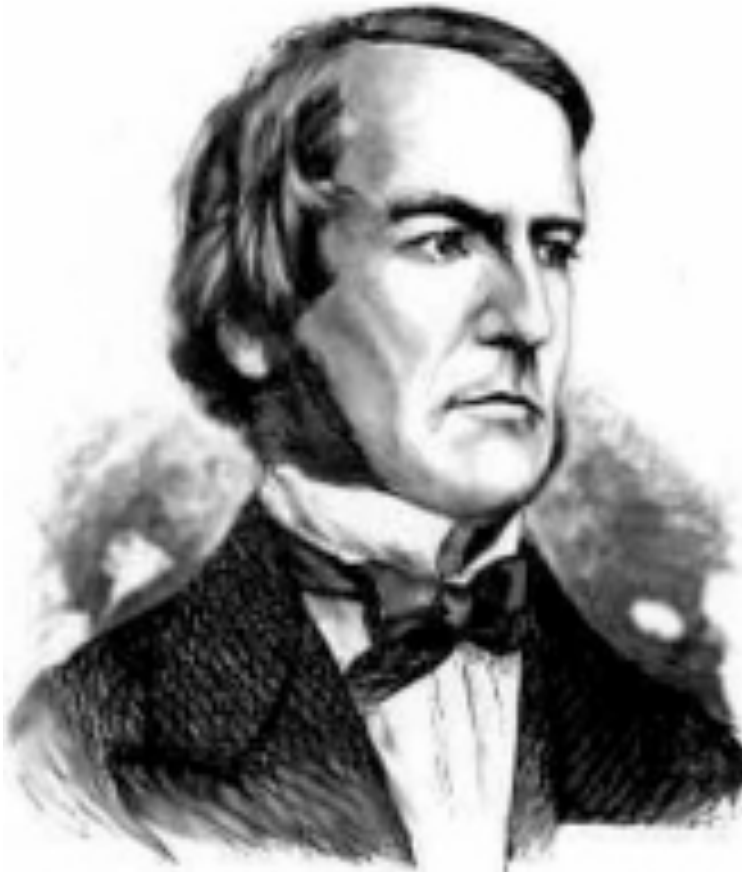




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## Boole'sche Algebra

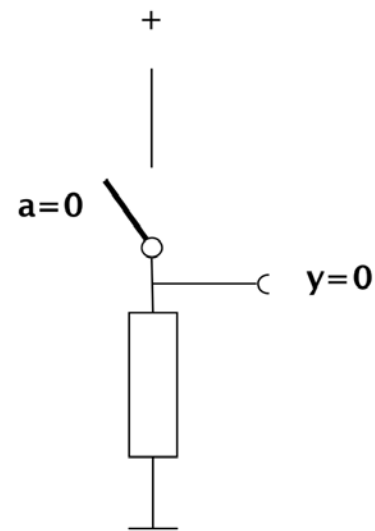
George Boole 1815 - 1864



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**Schalter a offen**



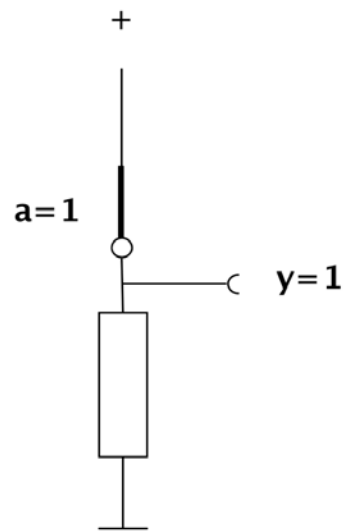
**Ausgang y  
ohne Spannung**



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**Schalter a  
geschlossen**



**Ausgang y  
unter Spannung**

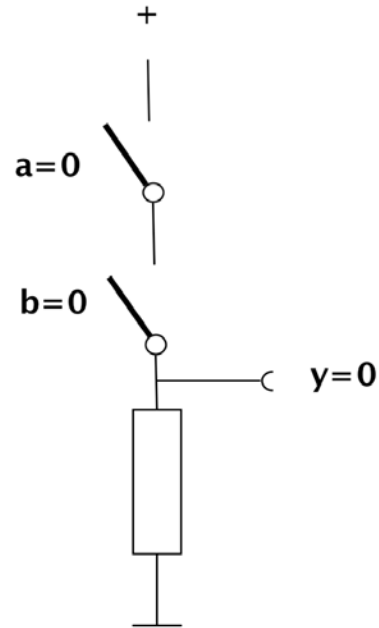


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**Schalter a offen**

**Schalter b offen**



**Ausgang y  
ohne Spannung**

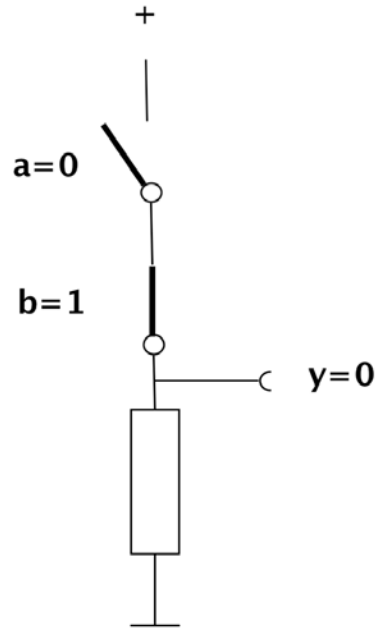


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**Schalter a offen**

**Schalter b  
geschlossen**



**Ausgang y  
ohne Spannung**

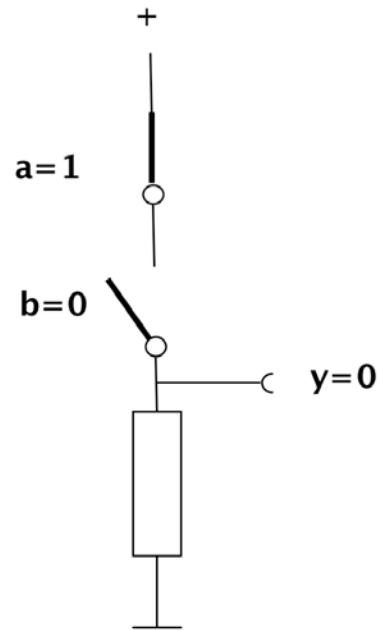


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**Schalter a  
geschlossen**

**Schalter b  
offen**



**Ausgang y  
ohne Spannung**

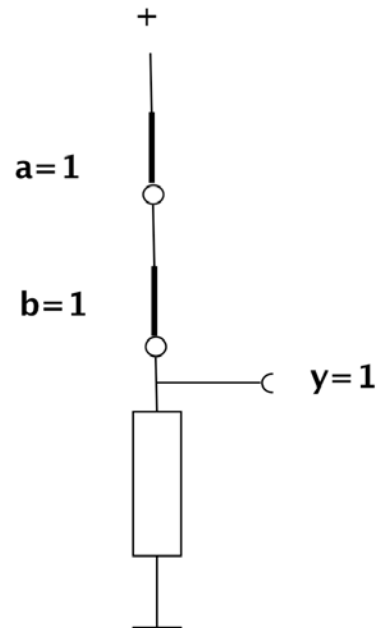


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**Schalter a  
geschlossen**

**Schalter b  
geschlossen**

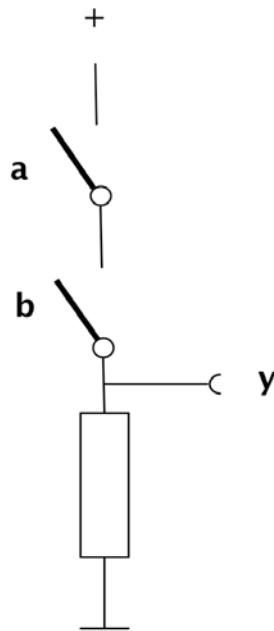


**Ausgang y  
unter Spannung**



## Konjunktion (and)

$$y = a \wedge b$$



**Serienschaltung**

<b>a</b>	<b>b</b>	<b>y</b>	
<b>0</b>	<b>0</b>	<b>0</b>	<b><math>0 \wedge 0 = 0</math></b>
<b>0</b>	<b>1</b>	<b>0</b>	<u><math>0 \wedge 1 = 0</math></u>
<b>1</b>	<b>0</b>	<b>0</b>	<u><math>1 \wedge 0 = 0</math></u>
<b>1</b>	<b>1</b>	<b>1</b>	<u><math>1 \wedge 1 = 1</math></u>
			<u><math>x \wedge 0 = 0</math></u>

$x \wedge 1 = x$  (neutrales Element 1)

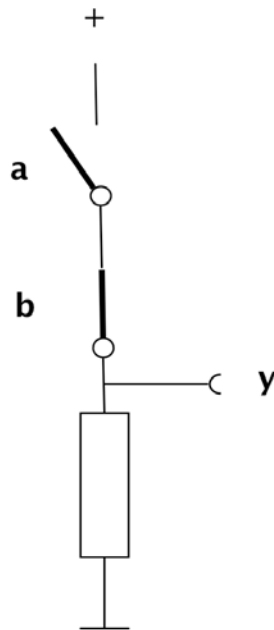
$x \wedge x = x$  (Idempotenz)





## Konjunktion (and)

$$y = a \wedge b$$



**Serienschaltung**

<b>a</b>	<b>b</b>	<b>y</b>	
<b>0</b>	<b>0</b>	<b>0</b>	<u><math>0 \wedge 0 = 0</math></u>
<b>0</b>	<b>1</b>	<b>0</b>	<u><math>0 \wedge 1 = 0</math></u>
<b>0</b>	<b>1</b>	<b>0</b>	<u><math>1 \wedge 0 = 0</math></u>
<b>1</b>	<b>0</b>	<b>0</b>	<u><math>1 \wedge 1 = 1</math></u>
<b>1</b>	<b>1</b>	<b>1</b>	

$$\underline{x \wedge 0 = 0}$$

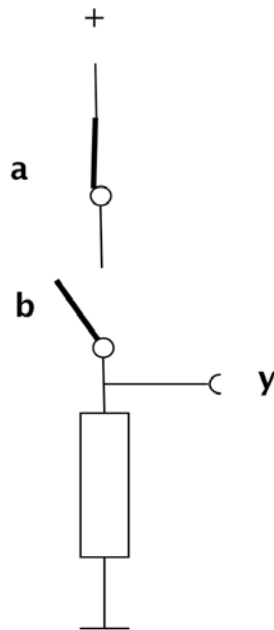
$$\underline{x \wedge 1 = x} \text{ (neutrales Element 1)}$$

$$\underline{x \wedge x = x} \text{ (Idempotenz)}$$



## Konjunktion (and)

$$y = a \wedge b$$



**Serienschaltung**

<b>a</b>	<b>b</b>	<b>y</b>	
<b>0</b>	<b>0</b>	<b>0</b>	<u><math>0 \wedge 0 = 0</math></u>
<b>0</b>	<b>1</b>	<b>0</b>	<u><math>0 \wedge 1 = 0</math></u>
<b>0</b>	<b>1</b>	<b>0</b>	<u><math>1 \wedge 0 = 0</math></u>
<b>1</b>	<b>0</b>	<b>0</b>	<u><math>1 \wedge 1 = 1</math></u>
<b>1</b>	<b>1</b>	<b>1</b>	

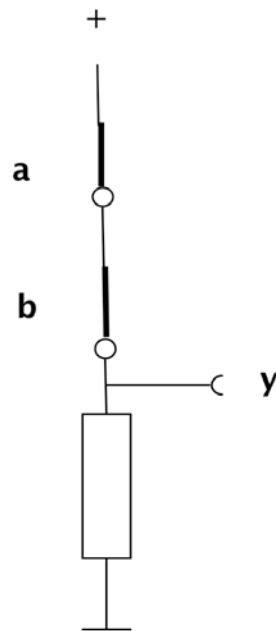
$$\underline{x \wedge 0 = 0}$$

$$\underline{x \wedge 1 = x} \text{ (neutrales Element 1)}$$

$$\underline{x \wedge x = x} \text{ (Idempotenz)}$$



## Konjunktion (and)



**Serienschaltung**

$$y = a \wedge b$$

<b>a</b>	<b>b</b>	<b>y</b>	
<b>0</b>	<b>0</b>	<b>0</b>	<u><math>0 \wedge 0 = 0</math></u>
<b>0</b>	<b>1</b>	<b>0</b>	<u><math>0 \wedge 1 = 0</math></u>
<b>0</b>	<b>1</b>	<b>0</b>	<u><math>1 \wedge 0 = 0</math></u>
<b>1</b>	<b>0</b>	<b>0</b>	<u><math>1 \wedge 1 = 1</math></u>
<b>1</b>	<b>1</b>	<b>1</b>	

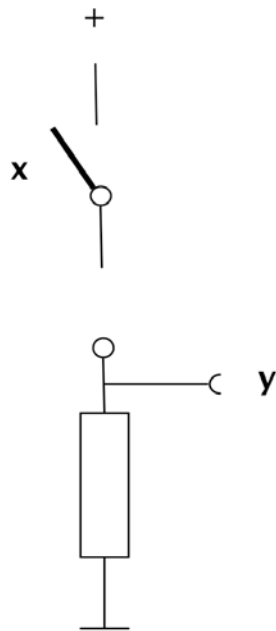
$$\underline{X \wedge 0 = 0}$$

$$\underline{X \wedge 1 = X} \text{ (neutrales Element 1)}$$

$$\underline{X \wedge X = X} \text{ (Idempotenz)}$$



## Konjunktion (and)



**Serienschaltung**

$$y = a \wedge b$$

a	b	y	
0	0	0	$0 \wedge 0 = 0$
0	1	0	$0 \wedge 1 = 0$
1	0	0	$1 \wedge 0 = 0$
1	1	1	$1 \wedge 1 = 1$

$$x \wedge 0 = 0$$

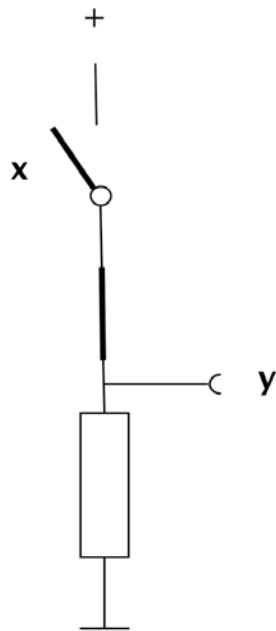
$$x \wedge 1 = x \text{ (neutrales Element 1)}$$

$$x \wedge x = x \text{ (Idempotenz)}$$



## Konjunktion (and)

$$y = a \wedge b$$



**Serienschaltung**

<b>a</b>	<b>b</b>	<b>y</b>
<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>1</b>	<b>0</b>
<b>1</b>	<b>0</b>	<b>0</b>
<b>1</b>	<b>1</b>	<b>1</b>

$$\underline{0 \wedge 0 = 0}$$

$$\underline{0 \wedge 1 = 0}$$

$$\underline{1 \wedge 0 = 0}$$

$$\underline{1 \wedge 1 = 1}$$

$$\underline{x \wedge 0 = 0}$$

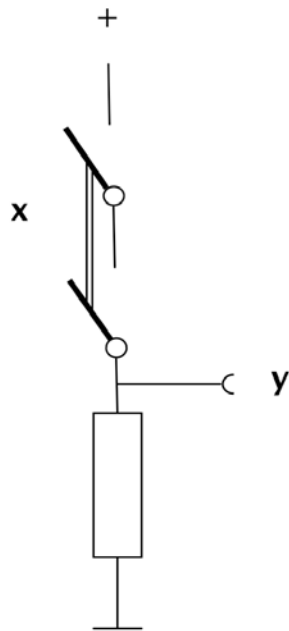
$$\underline{x \wedge 1 = x \text{ (neutrales Element 1)}}$$

$$\underline{x \wedge x = x \text{ (Idempotenz)}}$$



## Konjunktion (and)

$$y = a \wedge b$$



**Serienschaltung**

a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

$$\underline{0 \wedge 0 = 0}$$

$$\underline{0 \wedge 1 = 0}$$

$$\underline{1 \wedge 0 = 0}$$

$$\underline{1 \wedge 1 = 1}$$

$$\underline{x \wedge 0 = 0}$$

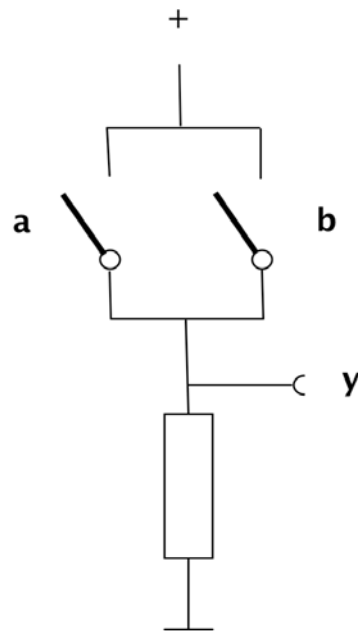
$$\underline{x \wedge 1 = x} \text{ (neutrales Element 1)}$$

$$\underline{x \wedge x = x} \text{ (Idempotenz)}$$



## Disjunktion (or)

$$y = a \vee b$$



Parallelschaltung

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$$0 \vee 0 = 0$$

$$0 \vee 1 = 1$$

$$1 \vee 0 = 1$$

$$1 \vee 1 = 1$$

$$X \vee 0 = X \text{ (neutrales Element 0)}$$

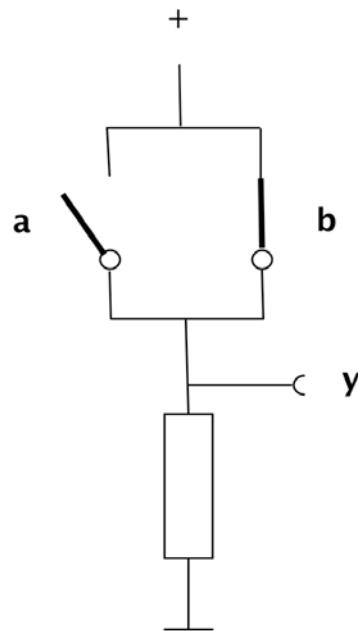
$$X \vee 1 = 1$$

$$X \vee X = X \text{ (Idempotenz)}$$



## Disjunktion (or)

$$y = a \vee b$$



Parallelschaltung

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$$\underline{0 \vee 0 = 0}$$

$$\underline{0 \vee 1 = 1}$$

$$\underline{1 \vee 0 = 1}$$

$$\underline{1 \vee 1 = 1}$$

$$\underline{X \vee 0 = X \text{ (neutrales Element 0)}}$$

$$\underline{X \vee 1 = 1}$$

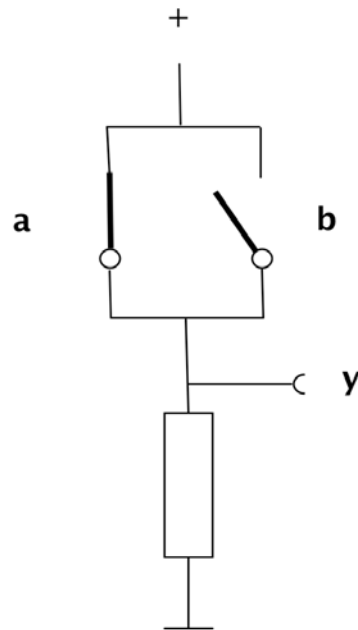
$$\underline{X \vee X = X \text{ (Idempotenz)}}$$





## Disjunktion (or)

$$y = a \vee b$$



Parallelschaltung

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$$0 \vee 0 = 0$$

$$0 \vee 1 = 1$$

$$1 \vee 0 = 1$$

$$1 \vee 1 = 1$$

$$X \vee 0 = X \text{ (neutrales Element 0)}$$

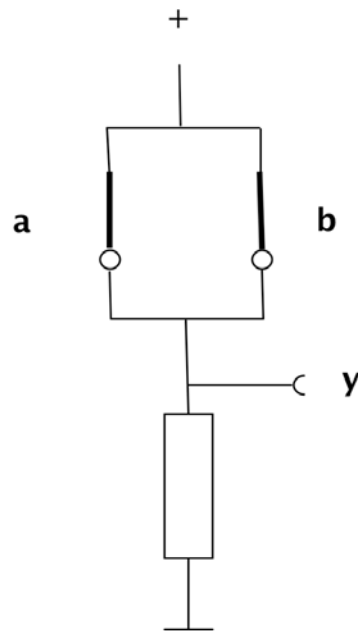
$$X \vee 1 = 1$$

$$X \vee X = X \text{ (Idempotenz)}$$



## Disjunktion (or)

$$y = a \vee b$$



Parallelschaltung

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$$0 \vee 0 = 0$$

$$0 \vee 1 = 1$$

$$1 \vee 0 = 1$$

$$1 \vee 1 = 1$$

$$X \vee 0 = X \text{ (neutrales Element 0)}$$

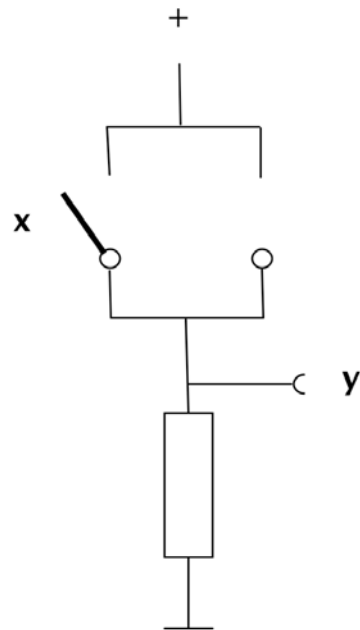
$$X \vee 1 = 1$$

$$X \vee X = X \text{ (Idempotenz)}$$



## Disjunktion (or)

$$y = a \vee b$$



Parallelschaltung

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$$\underline{0 \vee 0 = 0}$$

$$\underline{0 \vee 1 = 1}$$

$$\underline{1 \vee 0 = 1}$$

$$\underline{1 \vee 1 = 1}$$

$$\mathbf{X \vee 0 = X}$$
 (neutrales Element 0)

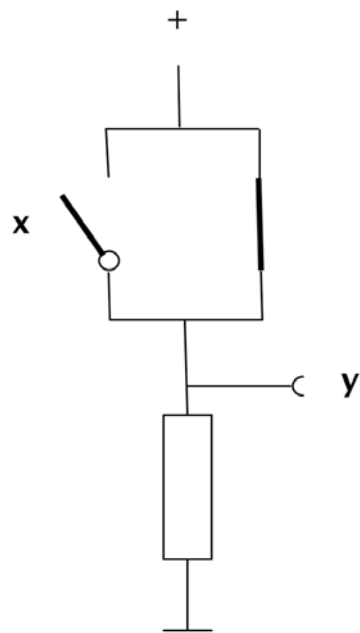
$$\underline{X \vee 1 = 1}$$

$$\underline{X \vee X = X}$$
 (Idempotenz)



## Disjunktion (or)

$$y = a \vee b$$



Parallelschaltung

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$$\underline{0 \vee 0 = 0}$$

$$\underline{0 \vee 1 = 1}$$

$$\underline{1 \vee 0 = 1}$$

$$\underline{1 \vee 1 = 1}$$

$$\underline{X \vee 0 = X \text{ (neutrales Element 0)}}$$

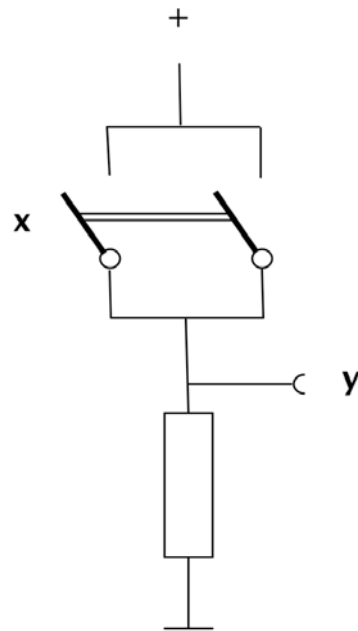
$$\underline{X \vee 1 = 1}$$

$$\underline{X \vee X = X \text{ (Idempotenz)}}$$



## Disjunktion (or)

$$y = a \vee b$$



Parallelschaltung

a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$$0 \vee 0 = 0$$

$$0 \vee 1 = 1$$

$$1 \vee 0 = 1$$

$$1 \vee 1 = 1$$

$$X \vee 0 = X \text{ (neutrales Element 0)}$$

$$X \vee 1 = 1$$

$$X \vee X = X \text{ (Idempotenz)}$$

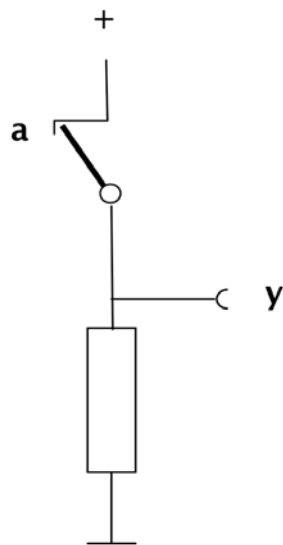


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## Negation (not)

$$y = \neg a$$



Ruhekontakt

a	y	
0	1	$\neg 0 = 1$
1	0	$\neg 1 = 0$



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## Kommutatives Gesetz

$$a \wedge b = b \wedge a$$

$$a \vee b = b \vee a$$



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## Assoziatives Gesetz

$$\mathbf{a \wedge (b \wedge c) = (a \wedge b) \wedge c}$$

$$\mathbf{a \vee (b \vee c) = (a \vee b) \vee c}$$





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## Distributives Gesetz

$$\mathbf{a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)}$$

$$\mathbf{a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)}$$



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## Gesetz der Verschmelzung

$$a \wedge (a \vee b) = a$$

$$a \vee (a \wedge b) = a$$



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## DeMorgan'sches Gesetz

$$\neg(a \wedge b) = \neg a \vee \neg b$$

$$\neg(a \vee b) = \neg a \wedge \neg b$$



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## Antivalenz

$$y = a \neq b$$

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

$$0 \neq 0 = 0$$

$$0 \neq 1 = 1$$

$$1 \neq 0 = 1$$

$$1 \neq 1 = 0$$

$$x \neq 0 = x$$

$$x \neq 1 = \neg x$$

$$x \neq x = 0$$



## Alle Boole'schen Funktionen mit zwei Parametern

<b>a</b> 0011 <b>b</b> 0101	
<b>0000</b>	$y_0 = 0$
<b>0001</b>	$y_1 = a \wedge b$
<b>0010</b>	$y_2 = a \wedge \neg b$
<b>0011</b>	$y_3 = a$
<b>0100</b>	$y_4 = \neg a \wedge b$
<b>0101</b>	$y_5 = b$
<b>0110</b>	$y_6 = a \neq b$
<b>0111</b>	$y_7 = a \vee b$

<b>a</b> 0011 <b>b</b> 0101	
<b>1111</b>	$y_{15} = 1$
<b>1110</b>	$y_{14} = \neg a \vee \neg b$
<b>1101</b>	$y_{13} = \neg a \vee b$
<b>1100</b>	$y_{12} = \neg a$
<b>1011</b>	$y_{11} = a \vee \neg b$
<b>1010</b>	$y_{10} = \neg b$
<b>1001</b>	$y_9 = a \equiv b$
<b>1000</b>	$y_8 = \neg a \wedge \neg b$



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## Spezielle Boole'sche Funktionen mit zwei Parametern

### Peirce-Funktion Nor

$$y_8 = \neg a \wedge \neg b = \neg(a \vee b)$$



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## Spezielle Boole'sche Funktionen mit zwei Parametern

### Sheffer-Funktion Nand

$$y_{14} = \neg a \vee \neg b = \neg(a \wedge b)$$



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## Spezielle Boole'sche Funktionen mit zwei Parametern

### Implikation

$$y_{11} = a \vee \neg b = b \Rightarrow a$$

$$y_{13} = \neg a \vee b = a \Rightarrow b$$





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## Implikation

$$a \Rightarrow b$$

“a impliziert b”

“aus a folgt b”

“wenn a gilt dann gilt auch b”

“a ist hinreichend für b”

“b ist notwendig für a”

a ... Prämisse b ... Conclusio

$$(a \Rightarrow b) \wedge (b \Rightarrow a) = (a \Leftrightarrow b)$$

$$(0 \Rightarrow x) = 1 \text{ "ex falso quodlibet"}$$



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## Spezielle Boole'sche Funktionen mit zwei Parametern

**Kontradiktion**

$$y_0 = 0$$

**Tautologie**

$$y_{15} = 1$$



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## Dualität

$$\neg f(x,y) = g(\neg x, \neg y)$$

**Beispiele für duale Funktionen:**

$\wedge, \vee$

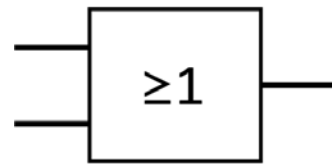
nand, nor



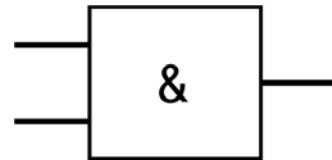
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## Gatter



**Oder-Gatter**



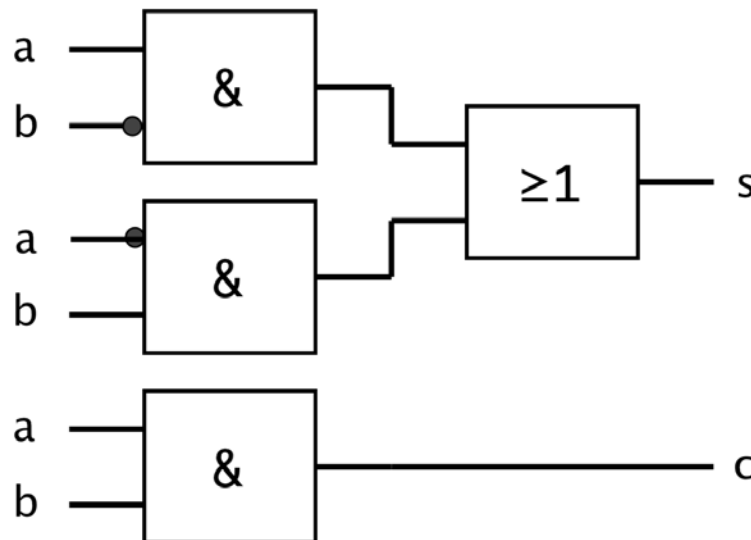
**Und-Gatter**



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## Halbaddierwerk



a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s = (a \wedge \neg b) \vee (\neg a \wedge b)$$

$$c = (a \wedge b)$$



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## Halbaddierwerk Umformung

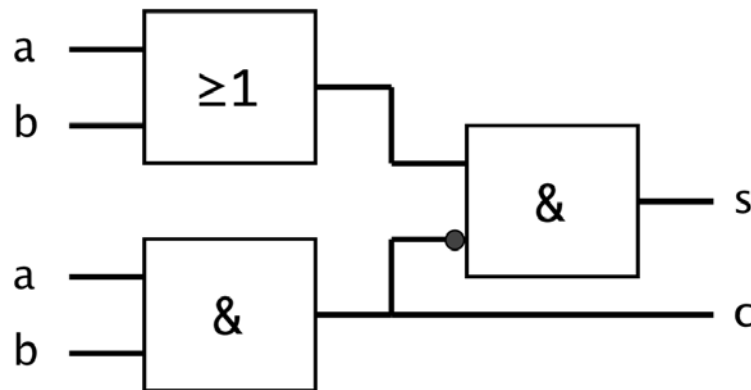
$$\begin{aligned} s &= (a \wedge \neg b) \vee (\neg a \wedge b) = \\ &= ((a \wedge \neg b) \vee \neg a) \wedge ((a \wedge \neg b) \vee b) = \\ &= (a \vee \neg a) \wedge (\neg b \vee \neg a) \wedge (a \vee b) \wedge (\neg b \vee b) = \\ &= (\neg b \vee \neg a) \wedge (a \vee b) = \\ &= (a \vee b) \wedge \neg (a \wedge b) \\ c &= (a \wedge b) \end{aligned}$$



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## Halbaddierwerk (optimiert)



a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s = (a \vee b) \wedge \neg(a \wedge b)$$

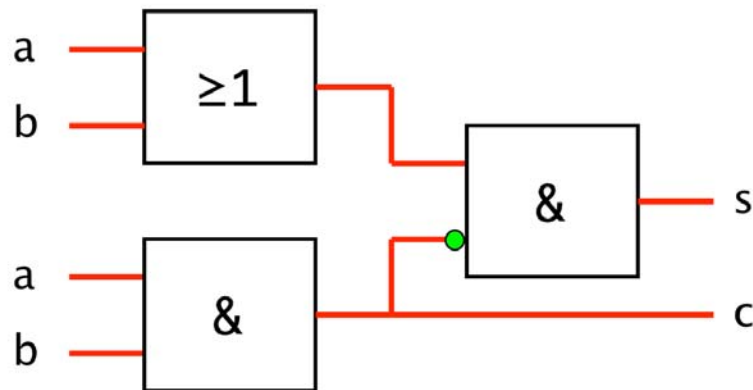
$$c = (a \wedge b)$$



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## Halbaddierwerk



a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s = (a \vee b) \wedge \neg(a \wedge b)$$

$$c = (a \wedge b)$$

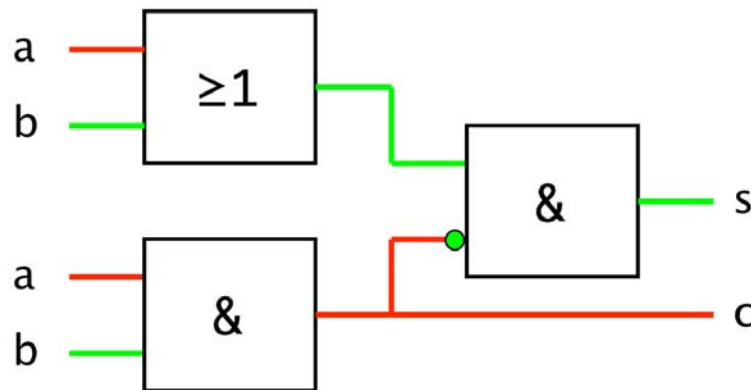




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## Halbaddierwerk



a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s = (a \vee b) \wedge \neg(a \wedge b)$$

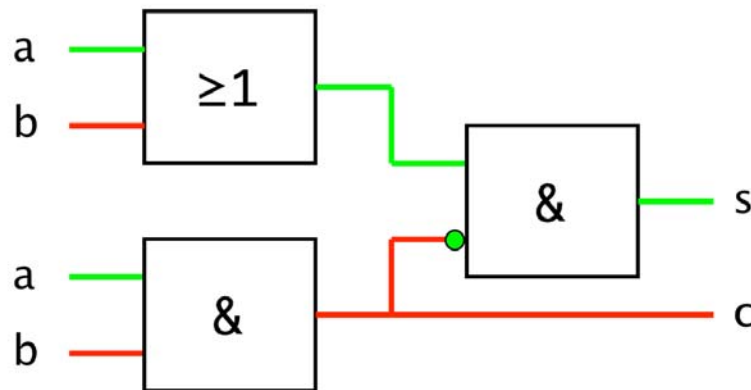
$$c = (a \wedge b)$$



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## Halbaddierwerk



a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s = (a \vee b) \wedge \neg(a \wedge b)$$

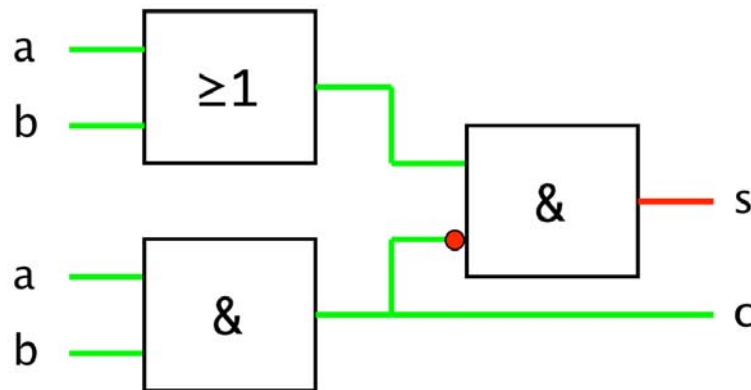
$$c = (a \wedge b)$$



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## Halbaddierwerk



a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s = (a \vee b) \wedge \neg(a \wedge b)$$

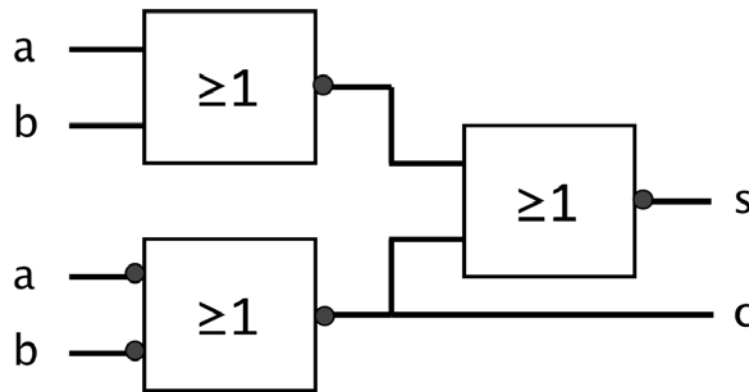
$$c = (a \wedge b)$$



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## Halbaddierwerk (aus NOR-Gattern)



a	b	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s = \neg(\neg(a \vee b) \vee \neg(\neg a \vee \neg b))$$

$$c = \neg(\neg a \vee \neg b)$$