Foundation of Computer Science 1

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Problem 1. Consider the binary relation $\approx_3$ over the integer numbers $\mathbb{Z}$ defined as:

$$a \approx_3 b \quad \text{if and only if } a - b \text{ is a multiple of 3} \qquad \text{(where } a, b \in \mathbb{Z}\text{)}$$

Prove that $\approx_3$ is an equivalence relation!

Problem 2.

(2.1) Consider the binary relation $\gg$ over the natural numbers $\mathbb{N}$ defined as:

$$a \gg b \quad \text{if and only if } b = a^r \text{ for some } r \in \mathbb{N} \qquad \text{(where } a, b \in \mathbb{N}\text{)}$$

(a) Prove that $\gg$ is a partial order!

(b) Consider the set $A = \{1, 2, 4, 16\} \subset \mathbb{N}$. Give three different upper bounds of A with respect to the relation $\gg$. What is $lub(A)$?

(c) Is $\gg$ an equivalence relation? Justify your answer!

(d) Is $\gg$ a total order? Justify your answer!

Problem 3. Let s_1, s_2 and s_3 be program statements, and consider Q be a predicate formula over program variables. What are the truth values of the following statements?

(3.1) $\text{wp}(s_1; s_2; s_3, Q) = \text{wp}(s_1; s_2, \text{wp}(s_3, Q))$;

(3.2) $\text{wp}(s_1; s_2; s_3, Q) = \text{wp}(s_2; s_1, \text{wp}(s_3, Q))$;

(3.2) $\text{wp}(\underline{\text{while}}\ (True)\ \underline{\text{do}}\ s_1, Q) \implies \text{wp}(s_1, \text{wp}(\underline{\text{while}}\ (True)\ \underline{\text{do}}\ s_1, Q))$;

Problem 4. Let x and y be program variables with values from the natural numbers $\mathbb{N}$.

(4.1) What is $\text{wp}(x := x + 1,\ x \leq 10)$?

(4.2) What is $\text{wp}(x := x + 1; y := y + x,\ x \leq 10)$?

(4.3) What is $\text{wp}(y := y + x; x := x + 1,\ x + y \leq 10)$?

(4.4) What is $\text{wp}(x := x + 1; y := y + x,\ x + y \leq 10)$?

(4.5) What is $\text{wp}(x := x + 1; y := y + x,\ True)$?

(4.6) What is $\text{wp}(x := x + 1; x := x - 1,\ x + y \leq 10)$?

(4.7) What is $\text{wp}(y := x - 1; x := y + 1,\ x + y \leq 10)$?

(4.8) What is $\text{wp}(\underline{\text{if}}\ (x > 5)\ \underline{\text{then}}\ x := x - 1\ \underline{\text{else}}\ x := x + 1,\ x + y \leq 10)$?

(4.9) What is $\text{wp}(\underline{\text{if}}\ (x > 5)\ \underline{\text{then}}\ x := x - 1; y := y - x\ \underline{\text{else}}\ x := x + 1; y := y + x,\ x + y \leq 10)$?

Problem 5. Let x and y be program variables with values from the integer numbers $\mathbb{Z}$. Consider the Hoare triple:

$$\{x = 1 \wedge y = 1\} \quad \underline{\text{while}}\ (x < 10)\ \underline{\text{do}}\ x := x + 1; y := y + 1\ \underline{\text{end while}} \quad \{x = 10 \wedge y = 10\},$$

annotated with the loop invariant $(x \leq 10 \wedge x = y)$.

What are the verification conditions of the above given Hoare triple?

Problem 6. Let x and y be program variables with values from the natural numbers $\mathbb{N}$. Consider the Hoare triple:

$$\{x = 1\} \quad \underline{\text{while}}\ (x < 10)\ \underline{\text{do}}\ x := x + 1\ \underline{\text{end while}} \quad \{x = 10\}.$$

What are the truth values of the following statements?

(6.1) $x \leq 10$ is an invariant;

(6.2) $x < 10$ is an invariant;

(6.3) $x = 10$ is an invariant.