Satisfiability example: Russian spy puzzle

There are three persons: Stirlitz, Müller, and Eismann. It is known that exactly one of them is Russian, while the other two are Germans. Moreover, every Russian must be a spy.

When Stirlitz meets Müller in a corridor, he makes the following joke: "you know, Müller, you are as German as I am Russian". It is known that Stirlitz always says the truth when he is joking.

We have to establish that Eismann is not a Russian spy.

How can we solve problems of this kind?

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Formalisation in propositional logic

Introduce propositions (atomic formulas, boolean variables) XY, where

$X \in \{R, G, S\}$ (denoting Russian, German, Spy)
$Y \in \{S, M, E\}$ (denoting Stirlitz, Müller, Eismann)

For example,

SE : Eismann is a Spy
RS : Stirlitz is Russian

Formalisation in propositional logic

There are three persons: Stirlitz, Müller, and Eismann. It is known that exactly one of them is Russian, while the other two are Germans.

$$(RS \wedge GM \wedge GE) \vee (GS \wedge RM \wedge GE) \vee (GS \wedge GM \wedge RE).$$

Moreover, every Russian must be a spy.

$$(RS \rightarrow SS) \wedge (RM \rightarrow SM) \wedge (RE \rightarrow SE).$$

When Stirlitz meets Müller in a corridor, he makes the following joke: "you know, Müller, you are as German as I am Russian".

$$RS \leftrightarrow GM.$$

We have to establish that Eismann is not a Russian spy.

$$RE \wedge SE.$$

Hidden:

$$(RS \leftrightarrow \neg GS) \wedge (RM \leftrightarrow \neg GM) \wedge (RE \leftrightarrow \neg GE).$$

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$$(RS \land GM \land GE) \lor (GS \land RM \land GE) \lor (GS \land GM \land RE).$$

Moreover, every Russian must be a spy.

$$(RS \to SS) \land (RM \to SM) \land (RE \to SE).$$

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Why satisfiability?

If this set of formulas is satisfiable in some situation *I*, then

1. All conditions of the puzzle are satisfied by *I*.
2. *I* also satisfies the property that Eismann is a Russian spy.

If we demonstrate that the problem is unsatisfiable (it is a contradiction), then we prove that Eismann cannot be a Russian spy.