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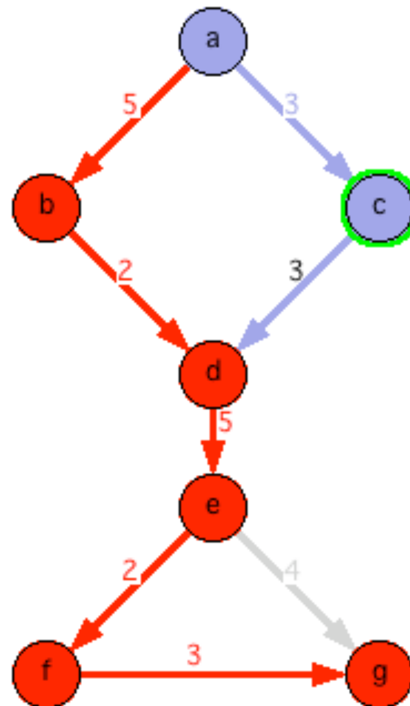

Graphen (1)

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Bezeichnungen

V	**Menge der Knoten (vertices)**
|V|	**Anzahl der Knoten**
E	**Menge der Kanten (edges)**
|E|	**Anzahl der Kanten**
G	**Graph G = (V,E)**
(v,w)	**Kante von v nach w**

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Eigenschaften

gerichtet (directed) — **ungerichtet (undirected)**
gewichtet (weighted) — **ungewichtet (unweighted)**
zyklisch (cyclic) — **zyklenfrei (acyclic)**

vollständig (complete)
bipartit (bipartite)

gerichtete Graphen:
- **schwach zusammenhängend (weakly connected)**
- **stark zusammenhängend (strongly connected)**

ungerichtete Graphen:
- **zusammenhängend (connected)**
- **zweifach zusammenhängend (biconnected)**

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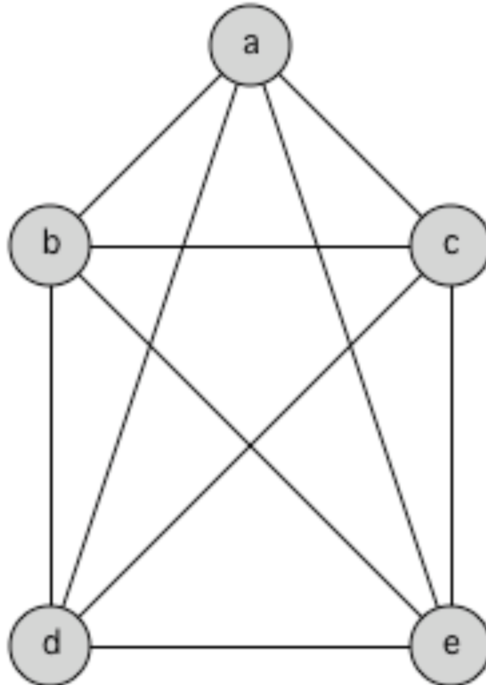
Beispiel:



- ungerichtet
- ungewichtet
- vollständig
- zweifach zusammenhängend
- zyklisch
- nicht planar

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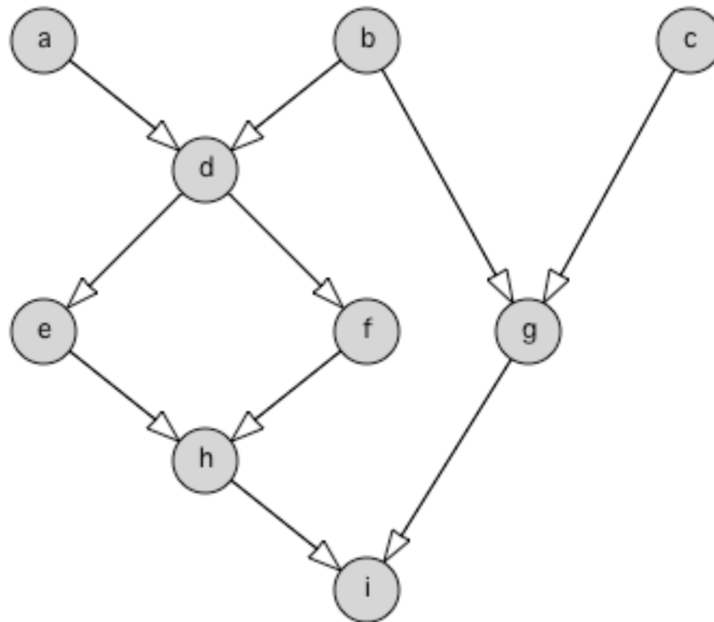
Beispiel:



- gerichtet
- ungewichtet
- schwach zusammenhängend
- zyklenfrei
- planar

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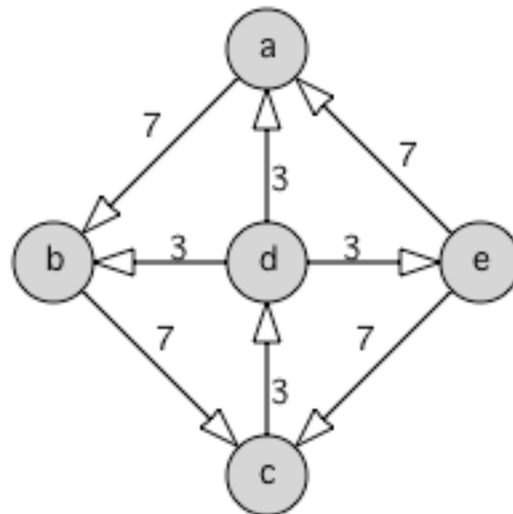
Beispiel:



- gerichtet
- gewichtet
- stark zusammenhängend
- zyklisch
- planar

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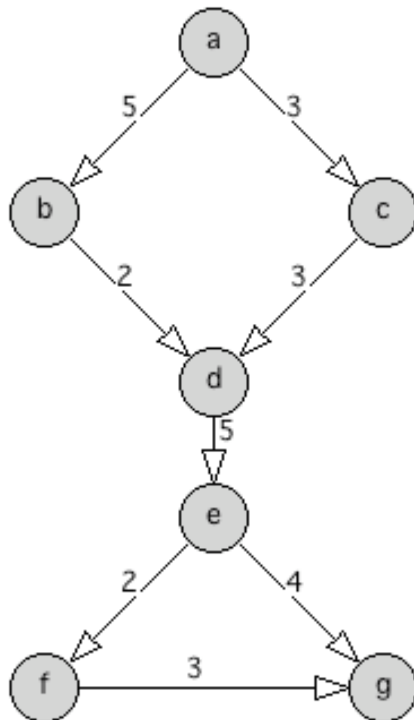
Adjazenzmatrix
Bsp: gerichteter und gewichteter Graph



	a	b	c	d	e	f	g
a		5	3				
b				2			
c				3			
d					5		
e						2	4
f							3
g							

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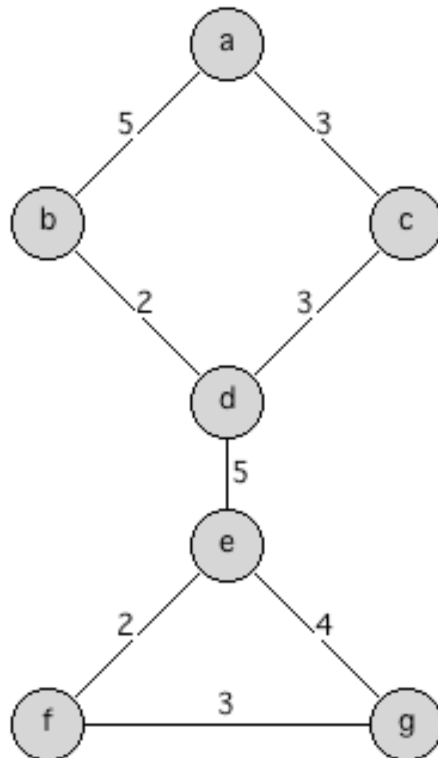
Adjazenzmatrix

Bsp: ungerichteter gewichteter Graph



	a	b	c	d	e	f	g
a		5	3				
b	5			2			
c	3			3			
d		2	3		5		
e				5		2	4
f					2		3
g					4	3	

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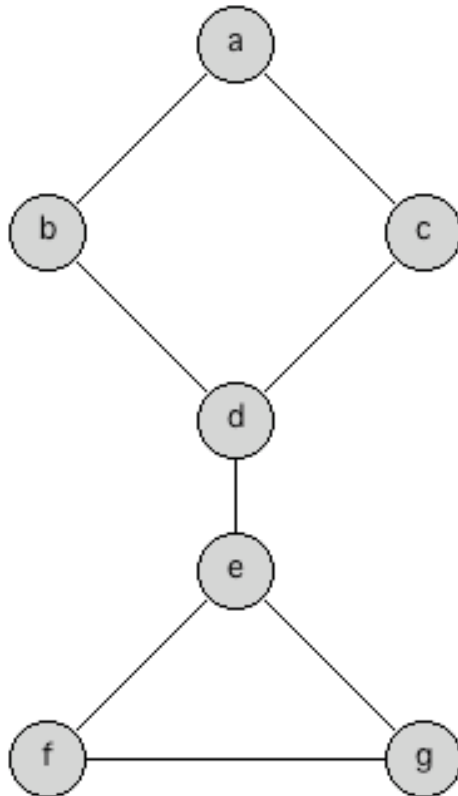
Adjazenzmatrix
Bsp: ungerichteter ungewichteter Graph



	a	b	c	d	e	f	g
a		1	1				
b	1			1			
c	1			1			
d		1	1		1		
e				1		1	1
f					1		1
g					1	1	

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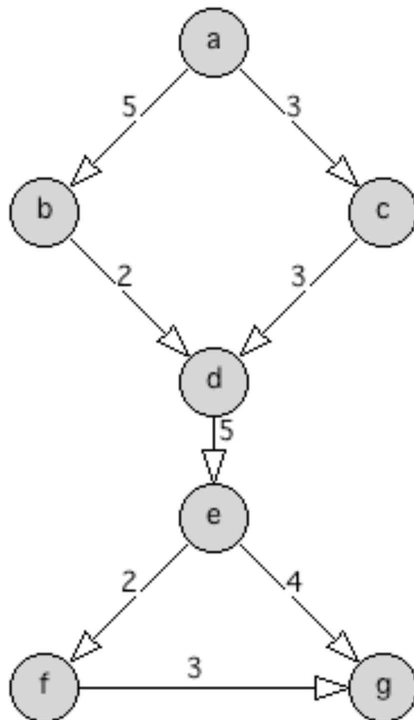
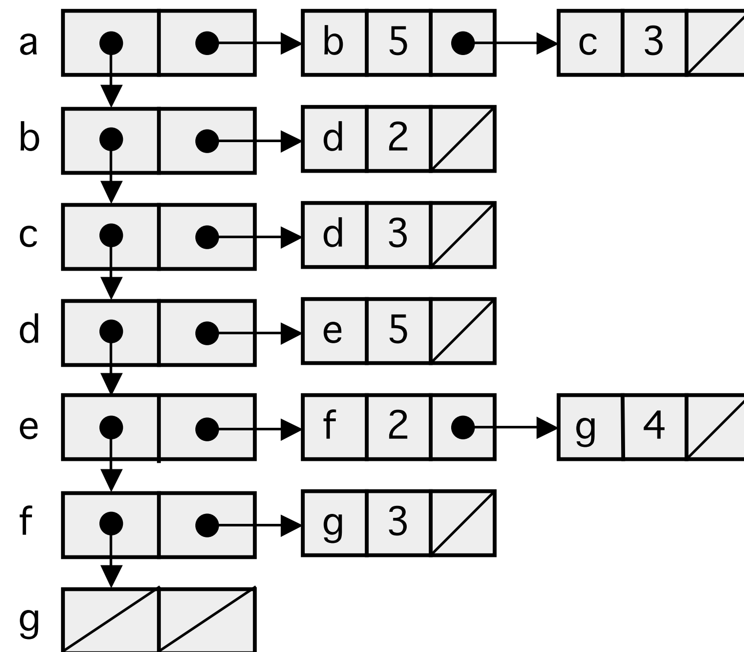
Adjazenzlisten
Bsp: gerichteter Graph





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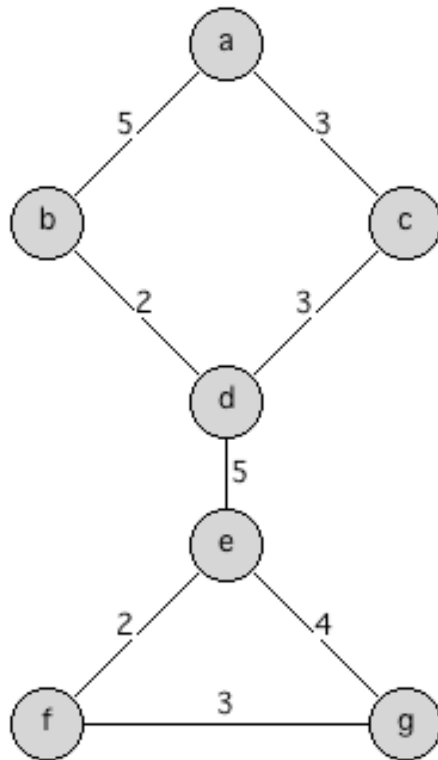
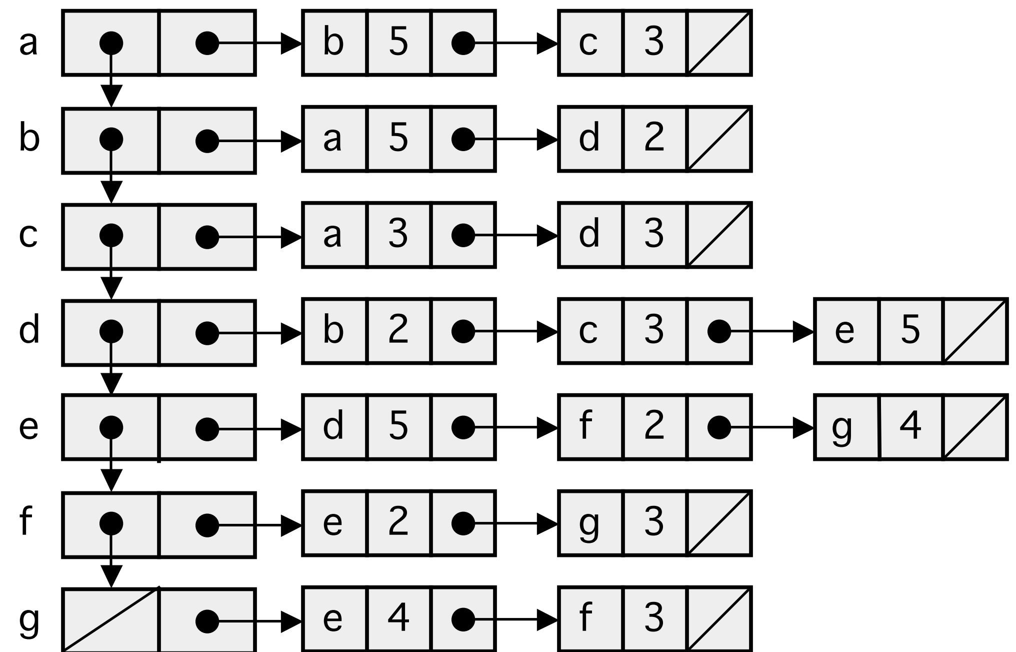
Adjazenzlisten
Bsp: ungerichteter Graph





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Implementierung eines Graphen

```
class Graph {
    Vector nodes = new Vector();
    void clearNodes() {
        for (int i=0; i<nodes.size(); i++) {
            Node node = (Node)nodes.elementAt(i);
            node.setVisited(false);
        }
    }
    void depthFirst()...
    void breadthFirst()...
}
```

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Implementierung der Knoten

```
class Node {
	boolean visited = false;
	Vector edges = new Vector();

	boolean isVisited() {return visited;}
	void setVisited(boolean p) {visited = p;}
	void depthFirst()...
	void breadthFirst()...
}
```

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Implementierung der Kanten

```
class Edge {
    int weight;
    Node to;
    Edge(int weight, Node to) {
        this.weight = weight;
        this.to= to;
    }
}
```

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Tiefensuche (rekursiv)

```
class Graph {
...
    void depthFirst() {
        clearNodes();
        for (int i=0; i<nodes.size(); i++) {
            Node node = (Node)nodes.elementAt(i);
            if (!node.isVisited())
                node.depthFirst();
        }
    }
}
```

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Tiefensuche (rekursiv)

$O(|V|+|E|)$

```
class Node {
...
    void depthFirst() {
        if (isVisited()) return;
        visit(this);
        setVisited(true);
        for (int i=0; i<edges.size(); i++) {
            Edge edge = (Edge)edges.elementAt(i);
            edge.to.depthFirst();
        }
    }
}
```

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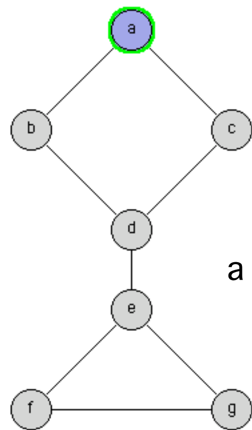
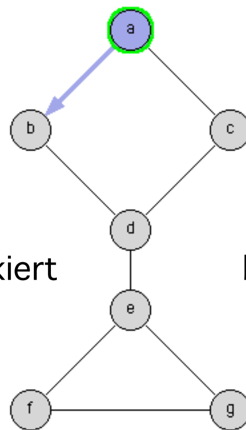
Tiefensuche (1)



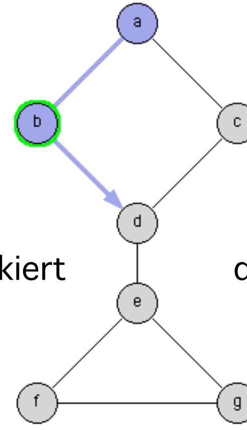
a wird markiert

1. Schritt:
Start mit a



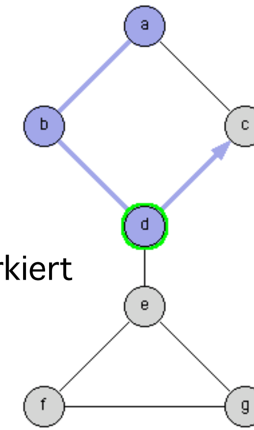
b wird markiert

2. Schritt: weiter
von a nach b



d wird markiert

3. Schritt: weiter
von b nach d



c wird markiert

4. Schritt: weiter
von d nach c

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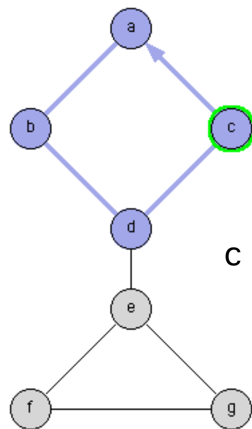
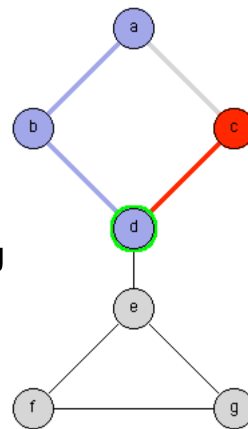
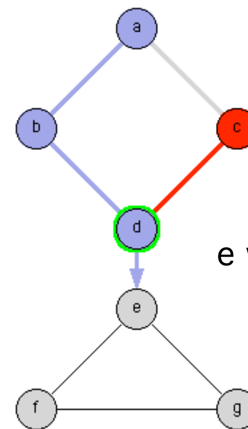
Tiefensuche (2)

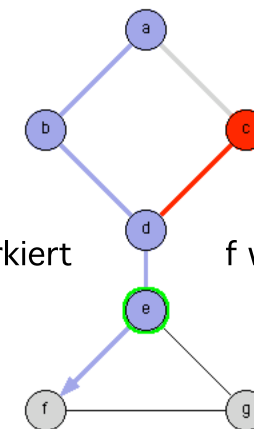


c ist fertig





e wird markiert



f wird markiert

5. Schritt: stop
a ist markiert!
Vorsicht Zyklus!

6. Schritt: zurück
von c nach d
(c,d) wird Baumkante

7. Schritt: weiter
von d nach e

8. Schritt: weiter
von e nach f

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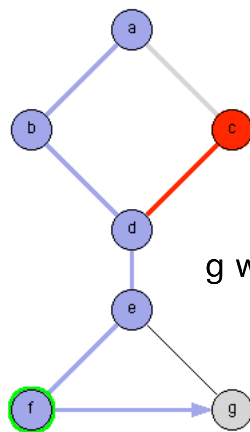
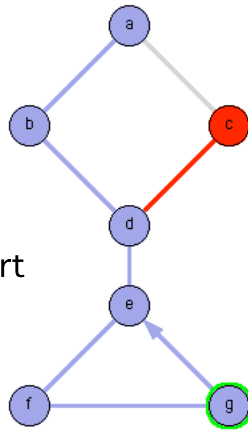
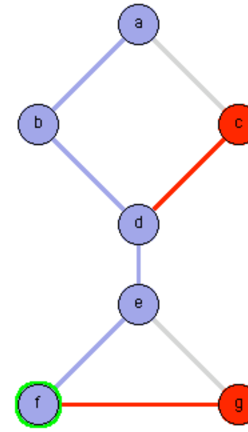
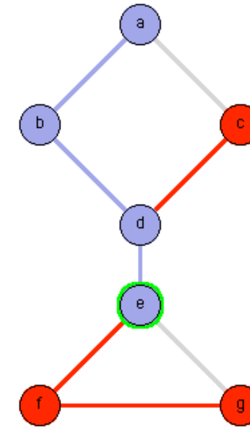
Tiefensuche (3)



g wird markiert







9. Schritt: weiter
von f nach g

10. Schritt: stop
e ist markiert!
Vorsicht Zyklus!

11. Schritt: zurück
von g nach f
(f,g) wird Baumkante

12. Schritt: zurück
von f nach e
(e,f) wird Baumkante

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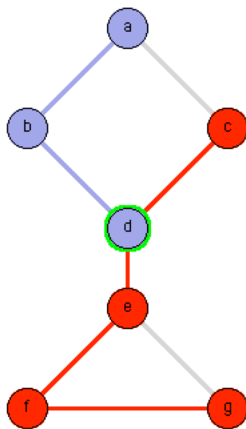
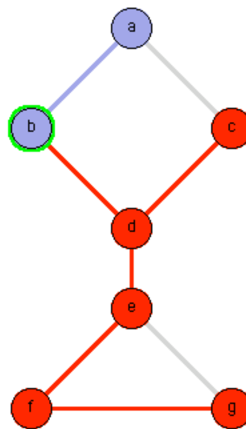
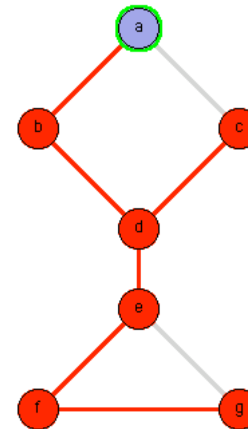
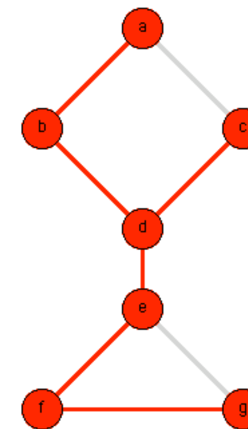
Tiefensuche (4)









13. Schritt: zurück
von e nach d
(d,e) wird Baumkante

14. Schritt: zurück
von d nach b
(b,d) wird Baumkante

15. Schritt: zurück
von b nach a
(a,b) wird Baumkante

16. Schritt: fertig
als Ergebnis entsteht
ein spannender Baum!

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Tiefensuche (iterativ)

```
void depthFirst() {
	Stack stack = new Stack();
	clearNodes();
	for (int i=0; i<nodes.size(); i++) {
		Node start = (Node)nodes.elementAt(i);
		if (!start.isVisited()) {
			start.setVisited(true);
			stack.push(start);
			do {
				Node node = (Node)stack.pop();
				visit(node);
				for (int j=node.edges.size()-1; j>=0; j--) {
					Edge edge = (Edge)node.edges.elementAt(j);
					if (!edge.to.isVisited()) {
						edge.to.setVisited(true);
						stack.push(edge.to);
					}
				}
			} while (!stack.isEmpty())
		}
	}
}
```

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Breitensuche (iterativ) O(|V|+|E|)

```
void breadthFirst() {
	Queue queue = new Queue();
	clearNodes();
	for (int i=0; i<nodes.size(); i++) {
		Node start = (Node)nodes.elementAt(i);
		if (!start.isVisited()) {
			start.setVisited(true);
			queue.put(start);
			do {
				Node node = (Node)queue.get();
				visit(node);
				for (int j=0; j<node.edges.size(); j++) {
					Edge edge = (Edge)node.edges.elementAt(j);
					if (!edge.to.isVisited()) {
						edge.to.setVisited(true);
						queue.put(edge.to);
					}
				}
			} while (!queue.isEmpty())
		}
	}
}
```

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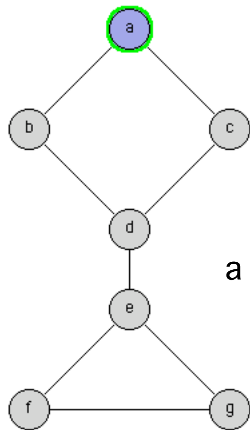
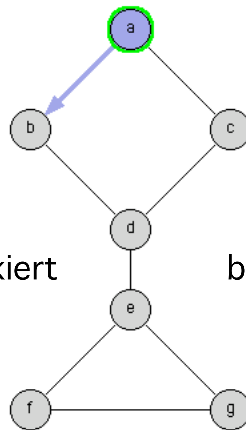
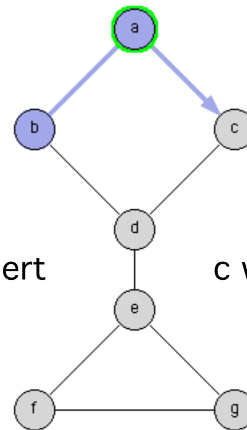
Breitensuche (1)



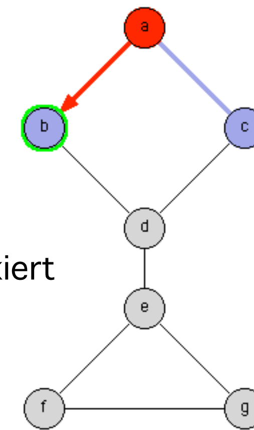
a wird markiert



b wird markiert



c wird markiert



1. Schritt:
Start mit a

2. Schritt:
[] ⇦ (a,b)

3. Schritt:
[(a,b)] ⇦ (a,c)

4. Schritt:
(a,b) ⇦ [(a,c)]
(a,b) wird Baumkante

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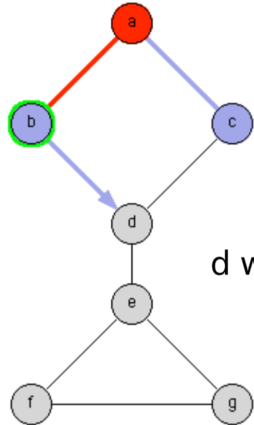
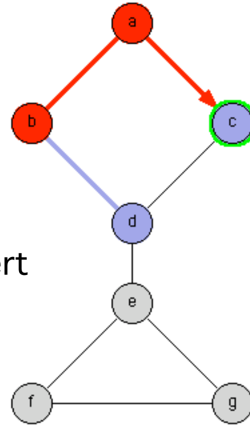
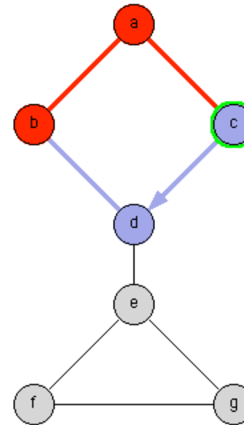
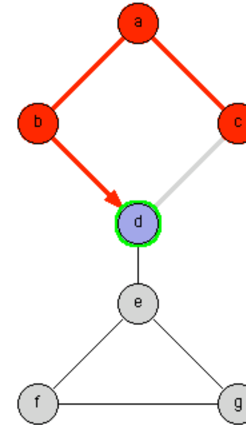
Breitensuche (2)



d wird markiert







5. Schritt:
[(a,c)] ⇦ (b,d)

6. Schritt:
(a,c) ⇦ [(b,d)]
(a,c) wird Baumkante

7. Schritt:
d ist markiert
Vorsicht Zyklus!

8. Schritt:
(b,d) ⇦ []
(b,d) wird Baumkante

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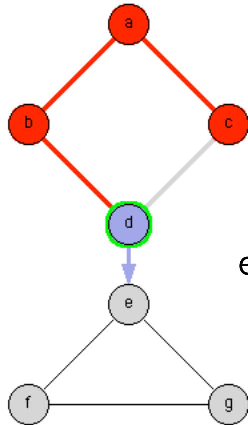
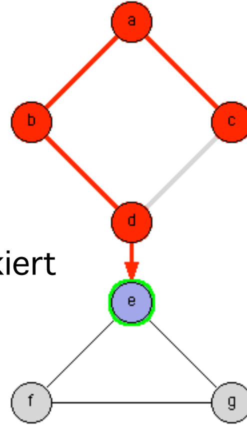
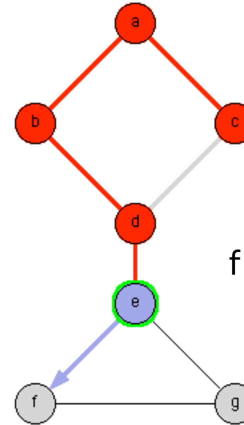
Breitensuche (3)

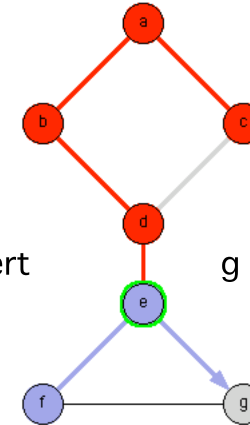


e wird markiert





f wird markiert



g wird markiert

9. Schritt:
[] ⇦ (d,e)

10. Schritt:
(d,e) ⇦ []
(d,e) wird Baumkante

11. Schritt:
[] ⇦ (e,f)

12. Schritt:
[(e,f)] ⇦ (e,g)

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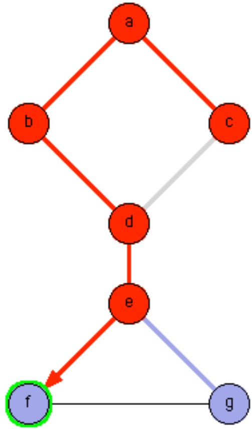
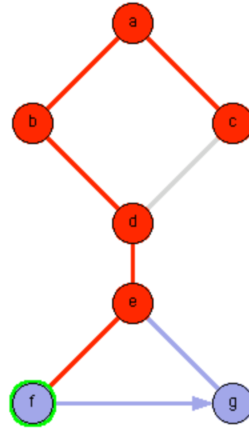
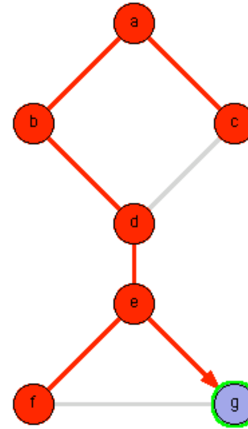
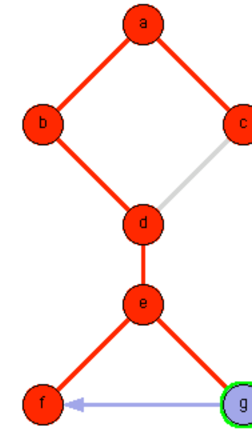
Breitensuche (4)









13. Schritt:
(e,f) ⇦ [(e,g)]
(e,f) wird Baumkante

14. Schritt:
g ist markiert
Vorsicht Zyklus!

15. Schritt:
(e,g) ⇦ []
(e,g) wird Baumkante

16. Schritt:
f ist markiert
Vorsicht Zyklus!

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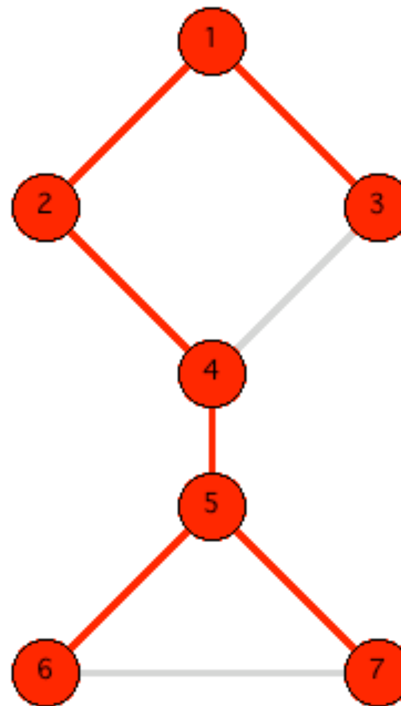
Spannender Baum als Ergebnis der Breitensuche



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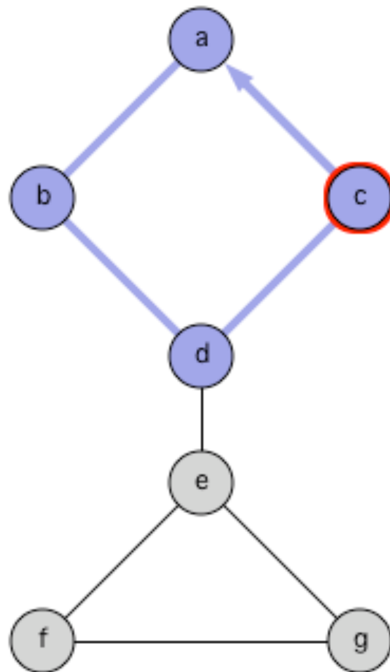
Traversieren eines ungerichteten Graphen:



Kanten die zu einem bereits markierten Knoten führen kennzeichnen einen Zyklus!

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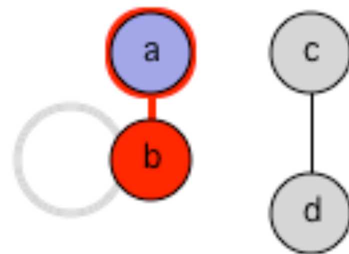
Traversieren eines ungerichteten Graphen:



Alle von einem Startknoten aus erreichbaren Knoten gehören zur gleichen Komponente!

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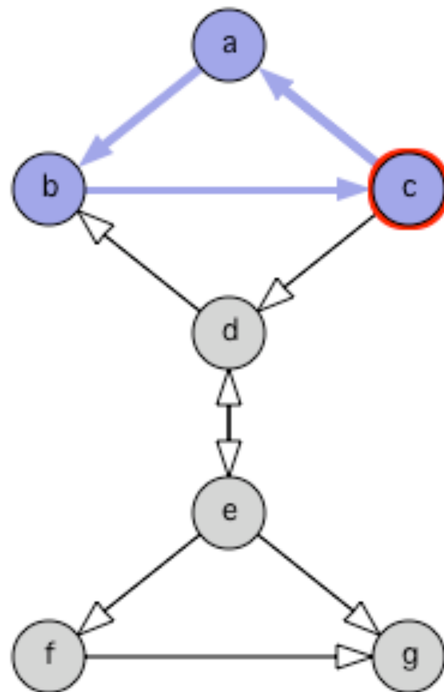
Department for Information Technology
University of Zurich



Rekursive Tiefensuche in gerichteten Graphen:

Kanten die zu einem aktiven (blau) markierten Knoten führen kennzeichnen einen Zyklus!

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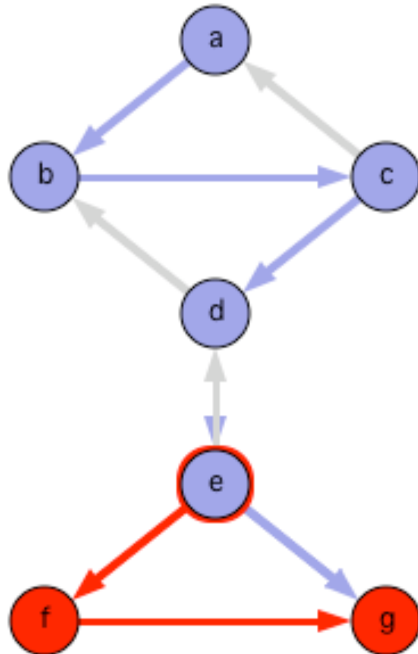

Rekursive Tiefensuche in gerichteten Graphen:

Kanten die zu einem (rot) markierten Baum-Knoten führen kennzeichnen keinen Zyklus!

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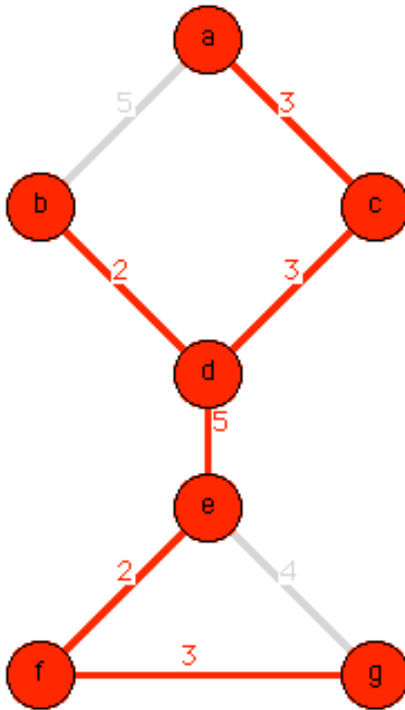
Minimum Spanning Tree (Dijkstra/Prim)



Ermittlung eines minimalen spannenden Baumes eines gewichteten Graphens nach Dijkstra/Prim

O(|E| log |V|)

Breitensuche unter Verwendung einer PriorityQueue (geringes Gewicht hat hohe Priorität) liefert einen minimalen spannenden Baum!

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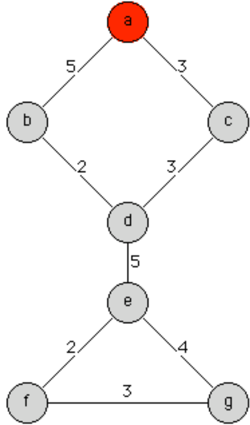
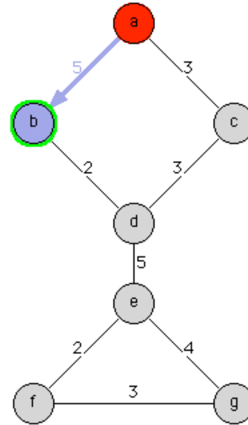
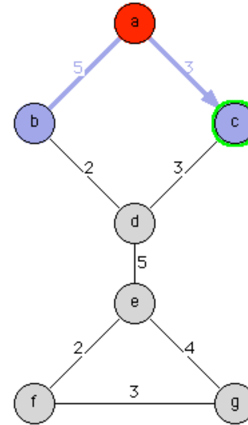
Dijkstra/Prim (1)



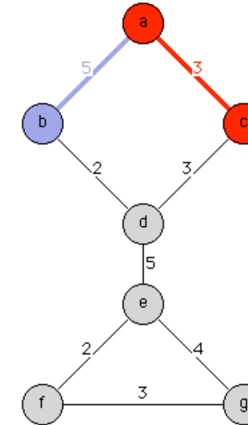
1. Schritt:
Start mit a



2. Schritt:
[] ⇦ (a,b,5)



3. Schritt:
[(a,b,5)] ⇦ (a,c,3)



4. Schritt:
(a,c,3) ⇦ [(a,b,5)]

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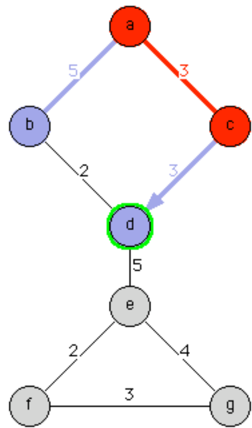
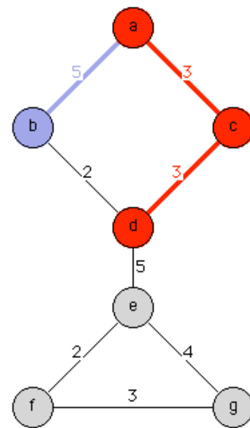
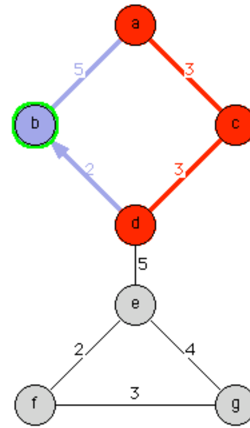
Dijkstra/Prim (2)



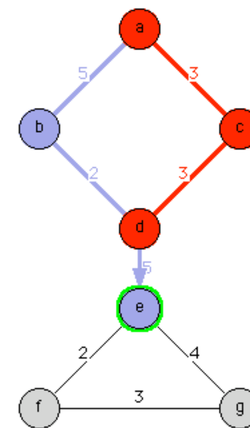
5. Schritt:
[(a,b,5)] ⇦ (c,d,3)



6. Schritt:
(c,d,3) ⇦ [(a,b,5)]



7. Schritt:
[(a,b,5)] ⇦ (d,b,2)
(d,b,2) ersetzt (a,b,5)



8. Schritt:
[(d,b,2)] ⇦ (d,e,5)

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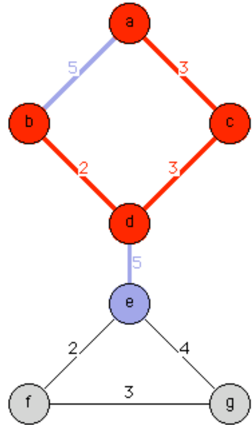
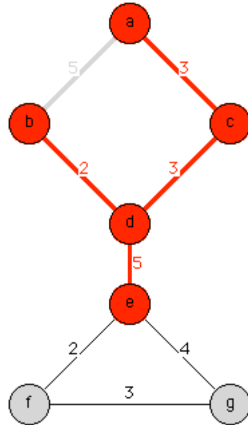
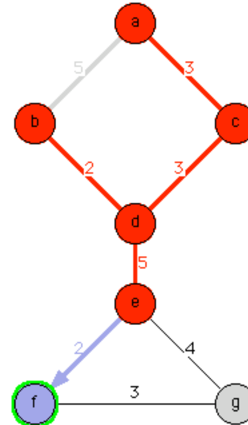
Dijkstra/Prim (3)



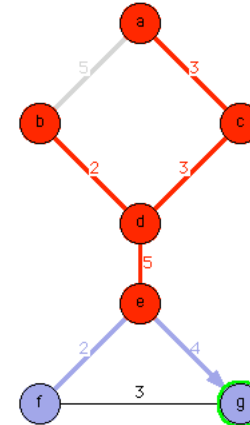
9. Schritt:
(d,b,2) ⇦ [(d,e,5)]



10. Schritt:
(d,e,5) ⇦ []



11. Schritt:
[] ⇦ (e,f,2)



12. Schritt:
[(e,f,2)] ⇦ (e,g,4)

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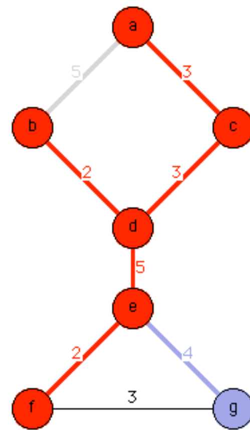
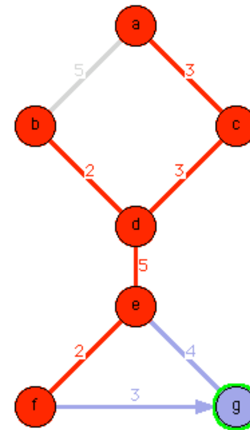
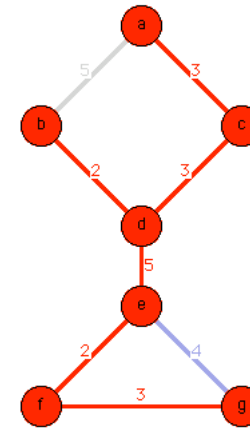
Dijkstra/Prim (4)







13. Schritt:
(e,f,2) ⇦ [(e,g,4)]

14. Schritt:
[(e,g,4)] ⇦ (f,g,3)
(f,g,3) ersetzt (e,g,4)

15. Schritt:
(f,g,3) ⇦ []
fertig

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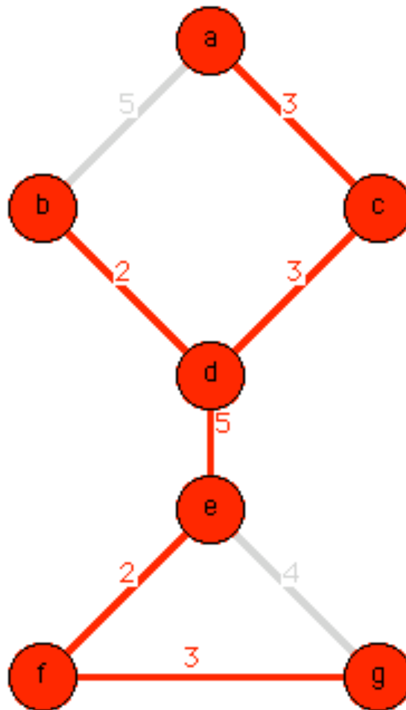
Minimum Spanning Tree (Kruskal)



Ermittlung eines minimalen spannenden Baumes eines gewichteten Graphens nach Kruskal

$O(|E| \log|E|)$

Entfernen sämtlicher Kanten (führt zu einem Graphen mit |V| isolierten Knoten).

Hinzufügen jener Kanten in nach Gewichten steigend sortierter Reihenfolge die die Anzahl der Komponenten verringern.

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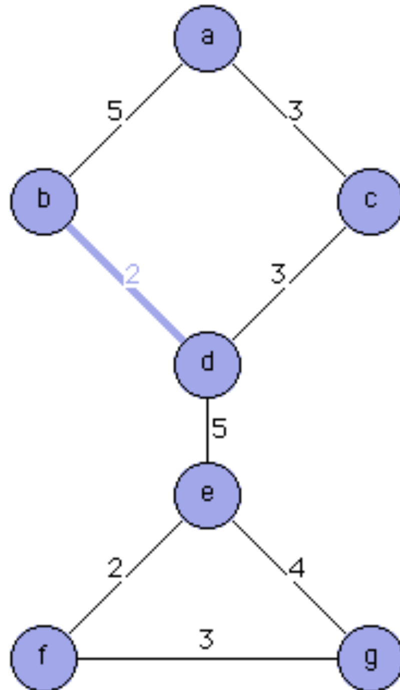
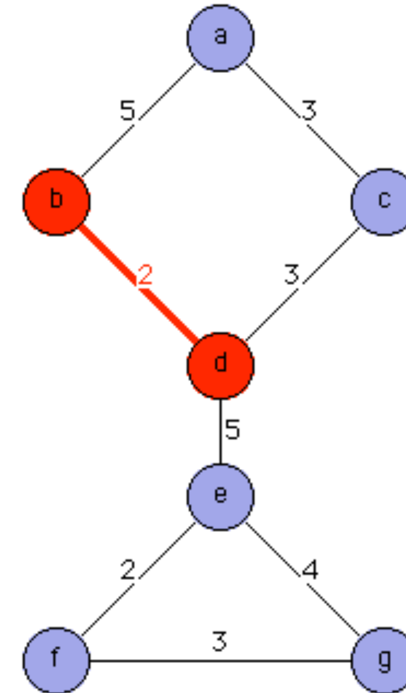
Kruskal (1)



b≠d ⇨ union(b,d)

Die Knoten b und d befinden sich in unterschiedlichen Komponenten.

Die Kante (b,d) verbindet die beiden isolierten Knoten b und d



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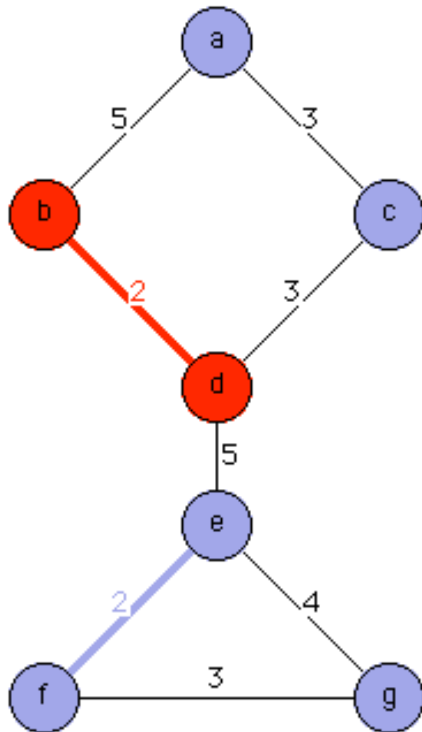
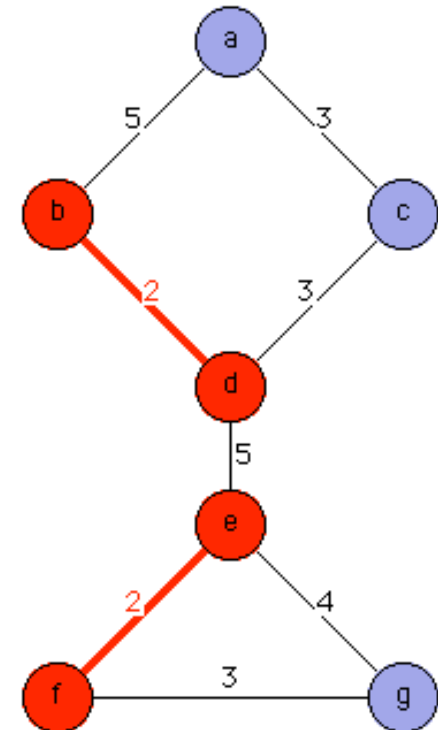
Kruskal (2)



e≠f ⇨ union(e,f)

Die Knoten e und f befinden sich in unterschiedlichen Komponenten.

Die Kante (e,f) verbindet die beiden isolierten Knoten e und f



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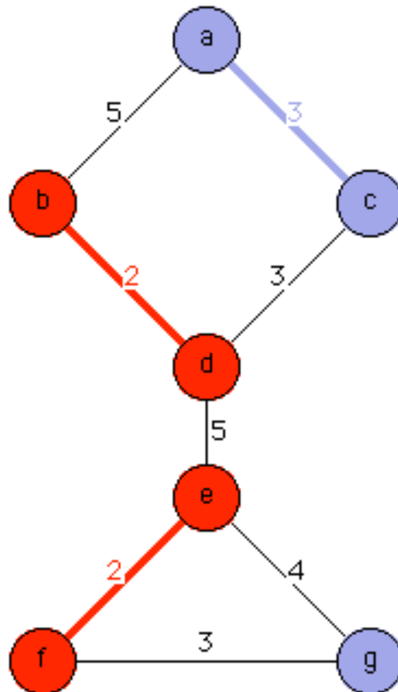
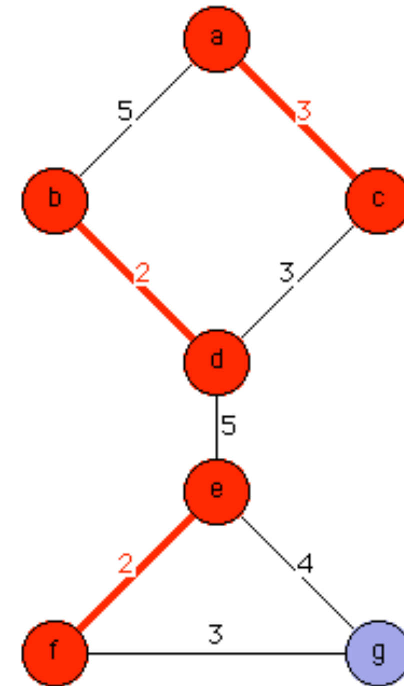
Kruskal (3)



a≠c ⇨ union(a,c)

Die Knoten a und c befinden sich in unterschiedlichen Komponenten.

Die Kante (a,c) verbindet die beiden isolierten Knoten a und c



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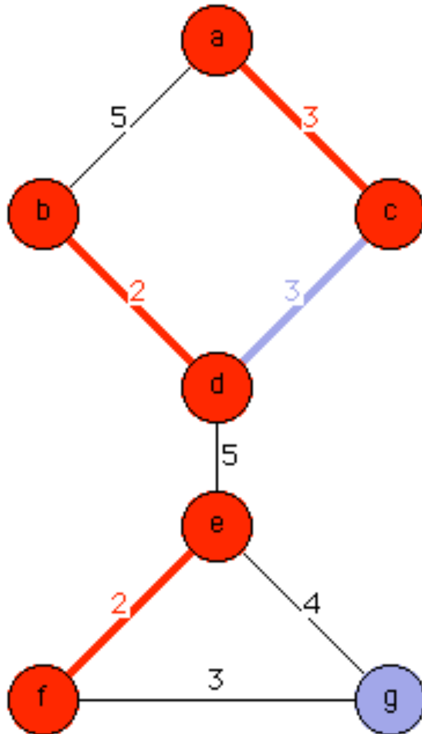
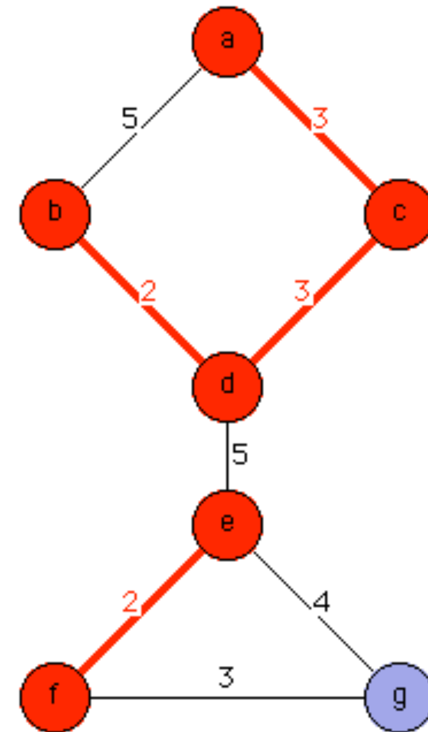
Kruskal (4)



c≠d ⇨ union(c,d)

Die Knoten c und d befinden sich in unterschiedlichen Komponenten.

Die Kante (c,d) verbindet die beiden Komponenten ac und bd



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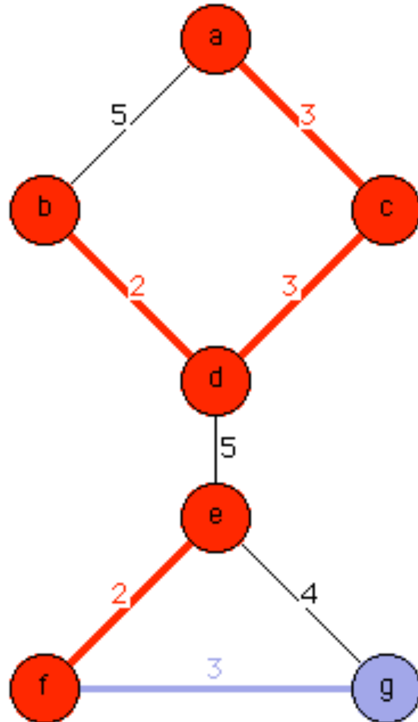
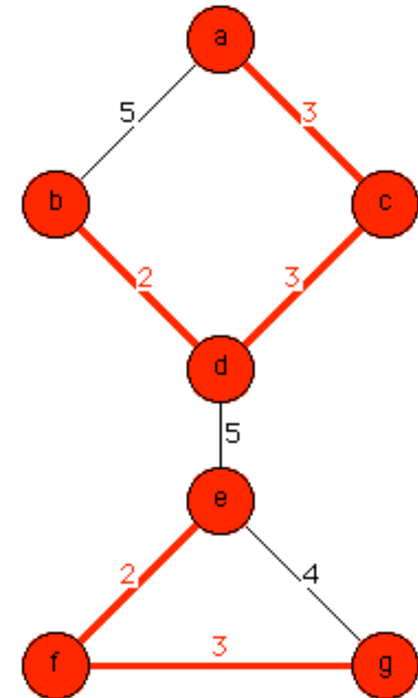
Kruskal (5)



f≠g ⇨ union(f,g)

Die Knoten f und g befinden sich in unterschiedlichen Komponenten.

Die Kante (f,g) verbindet die Komponente ef und den isolierten Knoten g



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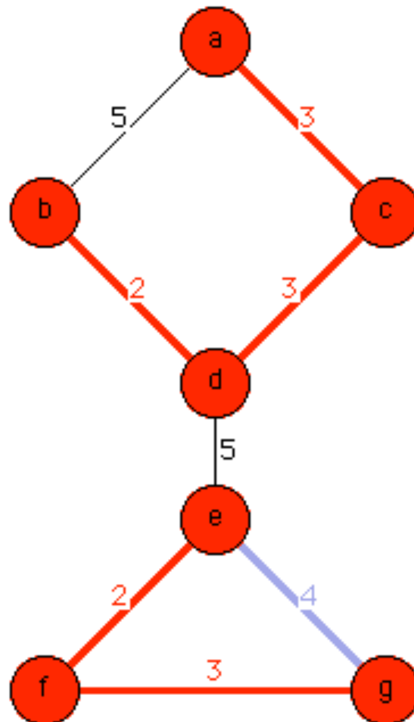
Kruskal (6)



e=g

Die Knoten e und g befinden sich in derselben Komponente efg.

Die Kante (e,g) ist nicht Bestandteil des minimalen spannenden Baumes!

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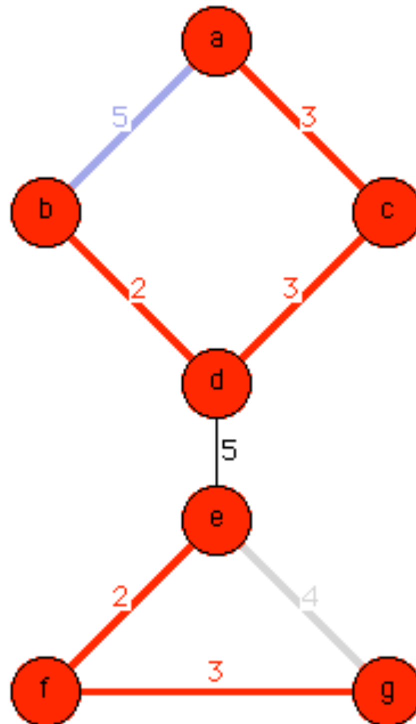
Kruskal (7)



a=b

Die Knoten a und b befinden sich in derselben Komponente abcd.

Die Kante (a,b) ist nicht Bestandteil des minimalen spannenden Baumes!

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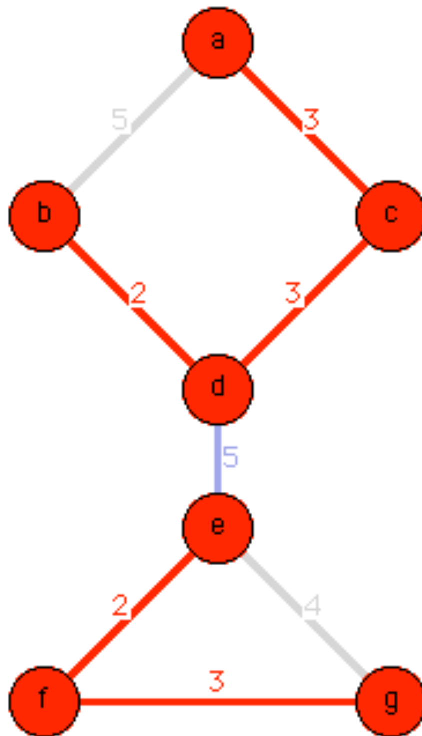
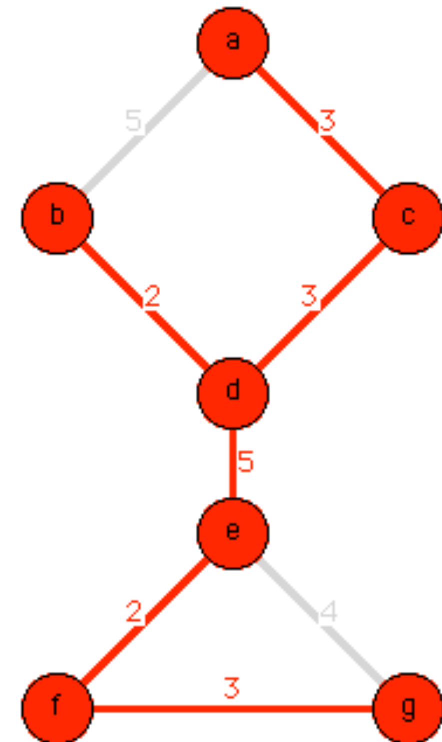
Kruskal (8)



d≠e ⇨ union(d,e)

Die Knoten d und e befinden sich in unterschiedlichen Komponenten.

Die Kante (d,e) verbindet die Komponente abcd und efg



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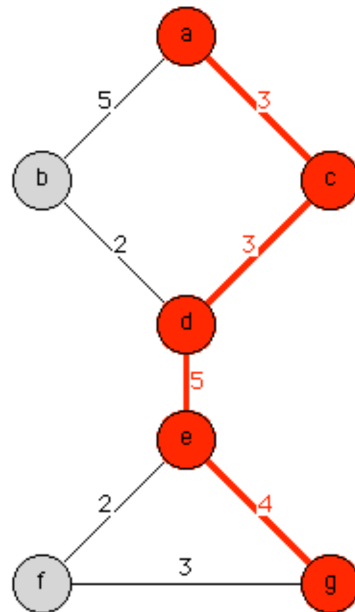

Ermittlung aller von einem Startknoten ausgehenden kürzesten Wege eines gewichteten Graphens nach Dijkstra

O(|E| log|V|)

Breitensuche unter Verwendung einer PriorityQueue (geringe Entfernung vom Startknoten hat hohe Priorität) ordnet jedem Knoten den kürzesten Weg vom Startknoten zu!

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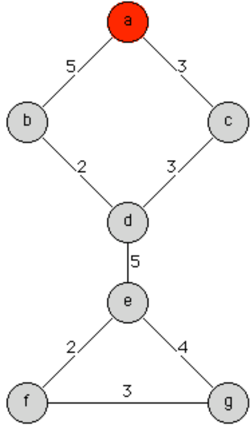
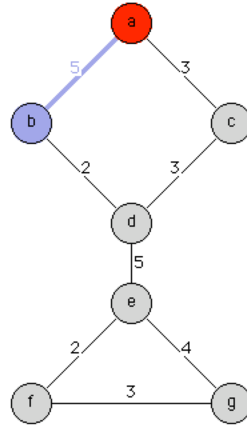
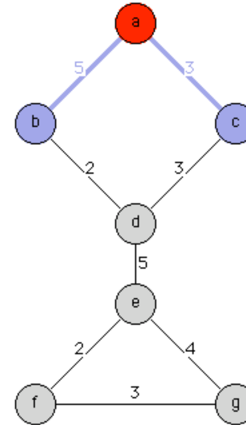
Dijkstra (1)



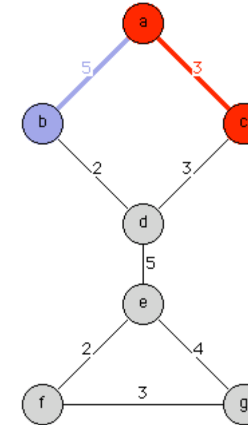
1. Schritt:
Start mit a



2. Schritt:
[] ⇦ (a,b,5)



3. Schritt:
[(a,b,5)] ⇦ (a,c,3)



4. Schritt:
(a,c,3) ⇦ [(a,b,5)]

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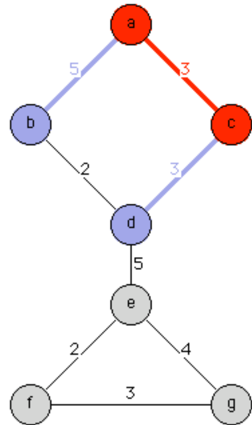
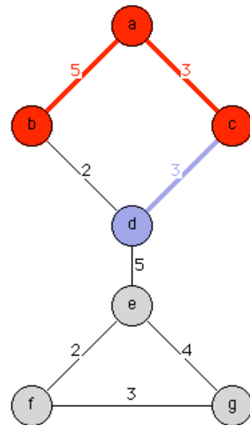
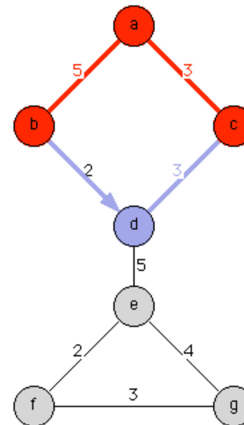
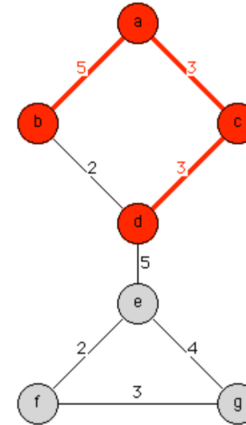
Dijkstra (2)









5. Schritt:
[(a,b,5)] ⇦ (c,d,6)

6. Schritt:
(a,b,5) ⇦ [(c,d,6)]

7. Schritt:
[(c,d,6)]
(b,d,7) > (c,d,6)

8. Schritt:
(c,d,6) ⇦ []

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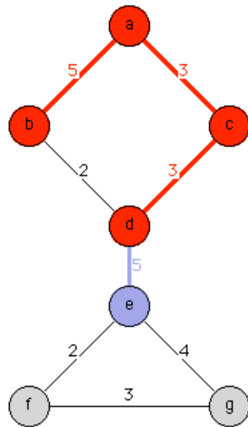
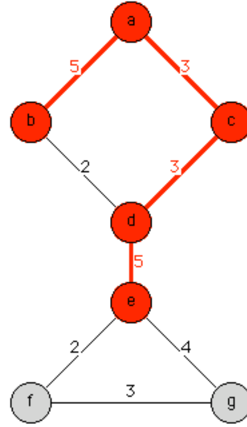
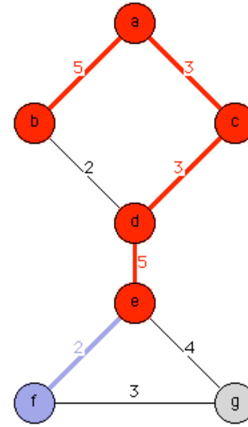
Dijkstra (3)



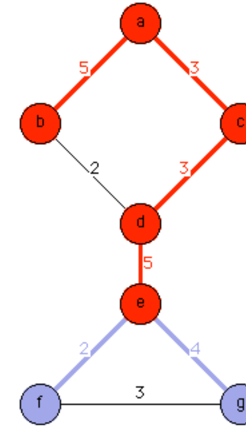
9. Schritt:
[] ⇦ (d,e,11)



10. Schritt:
(d,e,11) ⇦ []



11. Schritt:
[] ⇦ (e,f,13)



12. Schritt:
[(e,f,13)] ⇦ (e,g,15)

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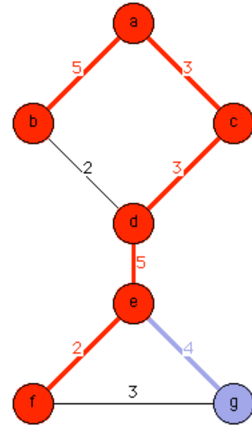
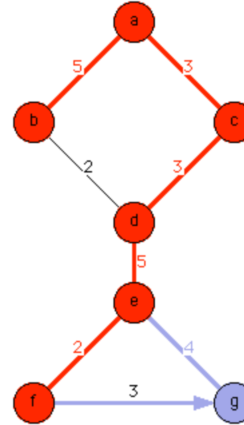
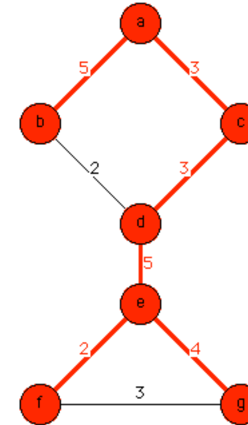
Dijkstra (4)



13. Schritt:
(e,f,13) ⇦ [(e,g,15)]



14. Schritt:
[(e,g,15)]
(f,g,16) > (e,g,15)



15. Schritt:
(e,g,15) ⇦ []
fertig

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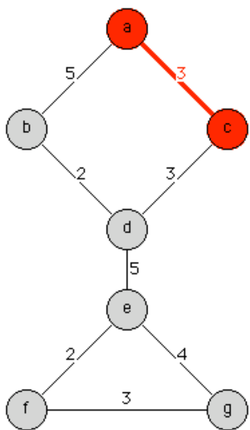
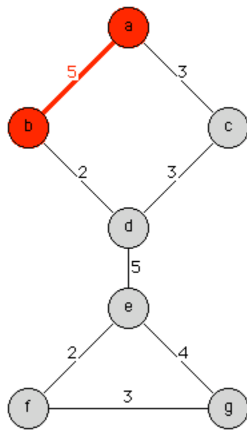
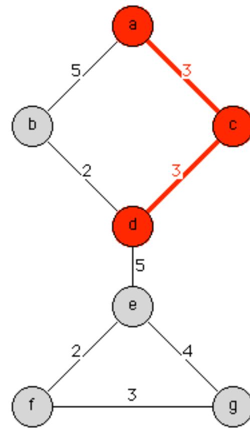
Dijkstra (5)
Alle kürzesten Wege von a:



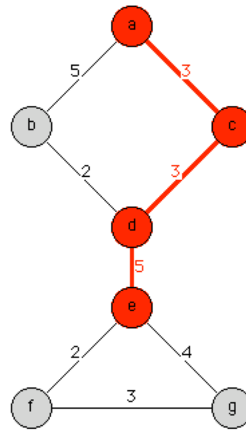
a-c 3



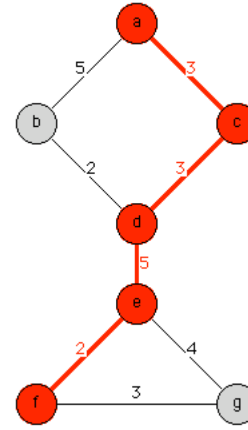
a-b 5



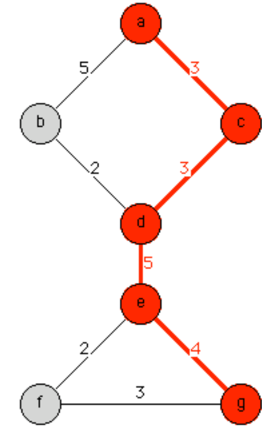
a-c-d 6



a-c-d-e 11



a-c-d-e-f 13



a-c-d-e-g 15

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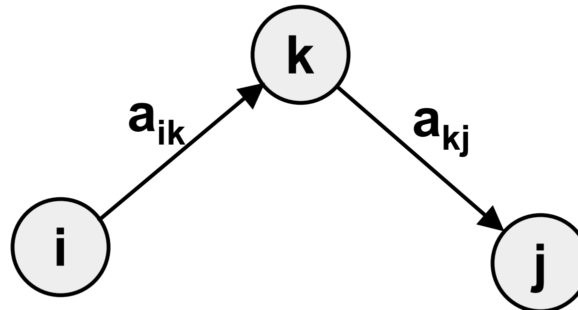
Transitive Hülle

Adjazenzmatrix a: **boolean a[][] = new boolean[n][n];**

Transitive Hülle: **All i,j,k: 0≤i,j,k<n: a[i][k] and a[k][j] ⇒ a[i][j]**



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Transitive Hülle
Algorithmus von Warshall
$O(|V|^3)$

```
for (int k=0; k<n; k++)
    for (int i=0; i<n; i++)
        for (int j=0; j<n; j++)
            if (a[i][k] && a[k][j]) a[i][j] = true;

            oder

            a[i][j] = a[i][j] || (a[i][k] && a[k][j]);
```

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Kürzeste Wege
Algorithmus von Floyd
$O(|V|^3)$

Adjazenzmatrix a: `double a[][] = new double[n][n];`

a_{ij} ... Entfernung von i nach j

$a_{ij} = \infty$ falls kein Weg von i nach j

```
for (int k=0; k<n; k++)
    for (int i=0; i<n; i++)
        for (int j=0; j<n; j++)
            a[i][j] = min(a[i][j], a[i][k] + a[k][j]);
```

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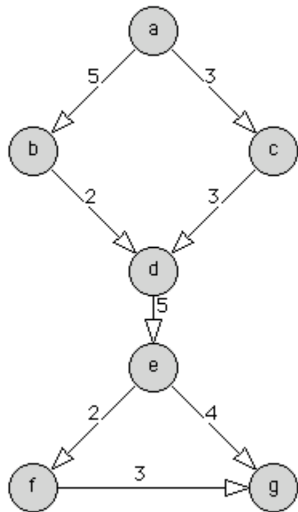
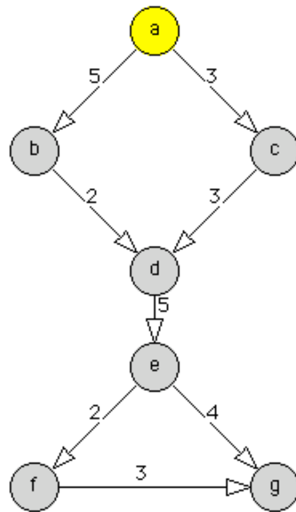
Floyd (1)



shortest paths:

	a	b	c	d	e	f	g
a		5	3				
b				2			
c				3			
d					5		
e						2	4
f							3
g							

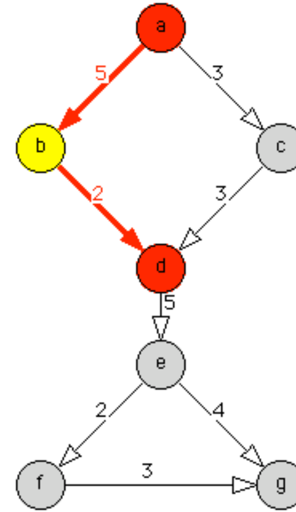
Adjazenzmatrix



shortest paths:

	a	b	c	d	e	f	g
a		5	3				
b				2			
c				3			
d					5		
e						2	4
f							3
g							

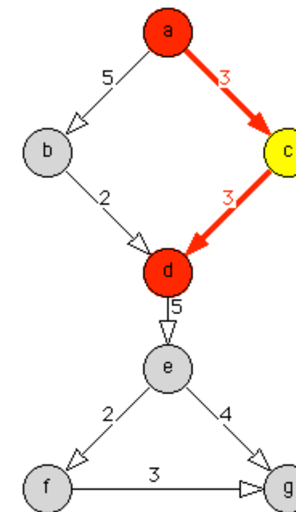
es führt kein Weg übera



shortest paths:

	a	b	c	d	e	f	g
a		5	3	7			
b				2			
c				3			
d					5		
e						2	4
f							3
g							

a - b -d (7)



shortest paths:

	a	b	c	d	e	f	g
a		5	3	6			
b				2			
c				3			
d					5		
e						2	4
f							3
g							

a - c -d (6)< a - b -d (7)

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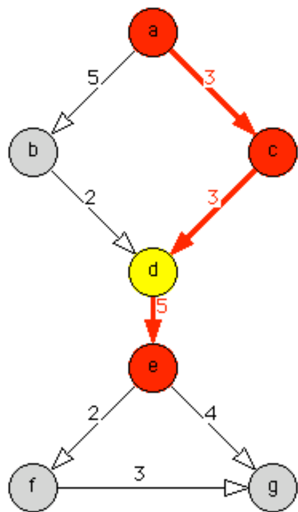
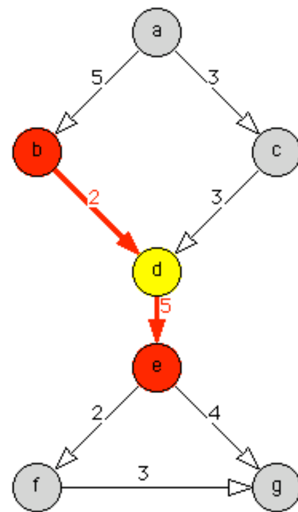
Floyd (2)



shortest paths:

	a	b	c	d	e	f	g
a		5	3	6	11		
b				2			
c				3			
d					5		
e						2	4
f							3
g							

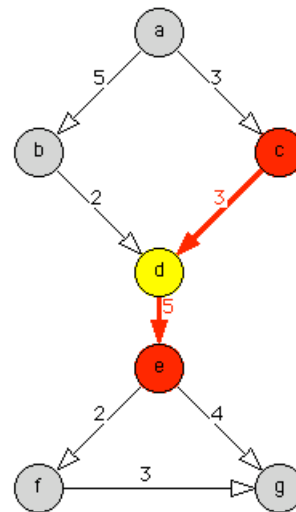
a -c -d -e (11)



shortest paths:

	a	b	c	d	e	f	g
a		5	3	6	11		
b				2	7		
c				3			
d					5		
e						2	4
f							3
g							

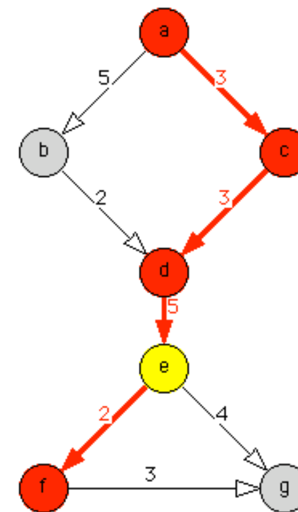
b -d -e (7)



shortest paths:

	a	b	c	d	e	f	g
a		5	3	6	11		
b				2	7		
c				3	8		
d					5		
e						2	4
f							3
g							

c -d -e (8)



shortest paths:

	a	b	c	d	e	f	g
a		5	3	6	11	13	
b				2	7		
c				3	8		
d					5		
e						2	4
f							3
g							

a - c -d -e -f (13)

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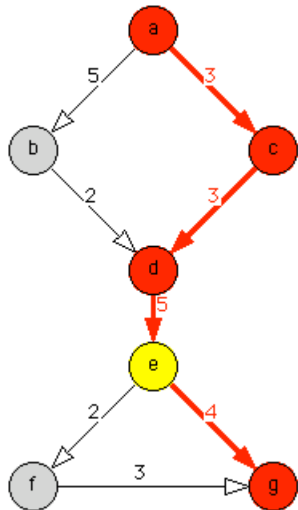
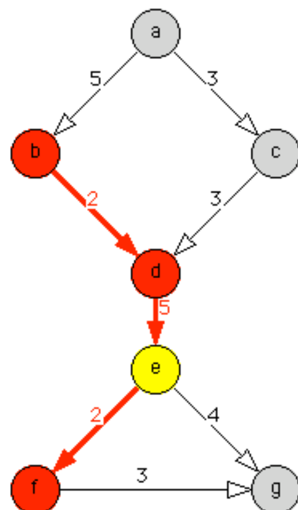
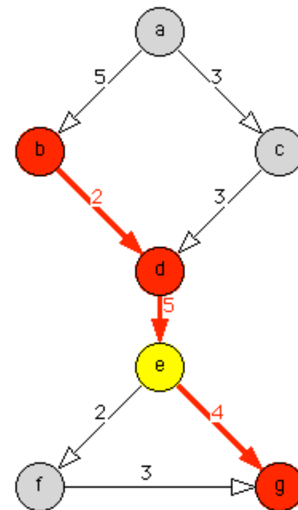
Floyd (3)



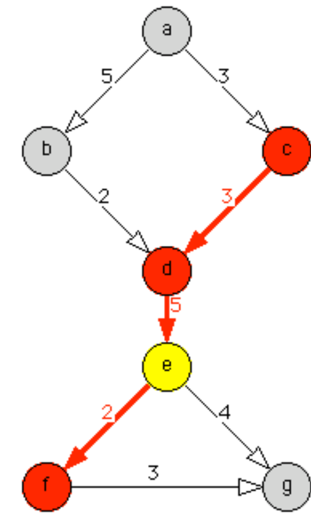
a - c -d -e -g (15)



b -d -e -f (9)



b -d -e -g (11)



c -d -e -f (10)

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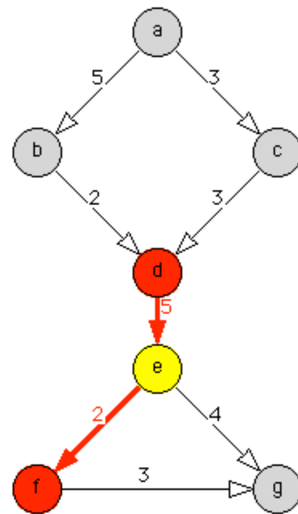
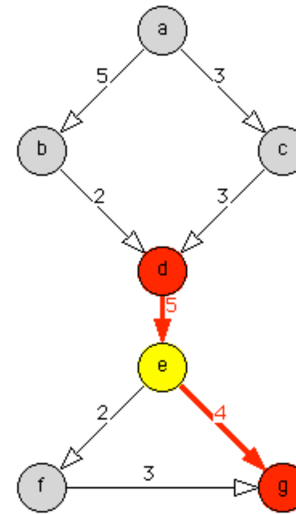
Floyd (4)



shortest paths:

	a	b	c	d	e	f	g
a		5	3	6	11	13	15
b				2	7	9	11
c				3	8	10	12
d					5	7	
e						2	4
f							3
g							

d -e -f (7)



shortest paths:

	a	b	c	d	e	f	g
a		5	3	6	11	13	15
b				2	7	9	11
c				3	8	10	12
d					5	7	9
e						2	4
f							3
g							

d -e -g (9)