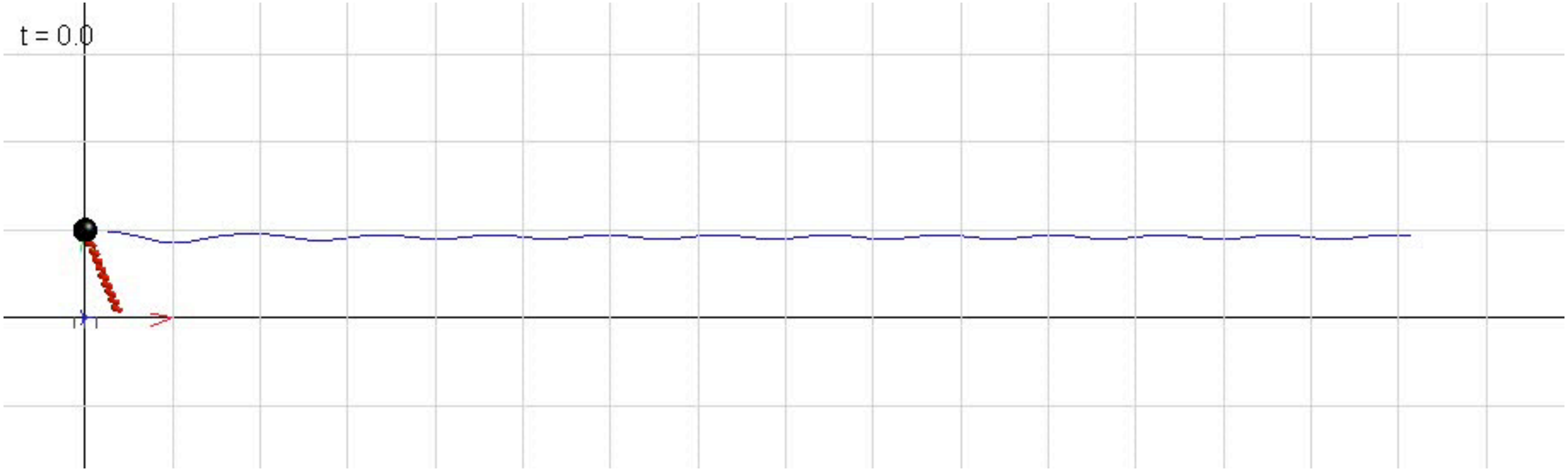


SLIP model

Solution

slip.m

slip_simu.mdl



Overview

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5. **Steps-to-fall map: A kind of sensitivity analysis**

Sensitivity analysis

What happens with respect to parameter variations?

1. We apply systematic search (equidistant grid)
   (popular: stiffness = 5 kN/m .. 40 kN/m, alpha = 40 deg .. 80 deg)
2. We apply constant initial values
   (popular: 1m height and 5m/s forward speed)
3. We record number of steps until model falls over or reaches some constant number of steps
   (popular: 50 steps maximum)
4. We plot that (e.g. with `pcolor` command)

That is called: **steps-to-fall map**

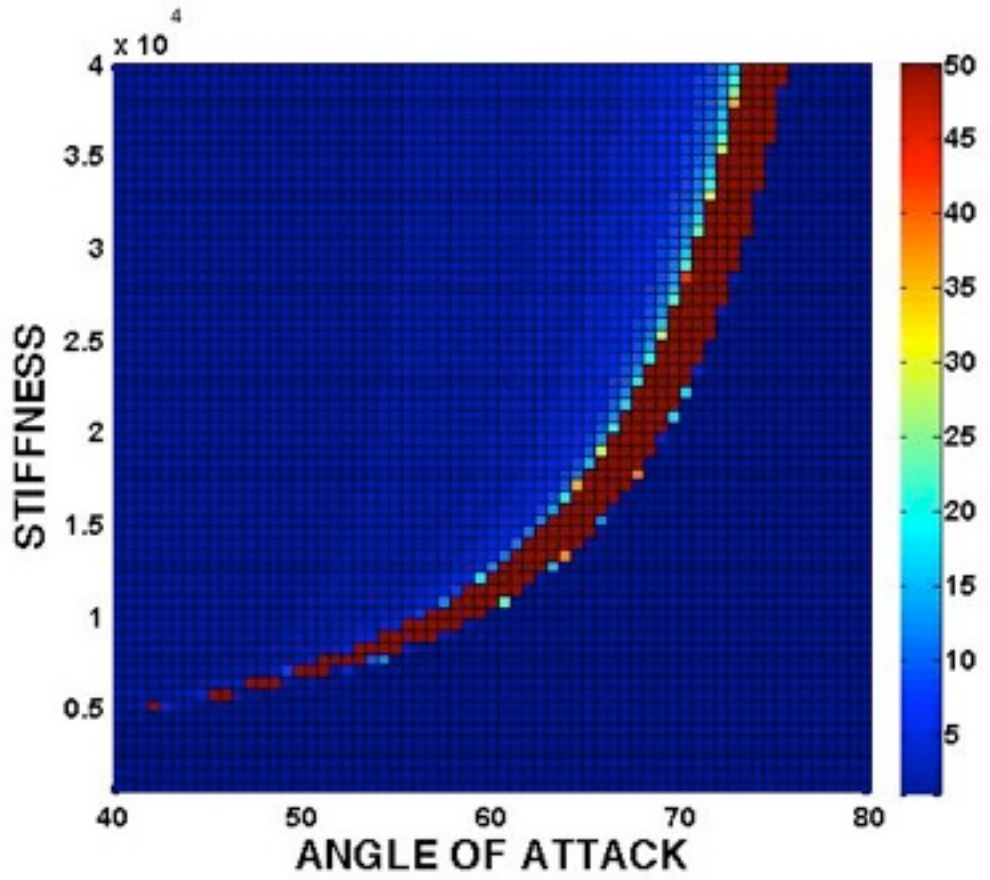
Steps-to-fall map

HandsOn

Goal:

1) Record a popular steps-to-fall map

What latin letter is similar to the identified domain of stability ?



It‘s a J !?

Steps-to-fall map

Solution

stf.m (main file)
slipFcn.m (function file)

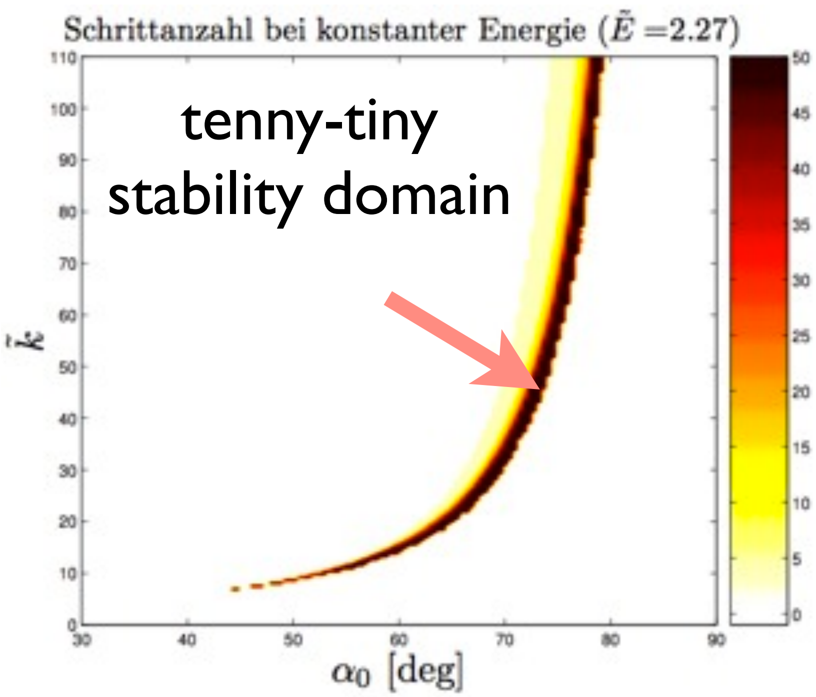
SLIP doping

Techniques to enhance stability

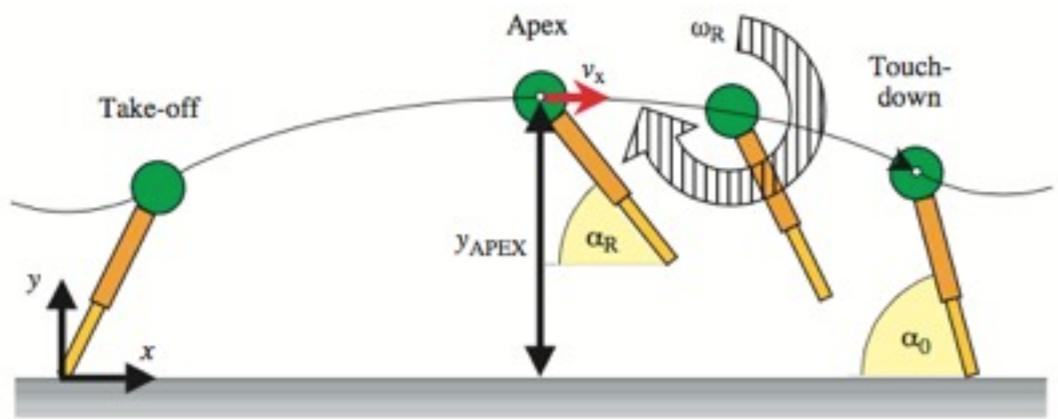
Stability Enhancers - powerful !

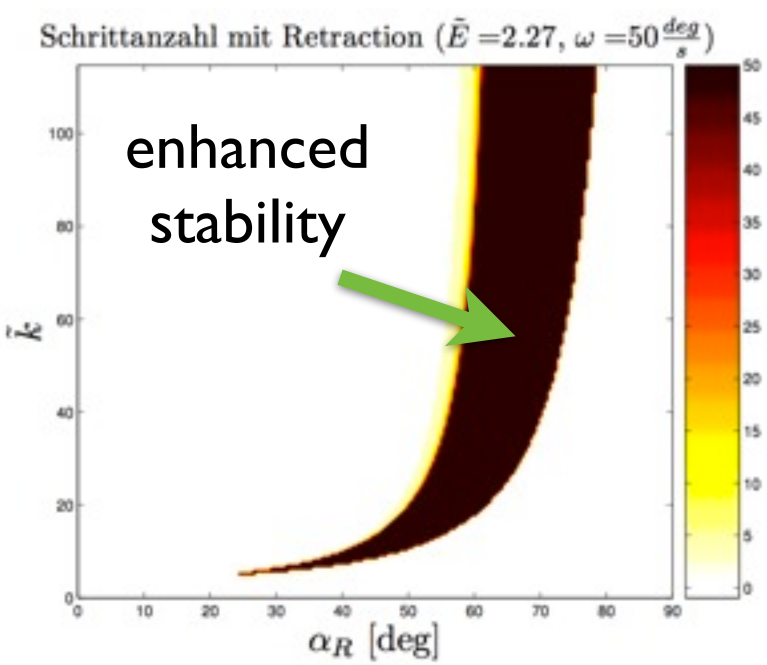
Retraction

Seyfarth et al., 2003
Exp.Biology



ENHANCER I

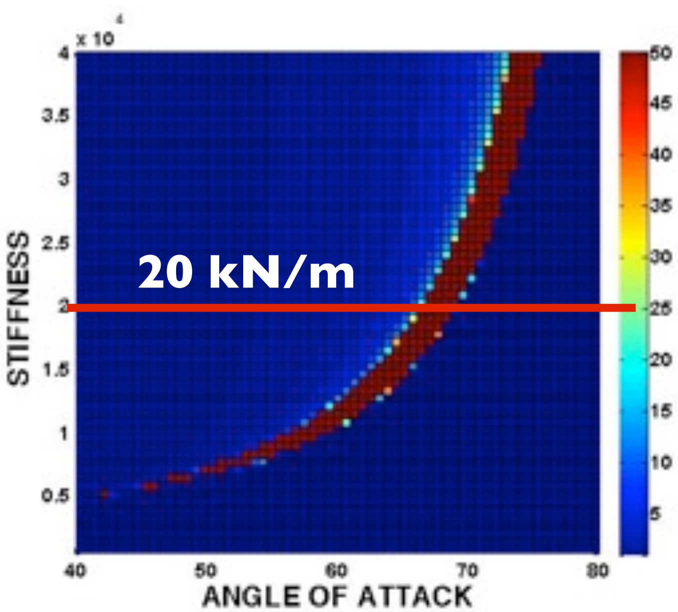




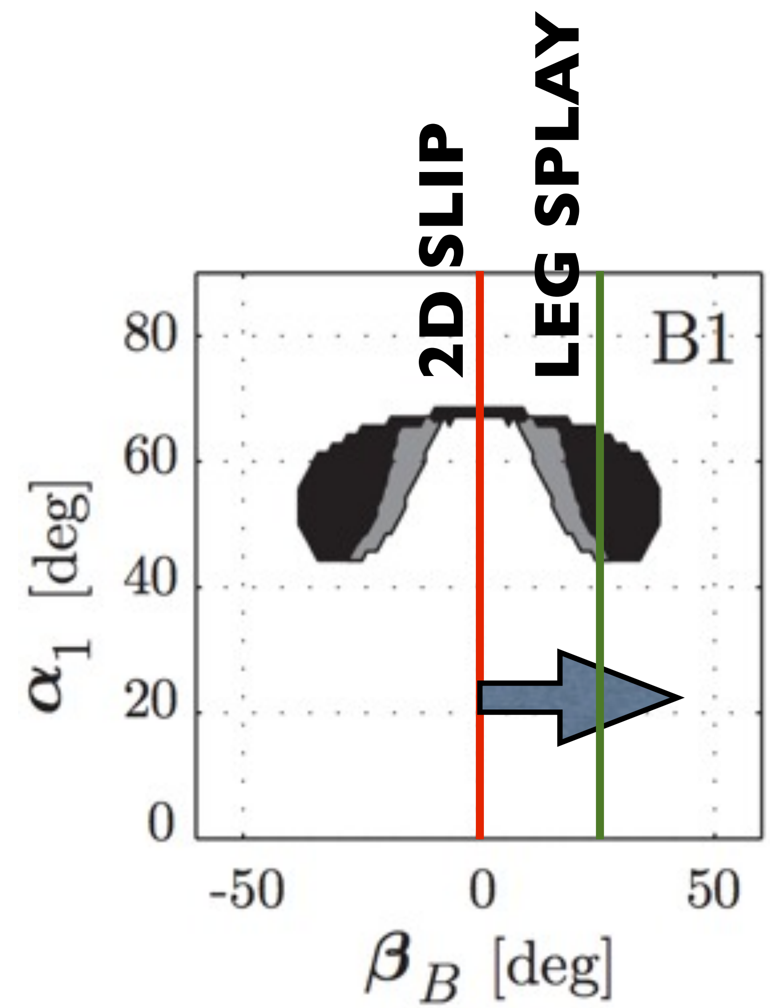
Stability Enhancers - powerful !

3D

Peuker et al., 2012
Bioinspiration & Biomimietics



ENHANCER 2

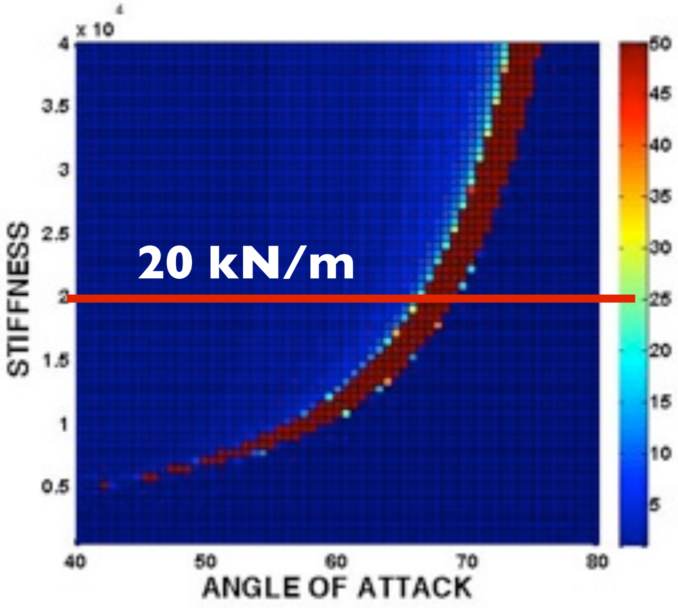


increase
stability
with
leg splay

Stability Enhancers - powerful !

3D Retraction

Peuker et al., 2012
Bioinspiration & Biomimietics



ENHANCER 3