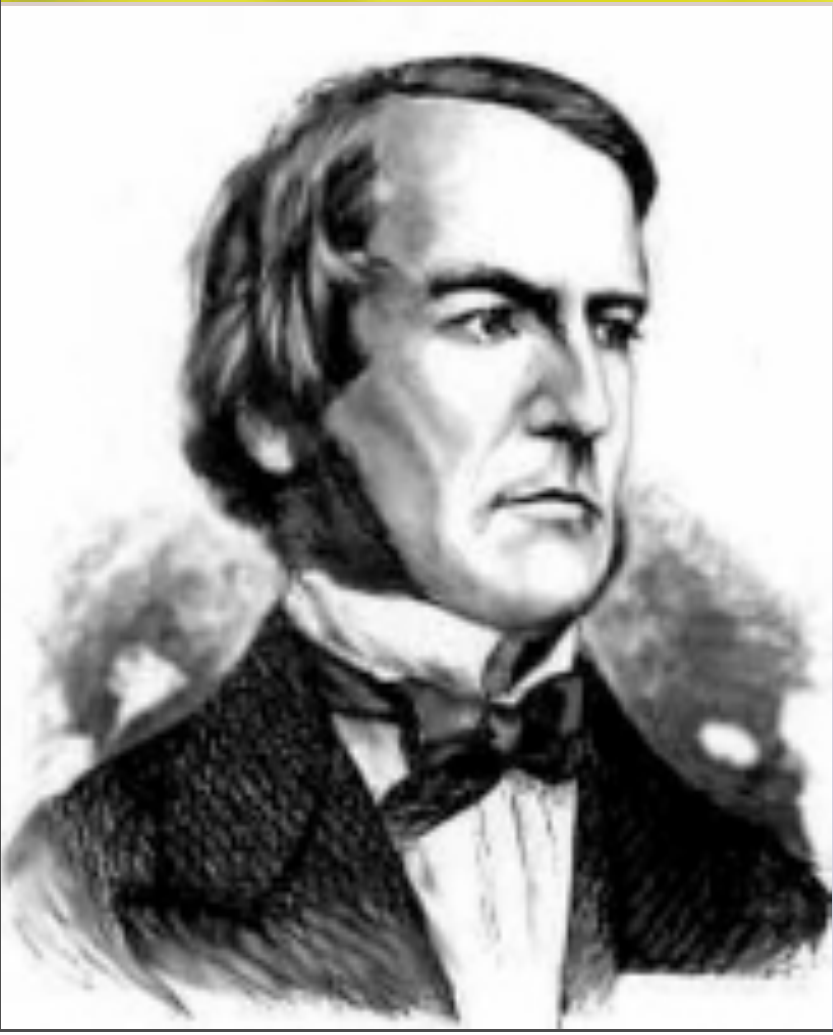
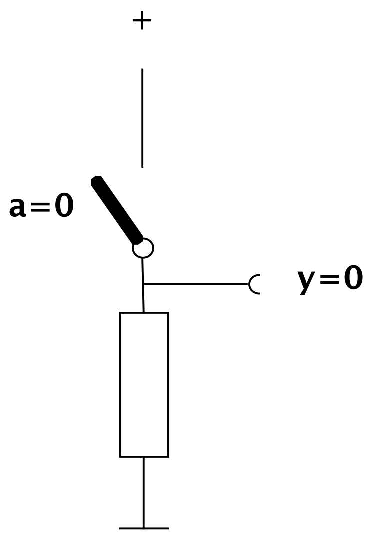
educational engineering lab
Department for Information Technology
University of Zurich



Boole'sche Algebra

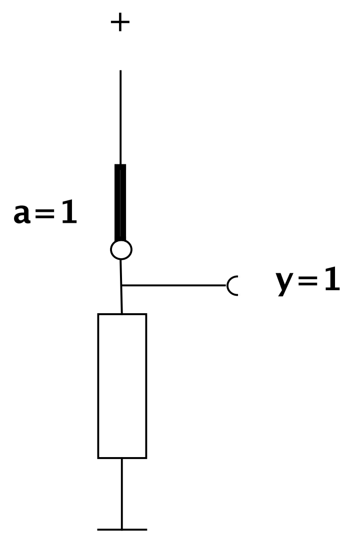
George Boole 1815 - 1864

Schalter a offen



Ausgang y ohne Spannung

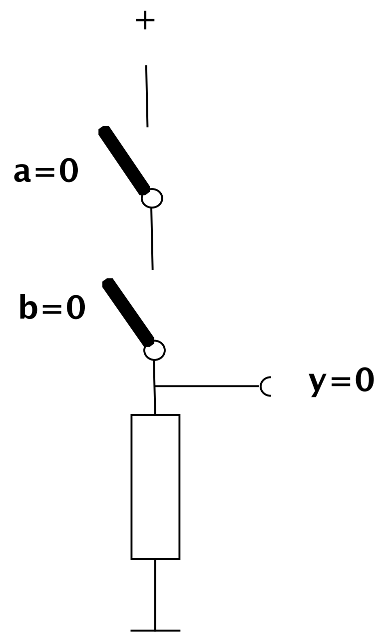
Schalter a geschlossen



Ausgang y unter Spannung

Schalter a offen

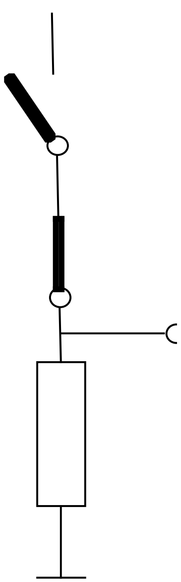
Schalter b offen



Ausgang y
ohne Spannung

Schalter a offen

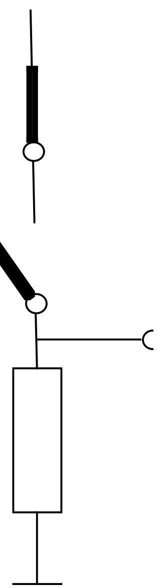
Schalter b geschlossen



Ausgang y ohne Spannung

Schalter a geschlossen

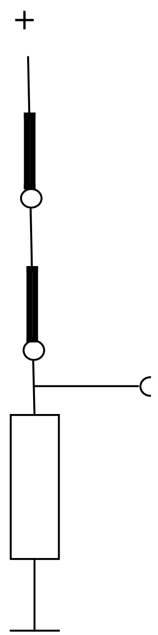
Schalter b offen



Ausgang y ohne Spannung

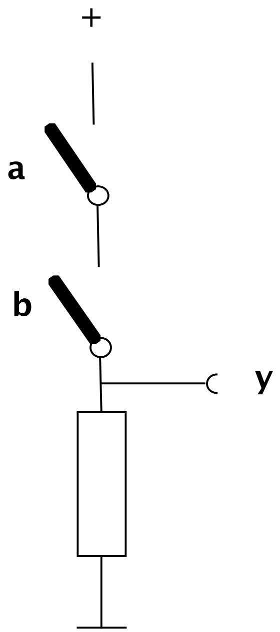
Schalter a geschlossen

Schalter b geschlossen



Ausgang y unter Spannung

Konjunktion (and)



Serienschaltung

$$y = a \wedge b$$

a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

$0 \wedge 0 = 0$

$0 \wedge 1 = 0$

$1 \wedge 0 = 0$

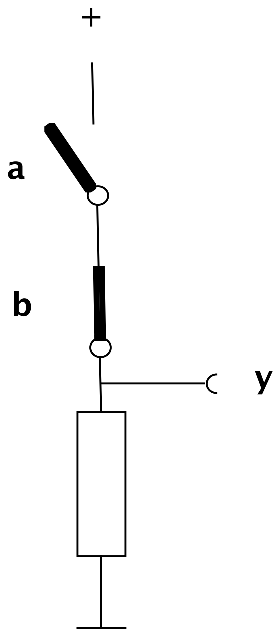
$1 \wedge 1 = 1$

$x \wedge 0 = 0$

$x \wedge 1 = x$ (neutrales Element 1)

$x \wedge x = x$ (Idempotenz)

Konjunktion (and)



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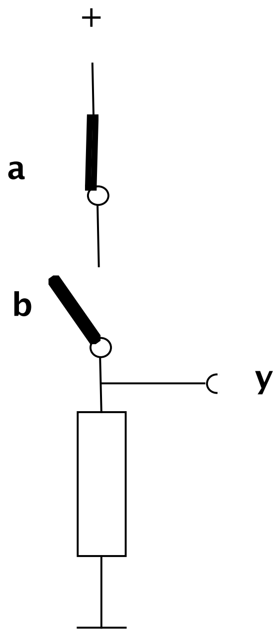
Konjunktion (and)



Serienschaltung

$$y = a \wedge b$$

a	b	y
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0	1	0
1	**0**	**0**
1	1	1

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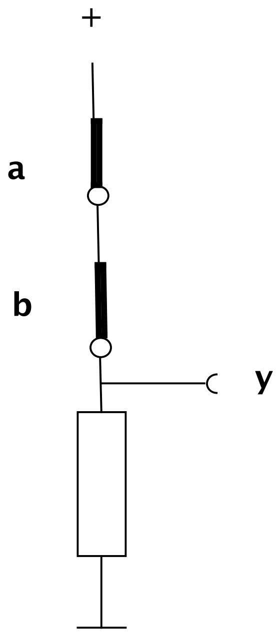
$x \wedge x = x$ (Idempotenz)

Konjunktion (and)



Serienschaltung

$$y = a \wedge b$$

a	b	y
0	0	0
0	1	0
1	0	0
1	**1**	**1**

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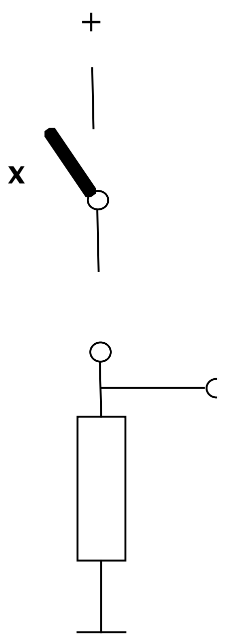
Konjunktion (and)



Serienschaltung

$$y = a \wedge b$$

a	b	y
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0	1	0
1	0	0
1	1	1

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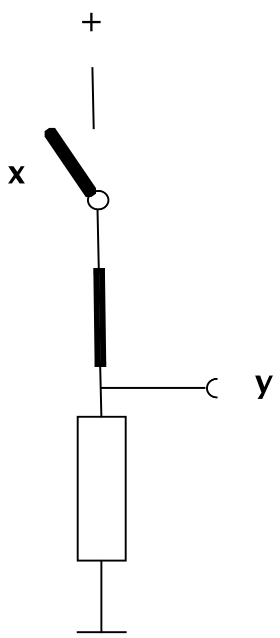
Konjunktion (and)



Serienschaltung

$$y = a \wedge b$$

a	b	y
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0	1	0
1	0	0
1	1	1

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$0 \wedge 1 = 0$

$1 \wedge 0 = 0$

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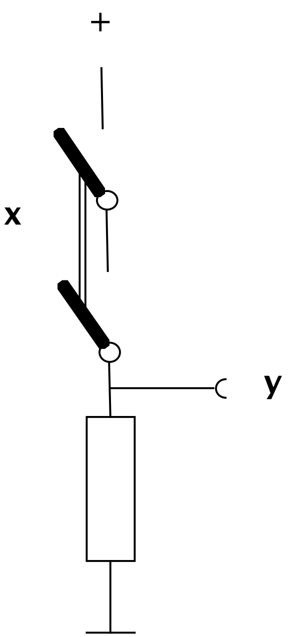
$x \wedge x = x$ (Idempotenz)

Konjunktion (and)



Serienschaltung

$$y = a \wedge b$$

a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

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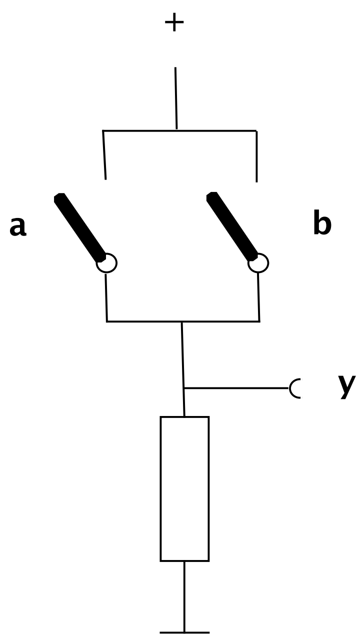
$1 \wedge 1 = 1$

$x \wedge 0 = 0$

$x \wedge 1 = x$ (neutrales Element 1)

$x \wedge x = x$ (Idempotenz)

Disjunktion (or)



Parallelschaltung

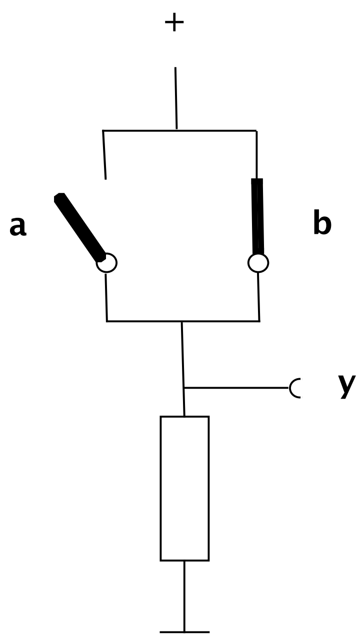
$$y = a \lor b$$

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$0 \lor 0 = 0$
$0 \lor 1 = 1$
$1 \lor 0 = 1$
$1 \lor 1 = 1$

$x \lor 0 = x$ (neutrales Element 0)
$x \lor 1 = 1$
$x \lor x = x$ (Idempotenz)

Disjunktion (or)



Parallelschaltung

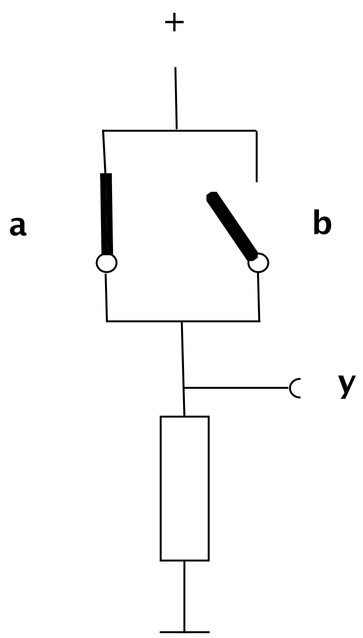
$$y = a \vee b$$

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$0 \vee 0 = 0$
$0 \vee 1 = 1$
$1 \vee 0 = 1$
$1 \vee 1 = 1$

$x \vee 0 = x$ (neutrales Element 0)
$x \vee 1 = 1$
$x \vee x = x$ (Idempotenz)

Disjunktion (or)



Parallelschaltung

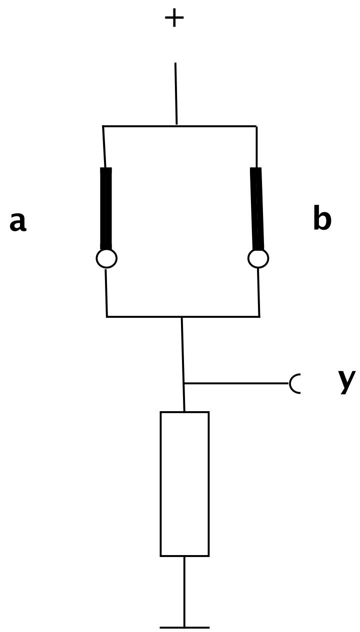
$$y = a \vee b$$

a	b	y
0	0	0
0	1	1
1	**0**	**1**
1	1	1

$0 \vee 0 = 0$
$0 \vee 1 = 1$
$\mathbf{1 \vee 0 = 1}$
$1 \vee 1 = 1$

$x \vee 0 = x$ (neutrales Element 0)
$x \vee 1 = 1$
$x \vee x = x$ (Idempotenz)

Disjunktion (or)



Parallelschaltung

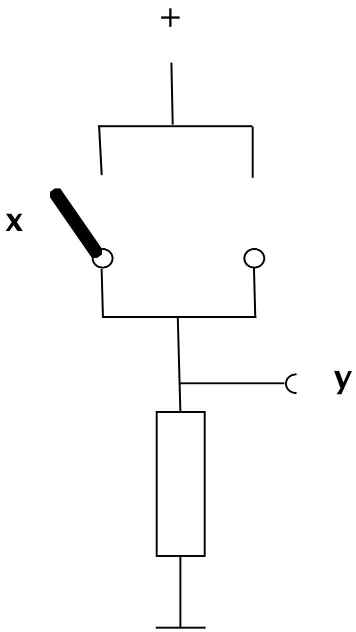
$$y = a \vee b$$

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$0 \vee 0 = 0$
$0 \vee 1 = 1$
$1 \vee 0 = 1$
$1 \vee 1 = 1$

$x \vee 0 = x$ (neutrales Element 0)
$x \vee 1 = 1$
$x \vee x = x$ (Idempotenz)

Disjunktion (or)



Parallelschaltung

$$y = a \vee b$$

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$\underline{0} \vee \underline{0 = 0}$

$\underline{0} \vee \underline{1 = 1}$

$\underline{1} \vee \underline{0 = 1}$

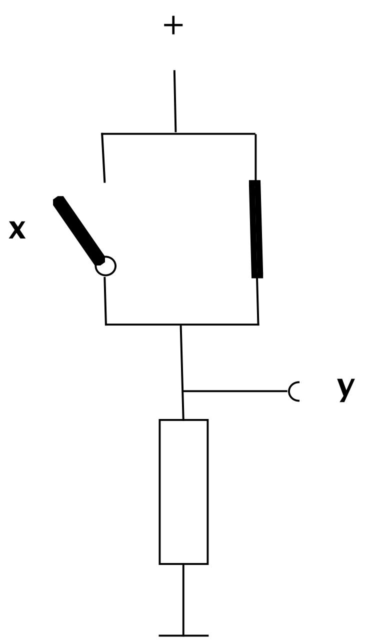
$\underline{1} \vee \underline{1 = 1}$

$x \vee 0 = x$ **(neutrales Element 0)**

$\underline{x} \vee \underline{1 = 1}$

$\underline{x} \vee \underline{x = x}$ **<u>(Idempotenz)</u>**

Disjunktion (or)



Parallelschaltung

$$y = a \vee b$$

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$0 \vee 0 = 0$

$0 \vee 1 = 1$

$1 \vee 0 = 1$

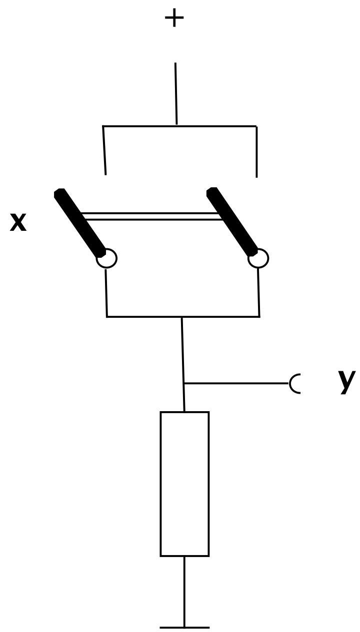
$1 \vee 1 = 1$

$x \vee 0 = x$ (neutrales Element 0)

$x \vee 1 = 1$

$x \vee x = x$ (Idempotenz)

Disjunktion (or)



Parallelschaltung

$$y = a \vee b$$

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$0 \vee 0 = 0$

$0 \vee 1 = 1$

$1 \vee 0 = 1$

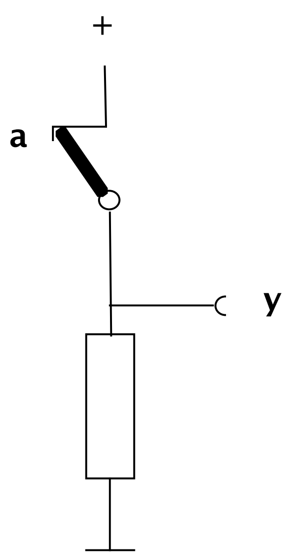
$1 \vee 1 = 1$

$x \vee 0 = x$ (neutrales Element 0)

$x \vee 1 = 1$

$x \vee x = x$ (Idempotenz)

Negation (not)



Ruhekontakt

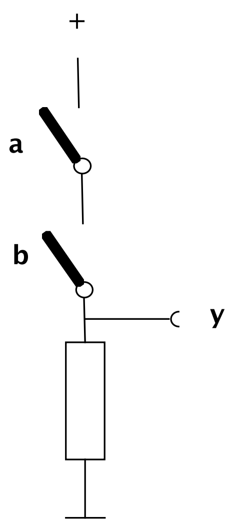
$$y = \neg a$$

a	y
0	1
1	0

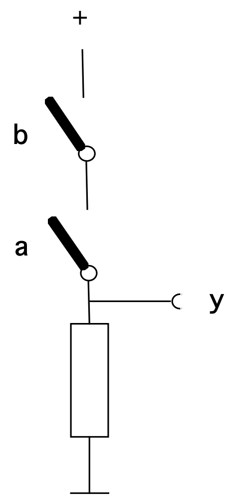
$\neg 0 = 1$

$\neg 1 = 0$

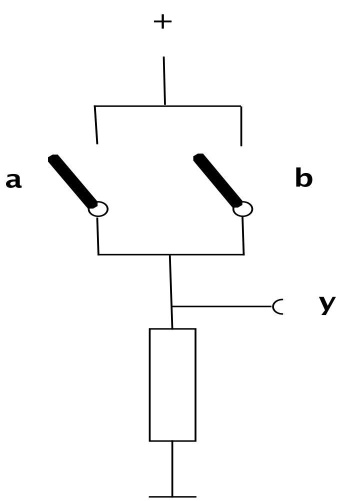
Kommutatives Gesetz der Konjunktion



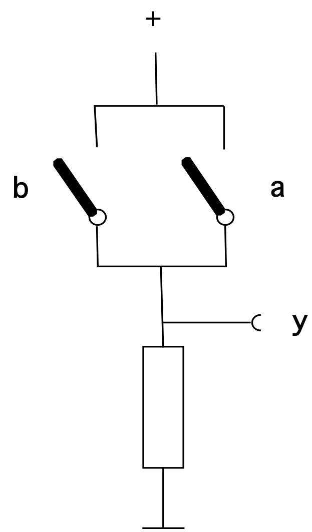
$$a \wedge b = b \wedge a$$



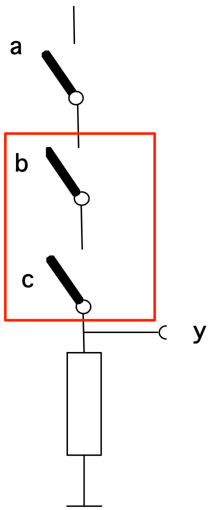
Kommutatives Gesetz der Disjunktion



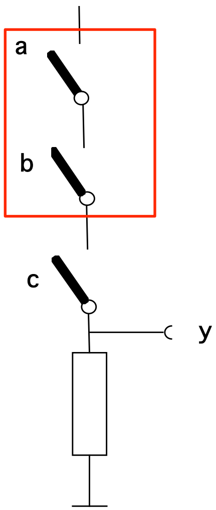
$$a \vee b = b \vee a$$



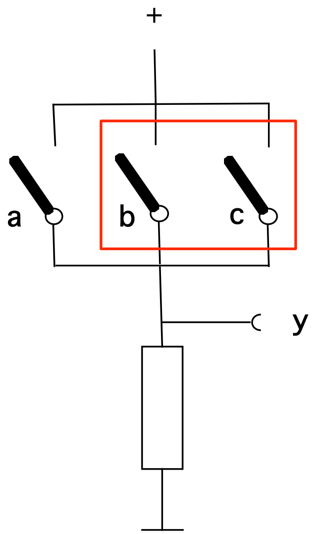
Assoziatives Gesetz der Konjunktion



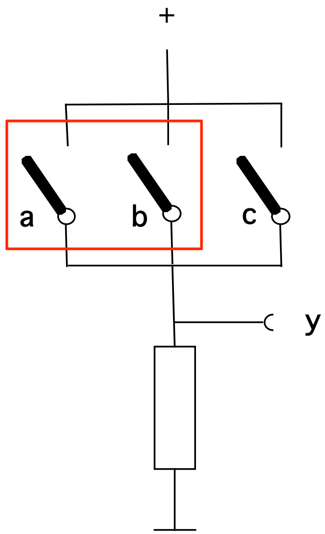
$$a \wedge (b \wedge c) = (a \wedge b) \wedge c$$



Assoziatives Gesetz der Disjunktion



$$a \vee (b \vee c) = (a \vee b) \vee c$$



Distributives Gesetz

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$
$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

Identitätsgesetz

$$a \wedge 1 = a$$

$$a \vee 0 = a$$

Null-/Einsgesetz

$$a \wedge 0 = 0$$

$$a \vee 1 = 1$$

Komplementärgesetz

$$a \wedge \neg a = 0$$
$$a \vee \neg a = 1$$

Idempotenzgesetz

$$a \wedge a = a$$
$$a \vee a = a$$

Gesetz der Verschmelzung

$$a \wedge (a \vee b) = a$$
$$a \vee (a \wedge b) = a$$

Doppeltes Negationsgesetz

$$\neg(\neg a) = a$$

DeMorgan'sches Gesetz

$$\neg(a \wedge b) = \neg a \vee \neg b$$
$$\neg(a \vee b) = \neg a \wedge \neg b$$

DeMorgan’sches Gesetz $\neg(a \wedge b) = \neg a \vee \neg b$

a	b	$a \wedge b$	$\neg(a \wedge b)$	$\neg a$	$\neg b$	$\neg a \vee \neg b$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

DeMorgan'sches Gesetz $\neg(a \vee b) = \neg a \wedge \neg b$

a	b	$a \vee b$	$\neg(a \vee b)$	$\neg a$	$\neg b$	$\neg a \wedge \neg b$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Boole'sche Algebra

Eine Boole'sche Algebra ist ein abgeschlossenes System, in dem zwei Operationen definiert sind, für die

- **kommutatives,**
- **assoziatives,**
- **distributives** und
- **Verschmelzungsgesetz**

gelten und in dem ein

- **Nullelement,** ein
- **Einselement** und zu jedem Element ein
- **Komplement**

existiert.

Beispiele für Boole'sche Algebren

- Mengenlehre
- Aussagenlogik
- Schaltalgebra

Mengenlehre

- Durchschnitt zweier Mengen a ∩ b (entspricht a ∧ b)
- Vereinigung zweier Mengen a ∪ b (entspricht a ∨ b)
- Komplement einer Menge a (entspricht ¬a)
- Leere Menge (entspricht 0)
- Gesamtmenge (entspricht 1)

Aussagenlogik

- Konjunktion zweier Aussagen (entspricht $a \wedge b$)
- Disjunktion zweier Aussagen (entspricht $a \vee b$)
- Negation einer Aussage (entspricht $\neg a$)
- Kontradiktion (entspricht 0)
- Tautologie (entspricht 1)

Schaltalgebra

- **Serienschaltung zweier Schalter** (entspricht $a \wedge b$)
- **Parallelschaltung zweier Schalter** (entspricht $a \vee b$)
- **Schalter mit Ruhekontakt** (entspricht $\neg a$)
- **Permanente Unterbrechung** (entspricht 0)
- **Permanente Verbindung** (entspricht 1)

Antivalenz $y = a \not\equiv b$

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

$0 \not\equiv 0 = 0$

$0 \not\equiv 1 = 1$

$1 \not\equiv 0 = 1$

$1 \not\equiv 1 = 0$

$x \not\equiv 0 = x$

$x \not\equiv 1 = \neg x$

$x \not\equiv x = 0$

Alle Boole'schen Funktionen mit zwei Parametern

a 0011 b 0101	
0000	$y_0 = 0$
0001	$y_1 = a \wedge b$
0010	$y_2 = a \wedge \neg b$
0011	$y_3 = a$
0100	$y_4 = \neg a \wedge b$
0101	$y_5 = b$
0110	$y_6 = a \not\equiv b$
0111	$y_7 = a \vee b$

a 0011 b 0101	
1111	$y_{15} = 1$
1110	$y_{14} = \neg a \vee \neg b$
1101	$y_{13} = \neg a \vee b$
1100	$y_{12} = \neg a$
1011	$y_{11} = a \vee \neg b$
1010	$y_{10} = \neg b$
1001	$y_9 = a \equiv b$
1000	$y_8 = \neg a \wedge \neg b$

Spezielle Boole'sche Funktionen mit zwei Parametern

Peirce-Funktion Nor

$$y_8 = \neg a \wedge \neg b = \neg(a \vee b)$$

Spezielle Boole'sche Funktionen mit zwei Parametern

Sheffer-Funktion Nand

$$y_{14} = \neg a \vee \neg b = \neg(a \wedge b)$$

Spezielle Boole'sche Funktionen mit zwei Parametern

Implikation

$$y_{11} = a \vee \neg b = b \Rightarrow a$$

$$y_{13} = \neg a \vee b = a \Rightarrow b$$

Implikation

$a \Rightarrow b$

“a impliziert b”

“aus a folgt b”

“wenn a gilt dann gilt auch b”

“a ist hinreichend für b”

“b ist notwendig für a”

$\neg a \vee b$

a ... Prämisse b ... Conclusio

$(a \Rightarrow b) \wedge (b \Rightarrow a) = (a \Leftrightarrow b)$

$(0 \Rightarrow x) = 1$ "ex falso quodlibet"

Spezielle Boole’sche Funktionen mit zwei Parametern

Kontradiktion

$$y_0 = 0$$

Tautologie

$$y_{15} = 1$$

Komplementäre Funktionen

$$f(x,y) = \neg g(x,y)$$

"f und g sind zueinaner komplementär"

Beispiele für komplementäre Funktionen:

$\wedge$, **nand**

$\vee$, **nor**

$\equiv$, $\not\equiv$

0, 1

Dualität

$$f(x,y) = \neg g(\neg x, \neg y)$$

"f und g sind zueinaner dual"

Beispiele für duale Funktionen:

$\wedge, \vee$

nand, nor

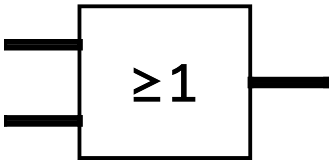
$\equiv, \not\equiv$

Dualität

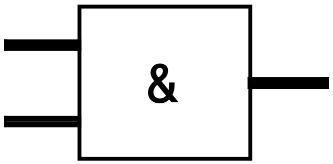
$$a \wedge b = \neg(\neg a \vee \neg b)$$

a	b	a ∧ b	¬a	¬b	¬a ∨ ¬b	¬(¬a ∨ ¬b)
0	0	0	1	1	1	0
0	1	0	1	0	1	0
1	0	0	0	1	1	0
1	1	1	0	0	0	1

Gatter

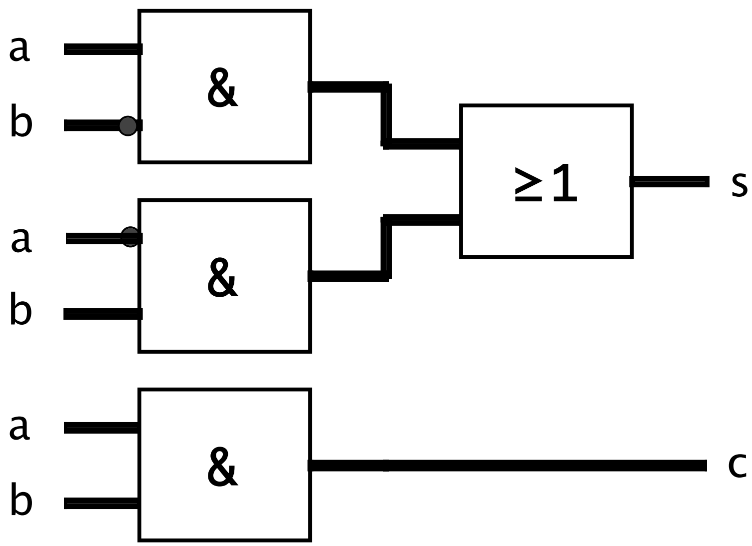


Oder-Gatter



Und-Gatter

Halbaddierwerk



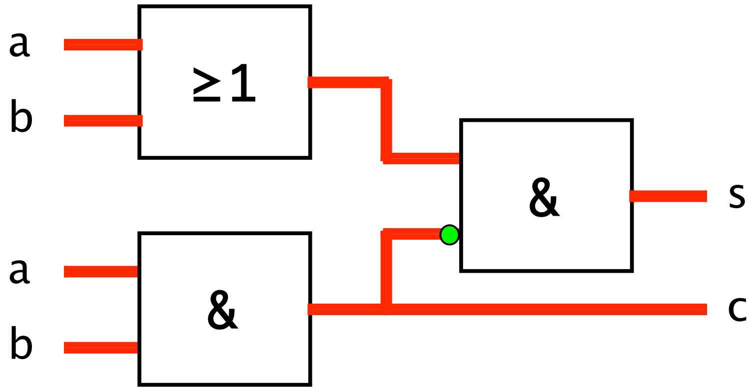
a	b	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$s = (\neg a \wedge b) \vee (a \wedge \neg b)$$
$$c = (a \wedge b)$$

Halbaddierwerk

$$
\begin{aligned}
s &= (\neg a \wedge b) \vee (a \wedge \neg b) = \\
&= ((\neg a \wedge b) \vee a) \wedge ((\neg a \wedge b) \vee \neg b) = \\
&= (\neg a \vee a) \wedge (b \vee a) \wedge (\neg a \vee \neg b) \wedge (b \vee \neg b) = \\
&= (b \vee a) \wedge (\neg a \vee \neg b) = \\
&= (a \vee b) \wedge \neg (a \wedge b) \\
c &= (a \wedge b)
\end{aligned}
$$

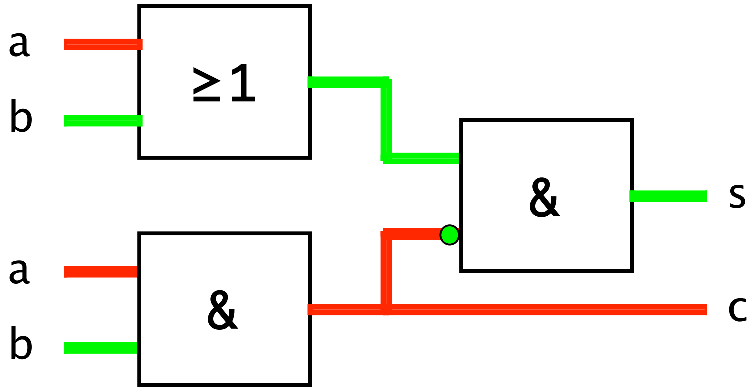
Halbaddierwerk



a	b	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$s = (a \vee b) \wedge \neg(a \wedge b)$
$c = (a \wedge b)$

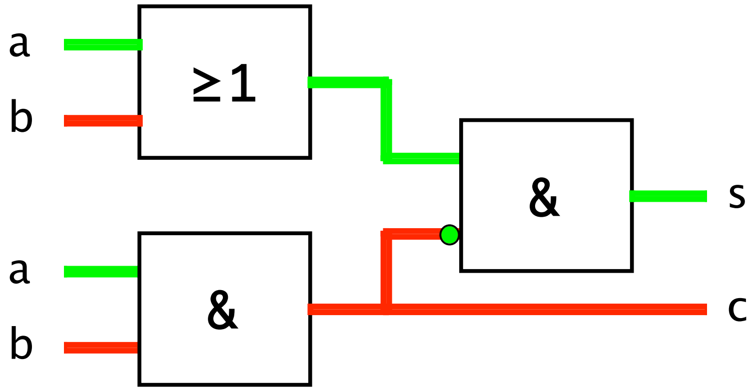
Halbaddierwerk



a	b	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$s = (a \lor b) \land \neg(a \land b)$$
$$c = (a \land b)$$

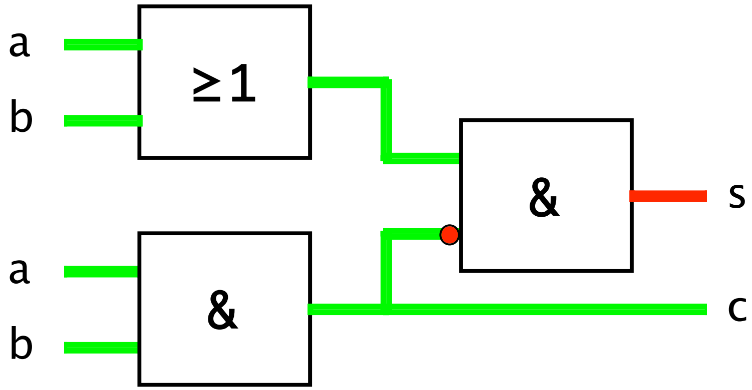
Halbaddierwerk



a	b	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$s = (a \vee b) \wedge \neg(a \wedge b)$
$c = (a \wedge b)$

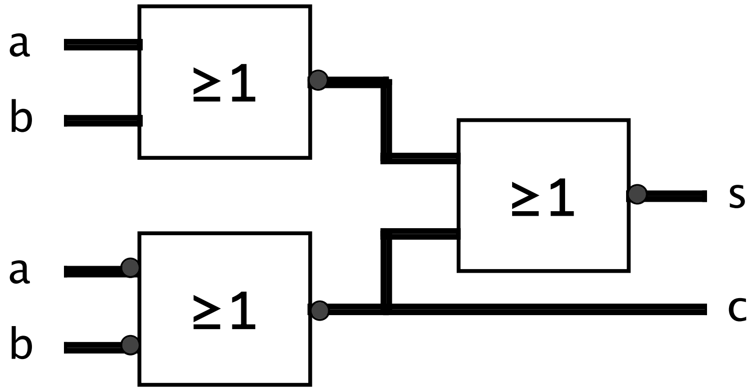
Halbaddierwerk



a	b	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$s = (a \vee b) \wedge \neg(a \wedge b)$$
$$c = (a \wedge b)$$

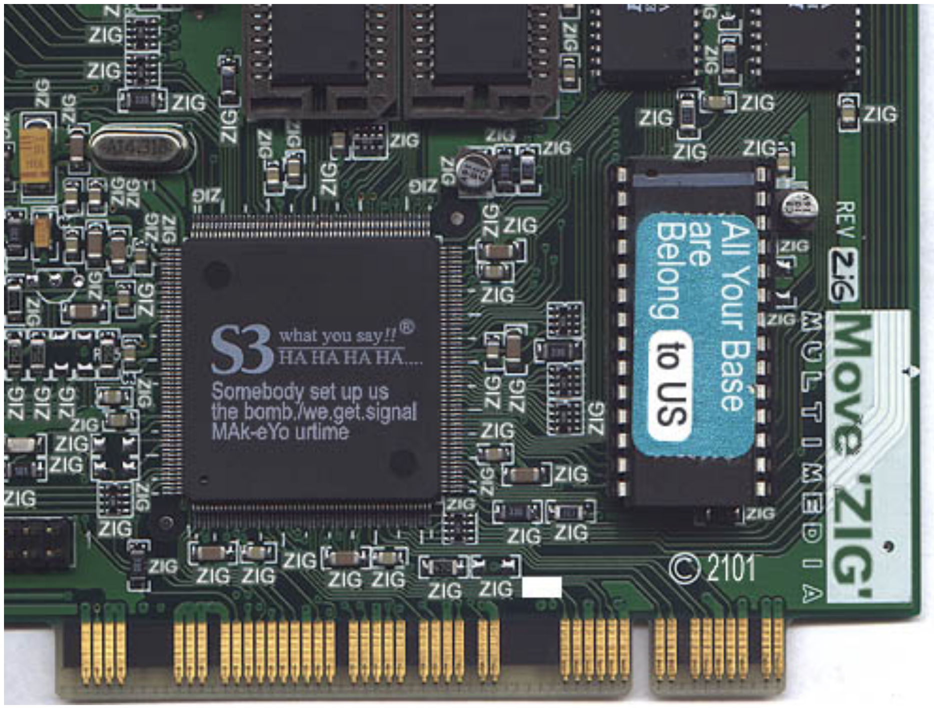
Halbaddierwerk



a	b	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$$s = \neg(\neg(a \vee b) \vee \neg(\neg a \vee \neg b))$$
$$c = \neg(\neg a \vee \neg b)$$

Prozessor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