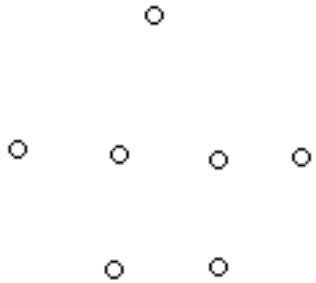
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Geometrische Algorithmen

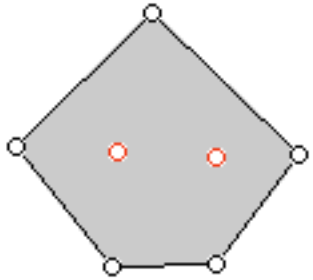
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Beispiel:
Konvexe Hülle einer 2D-Punktmenge (1)

Die Konvexe Hülle ist ein konvexes Polygon mit minimalem Umfang das sämtliche gegebenen Punkte einschliesst.



Punktmenge

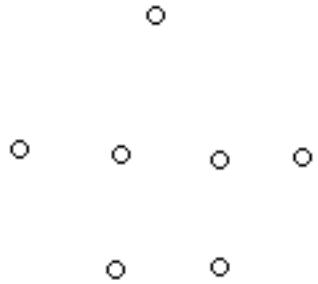


Konvexe Hülle

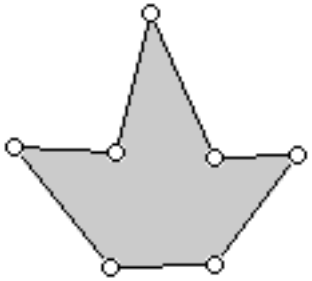
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Konvexe Hülle einer 2D-Punktmenge (2)
Graham-Scan O(N log N)

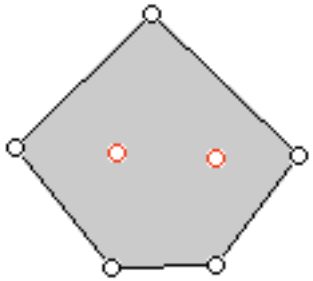
1) **Bilde ein Polygon das die gegebenen Punkte ohne Überschneidungen verbindet**
2) **Eliminiere sämtliche "einspringenden" Eckpunkte**



Punktmenge



Polygon



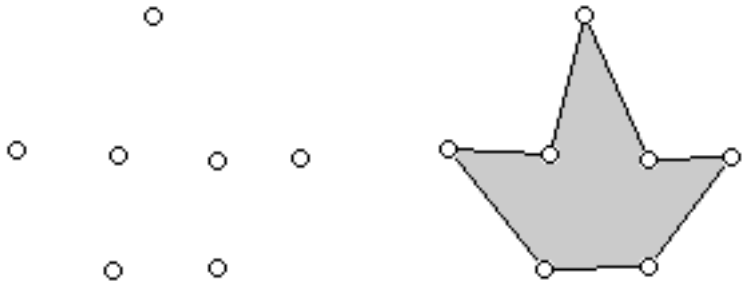
Konvexe Hülle

Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Konvexe Hülle einer 2D-Punktmenge (3)
Polygon

Algorithmus zur Bildung eines Polygons:

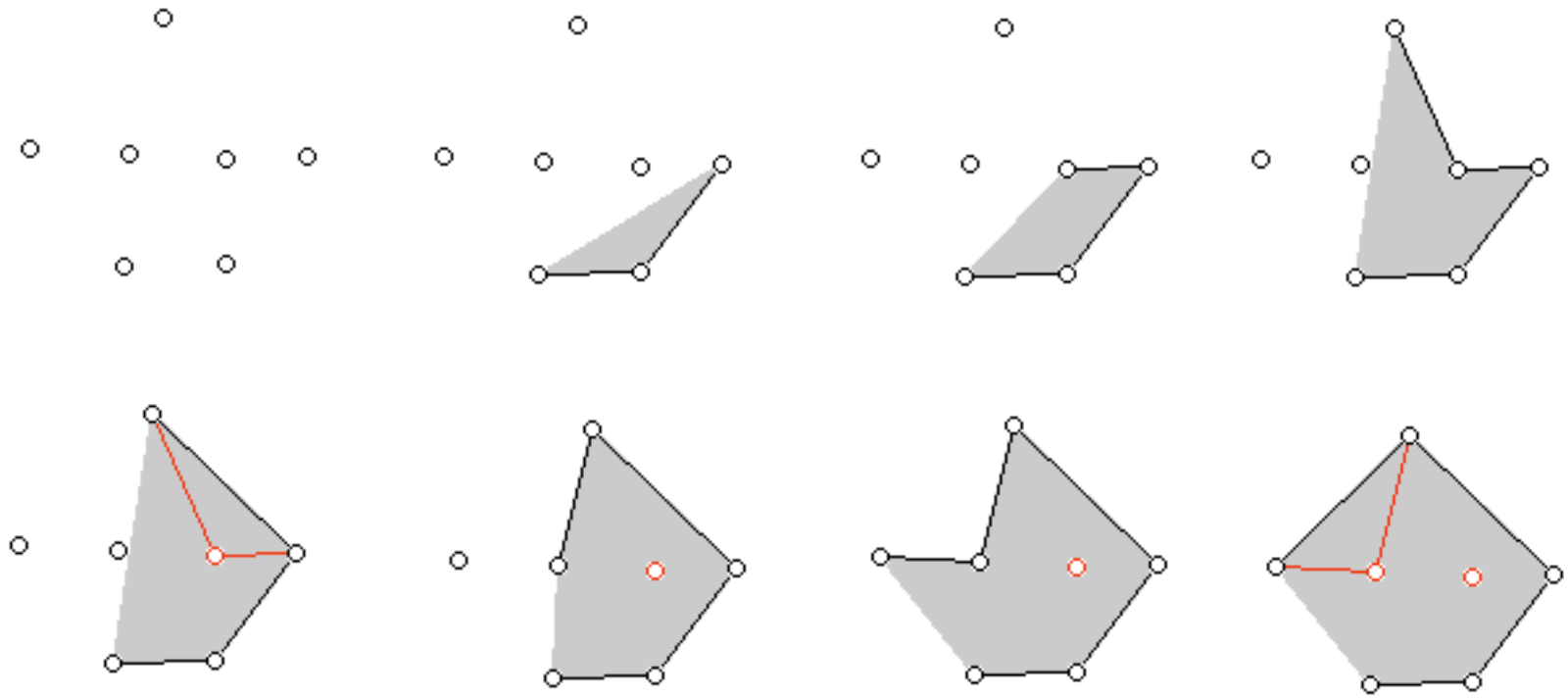
1) **Wähle einen Startpunkt (zB den untersten Punkt) und berechne zu jedem weiteren Punkt den Winkel den die Verbindung dieses Punktes und dem Startpunkt mit der Horizontalen einschliesst.**
2) **Verbinde die Punkte in steigender Reihenfolge dieser Winkel**



Punktmenge **Polygon**

Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Konvexe Hülle einer 2D-Punktmenge (4)
Eliminierung einspringender Eckpunkte



Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Theta-Funktion

```
double theta(Point p) {
    double t;
    int dx = this.x-p.x;
    int dy = p.y-this.y;
    if ((dx==0)&(dy==0)) t = 0;
    else t =
    (double)dy/(double)(Math.abs(dx)+Math.abs(dy));
    if (dx<0) t = 2-t;
    else if (dy<0) t = 4+t;
    return t;
}
```

Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Orientierung eines Dreiecks

```
public boolean convex(Point p1, Point p2, Point p3) {
    int dx1 = p2.x-p1.x;
    int dx2 = p3.x-p2.x;
    int dy1 = p1.y-p2.y;
    int dy2 = p2.y-p3.y;
    return dy2*dx1>dy1*dx2;
}
```

Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Attribute Geometrischer Algorithmen

- Dimension
- Orientierung
- Orthogonalität
- Queryobjekt

Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Klassifizierung Geometrischer Algorithmen

- Bereichssuche
- Schnittprobleme
- Einschlussprobleme
- Distanzprobleme

Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

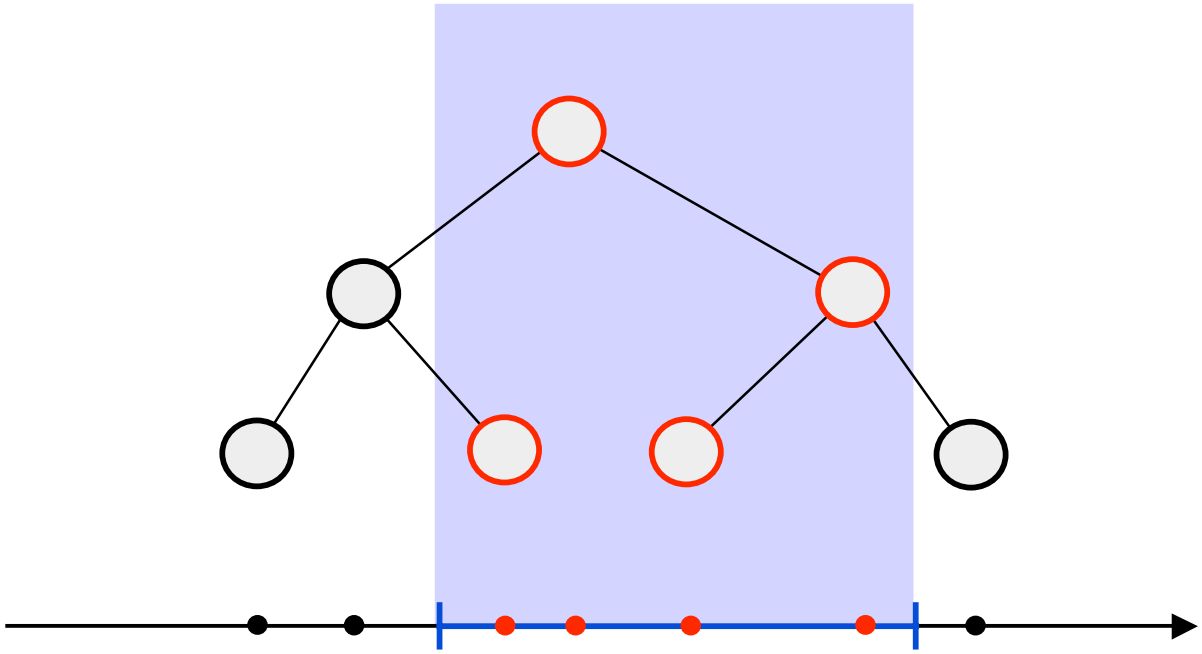
Bereichssuche (Range Searching) eindimensional O(R+log N)

Geg: eine Menge X von x-Koordinaten, N = |X|
ein Query-Intervall $[x_l,x_r]$

Ges: $\{x \in X: x \in [x_l,x_r]\}$

Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

1D-Suchbaum



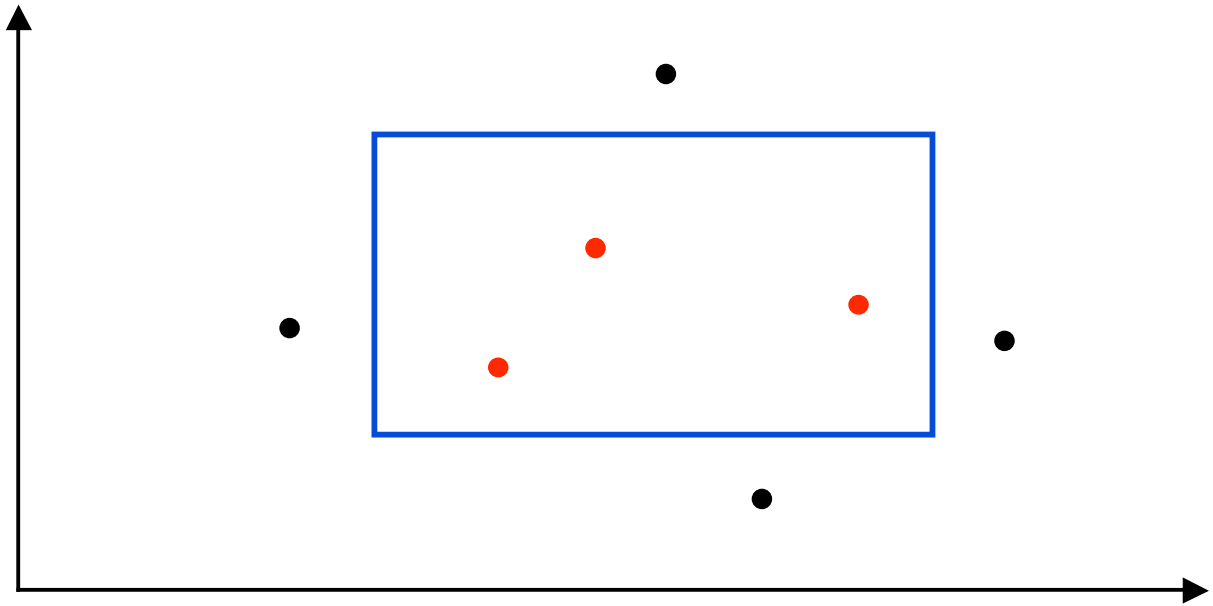
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Range Searching Implementierung eindimensional

```
void range(Interval x, Visitor v) {
    boolean tl = x.l <= key;
    boolean tr = key <= x.r;
    if (tl) left.range(x,v);
    if (tl&tr) v.action(key);
    if (tr) right.range(x,v);
}
```

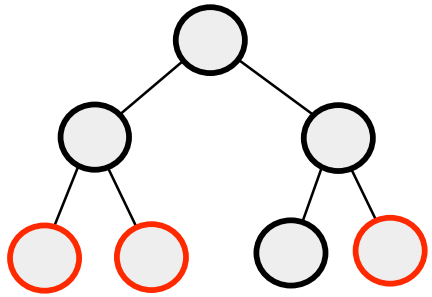
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

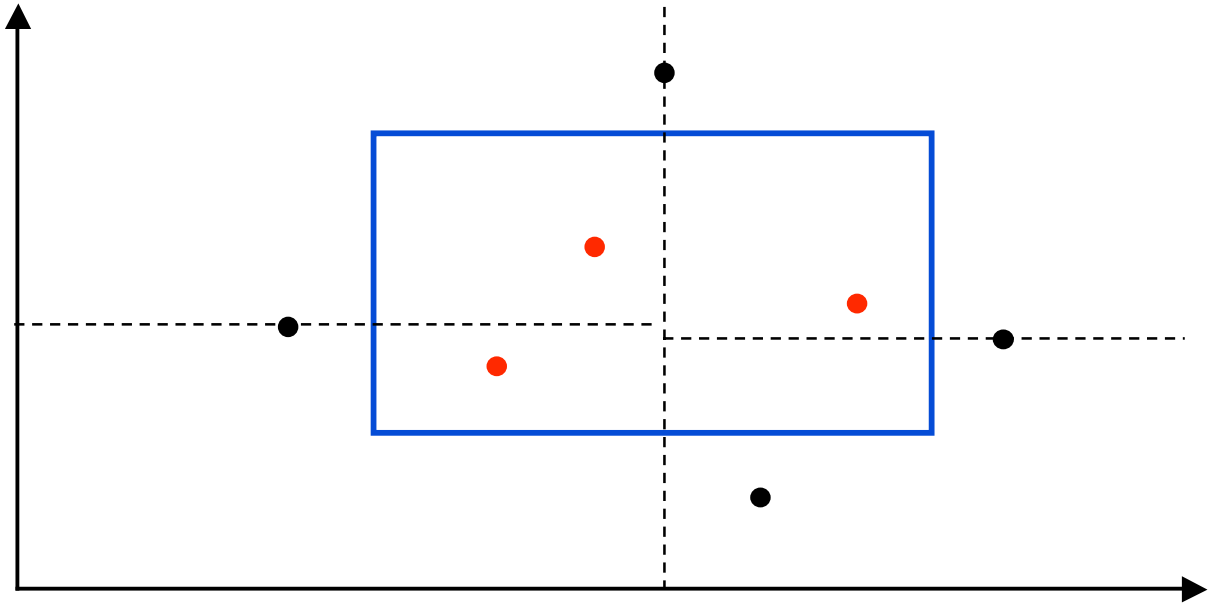
Bereichssuche (Range Searching) zweidimensional O(R+log N)



Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

2D-Suchbaum





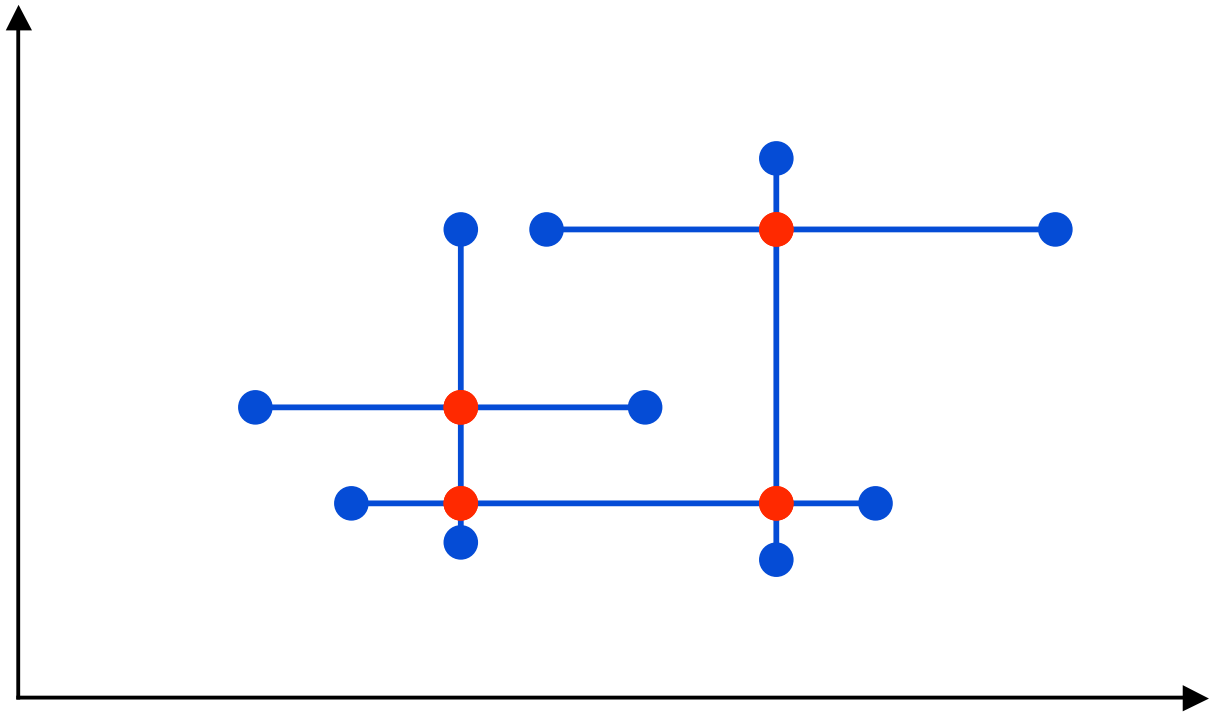
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Range Searching Implementierung zweidimensional (2D-Tree)

```
void range(Rectangle r, Visitor v, int level) {
    boolean tl, tr;
    if (level%2 == 0) {
        tl = r.x <= key.x;
        tr = key.x <= r.x+r.width;
    } else {
        tl = r.y <= key.y;
        tr = key.y <= r.y+.height;
    }
    if (tl) left.range(r,v,level+1);
    if (r.contains(key)) v.action(key);
    if (tr) right.range(r,v,level+1);
}
```

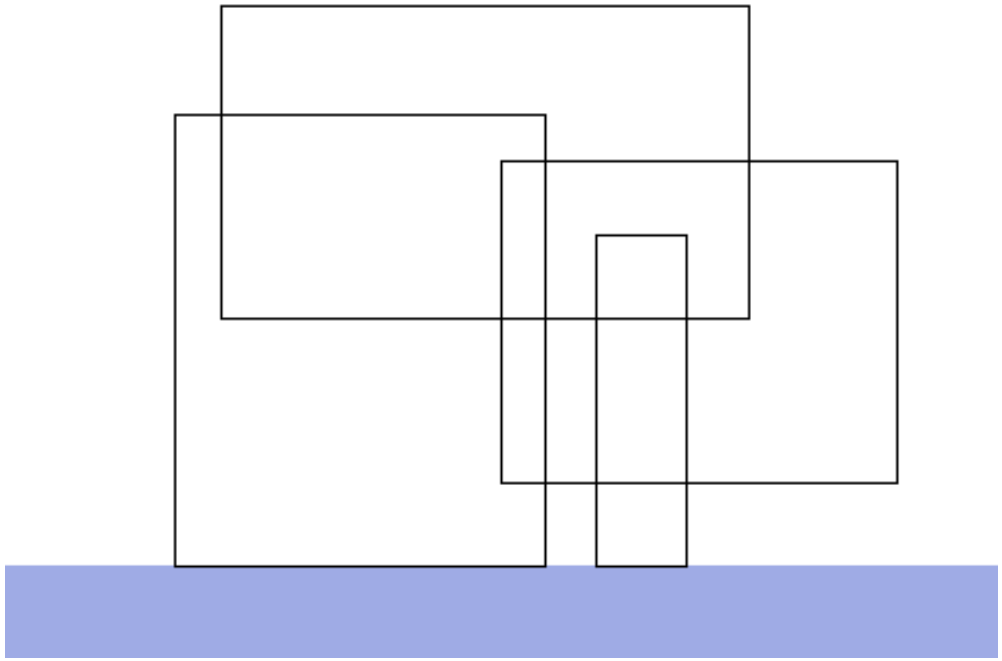
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Segmentschnitt (Segment Intersection) orthogonal O(I+N log N)



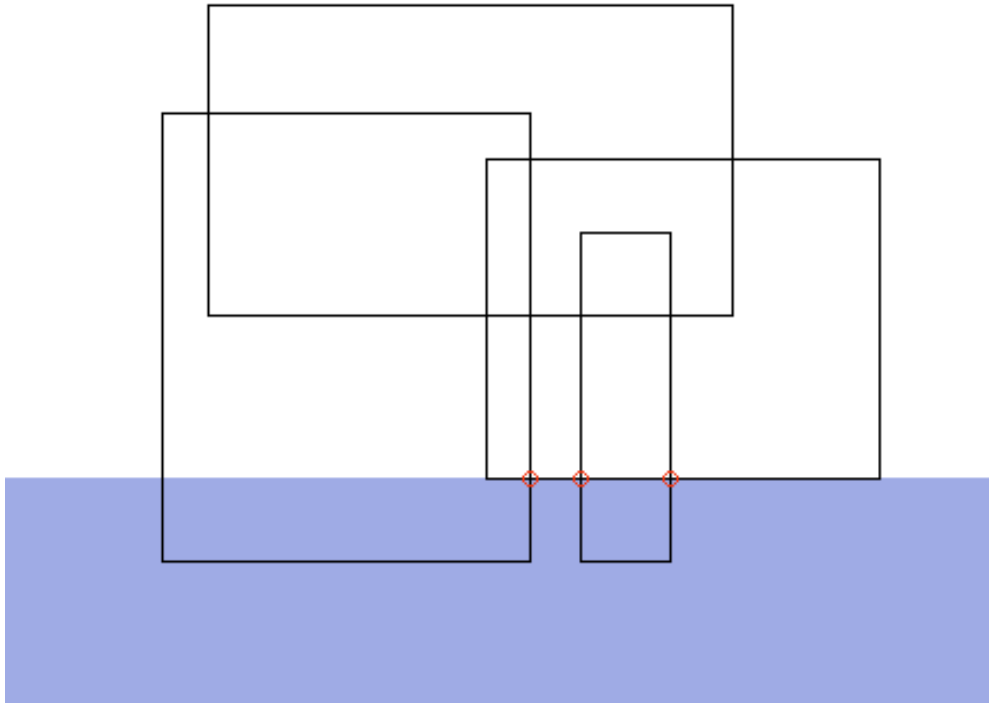
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Rechteckschnitt mittels Sweep-Line (1)



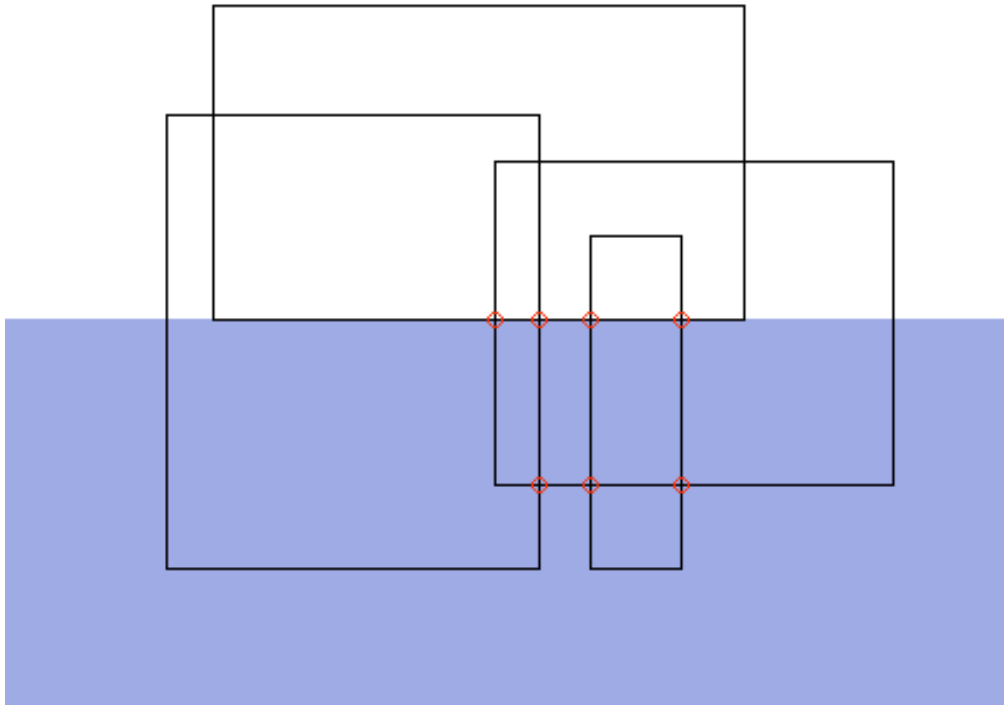
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Rechteckschnitt mittels Sweep-Line (2)



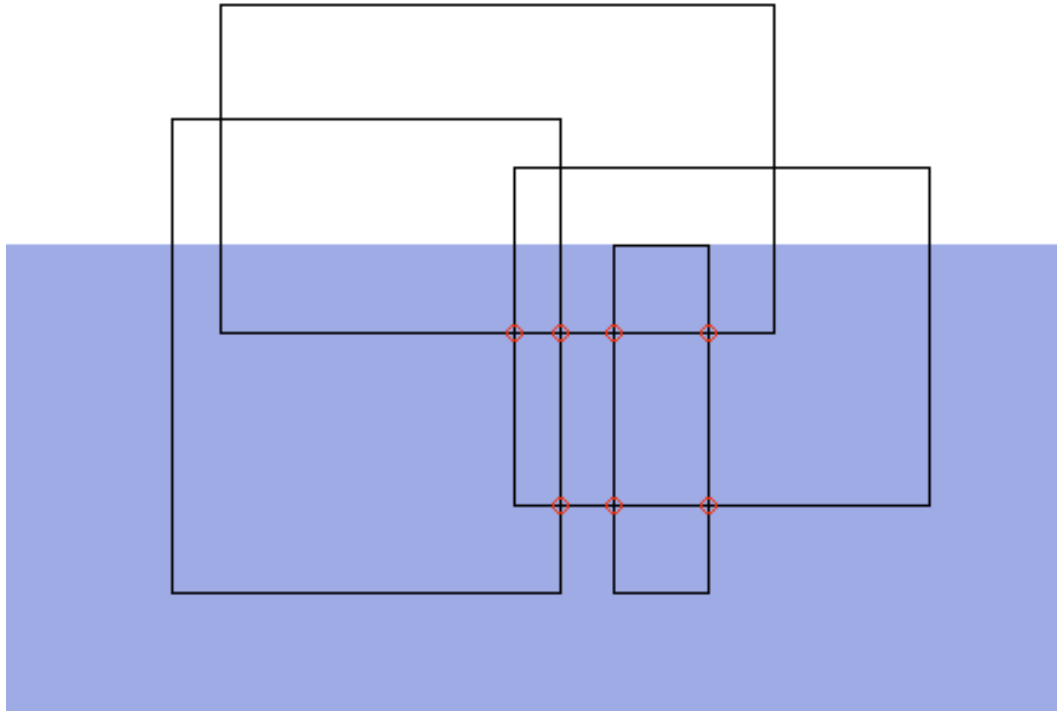
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Rechteckschnitt mittels Sweep-Line (3)



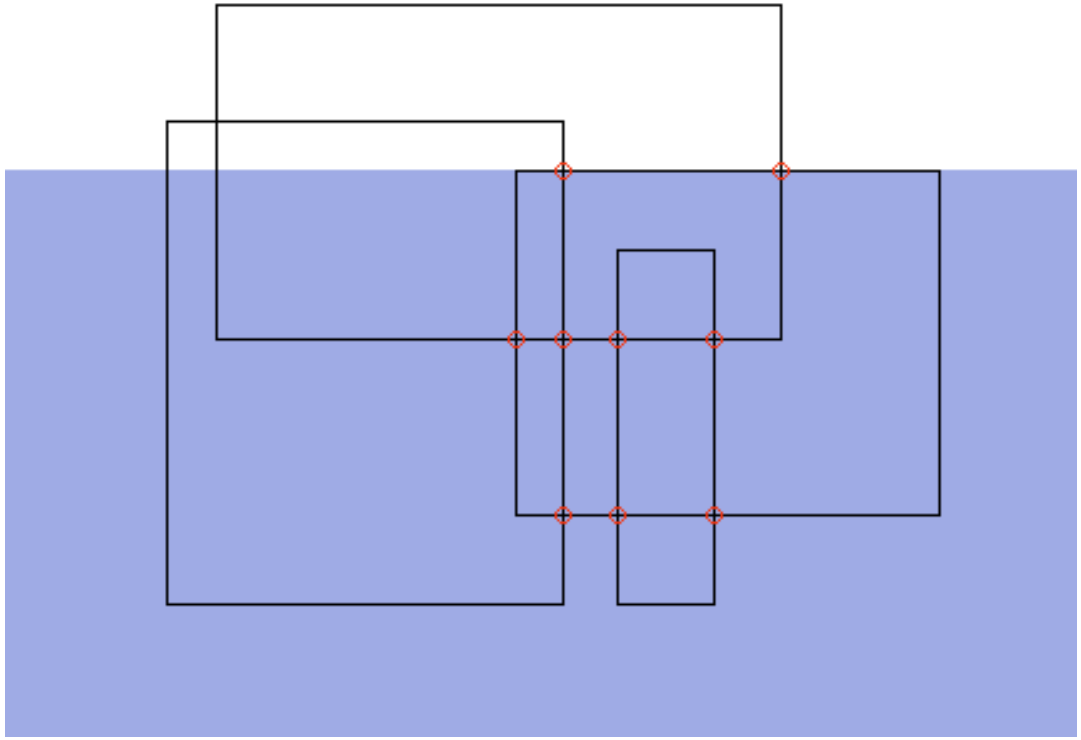
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Rechteckschnitt mittels Sweep-Line (4)



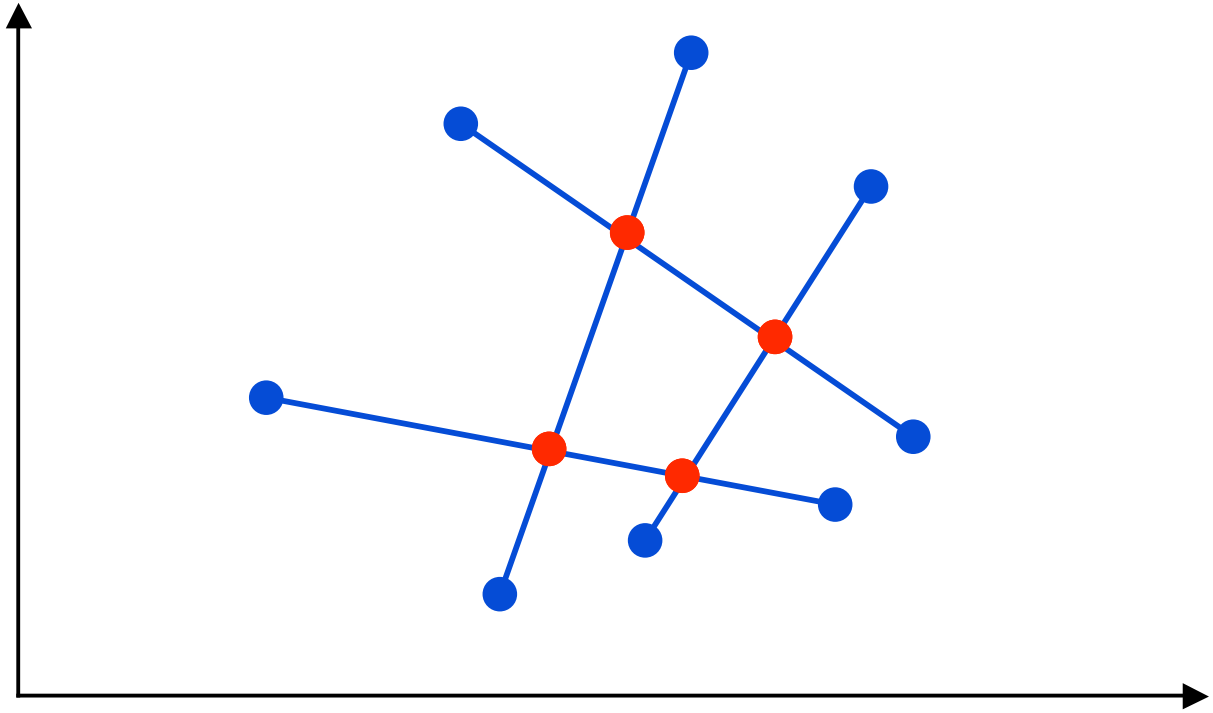
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Rechteckschnitt mittels Sweep-Line (5)



Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Segmentschnitt (Segment Intersection) nicht-orthogonal O((I+N) log N)

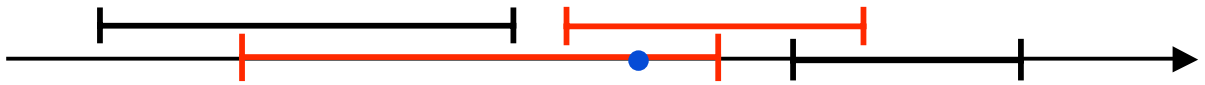


Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Punkteinschluss (Point Inclusion)
O(log N)

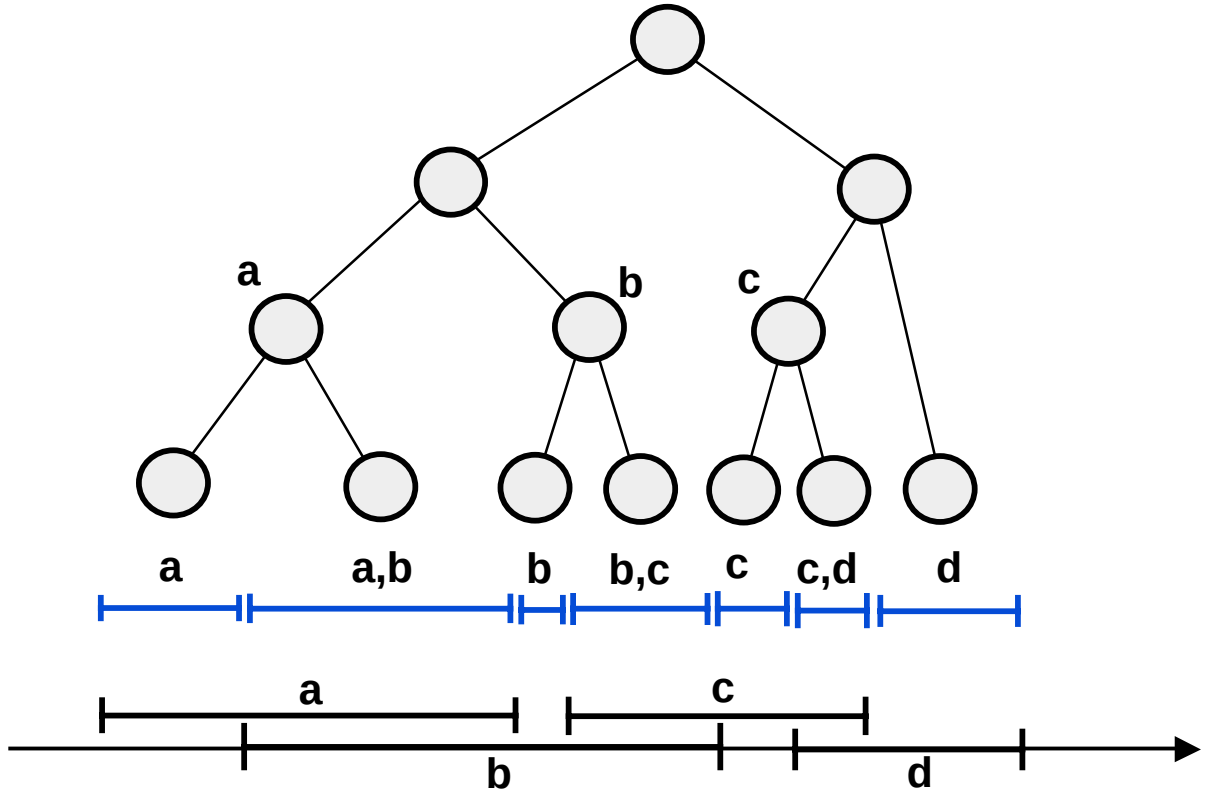
Geg: mehrere Segmente und ein Punkt

Ges: alle Segmente die den Punkt einschliessen



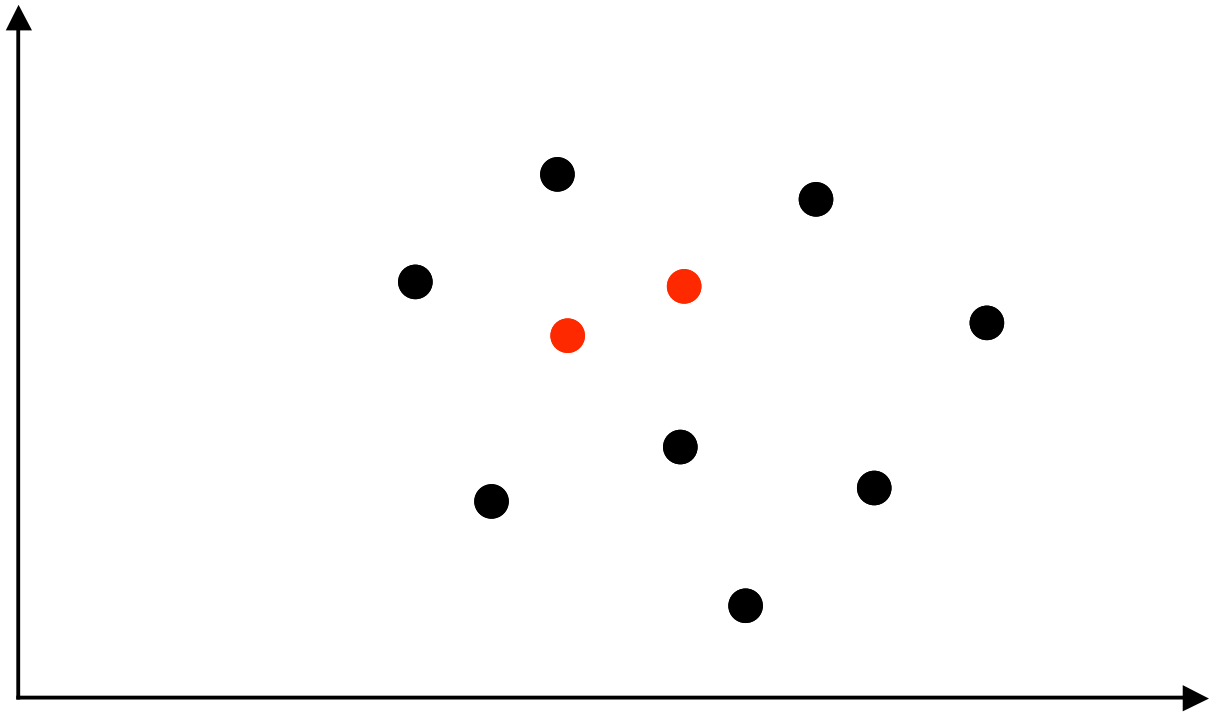
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Segmentbaum (Segment Tree)



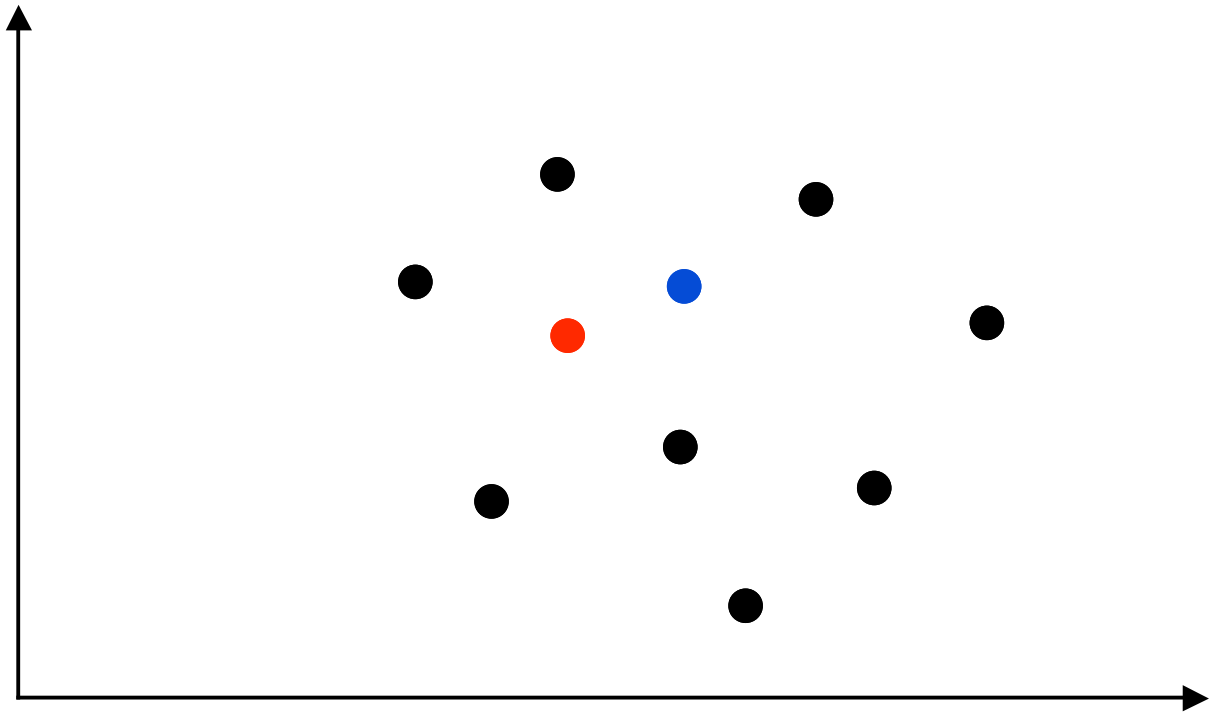
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Closest Pair
O(N log N)



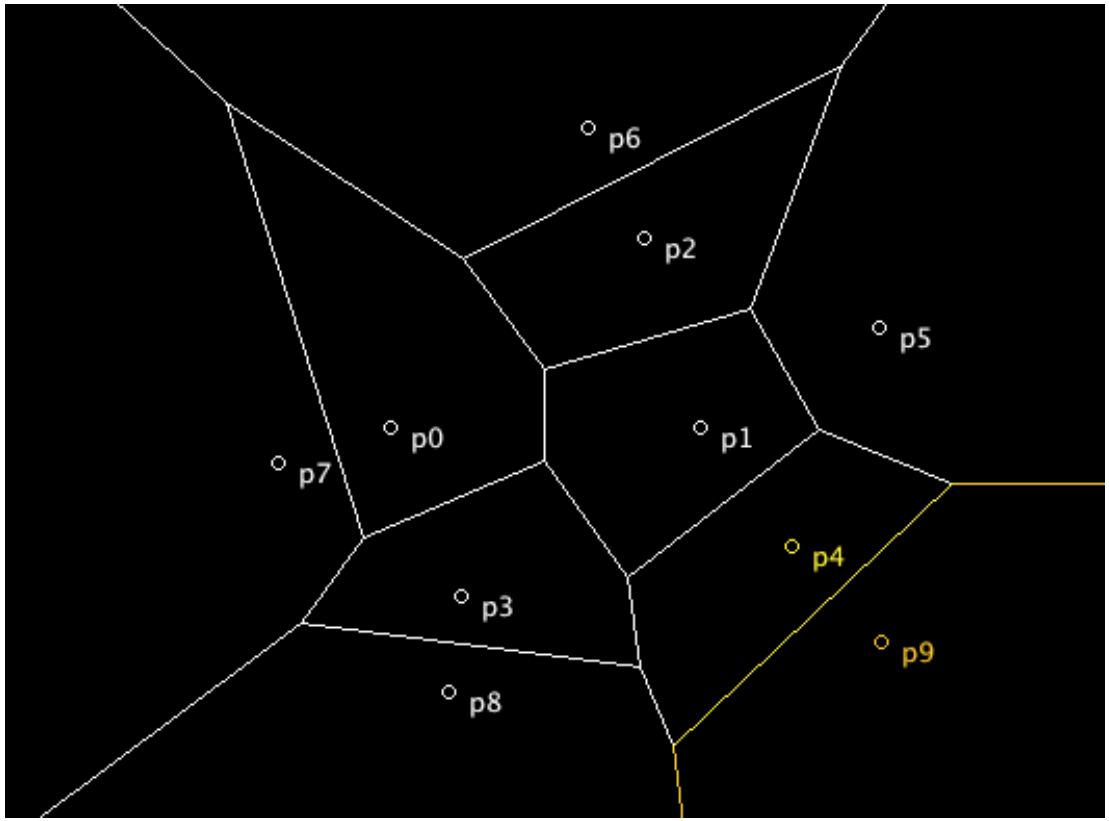
Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Nearest Neighbour
O(log N)



Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Voronoi Diagramm
O(N log N)



Helmut Schauer
Educational Engineering Lab
Department for Information Technology
University of Zurich

Delaunay Triangulierung
O(N log N)