Does Algorithmic Trading Increase Volatility?
Empirical Evidence from the
Fully-Electronic Trading Platform Xetra

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ABSTRACT

Being equipped with a unique high-frequency dataset that enables us to precisely identify algorithmic trading (i.e. computer-generated) activity, we provide strong evidence that algorithmic trading does not exceedingly increases volatility, at least not more than human traders do. Our empirical analyses cover several potential reasons why algorithmic trading could increase volatility. For example, we address whether or not algorithmic traders follow less diverse trading strategies than humans. Moreover, we investigate whether or not algorithmic traders withdraw liquidity from the market during periods of high volatility.

Keywords

Financial markets, algorithmic trading, volatility.

1. INTRODUCTION

Today, up to 40% of executed trading volume on major exchanges includes an algorithmic trader, i.e. a computer, as a trading counterparty [6][11]. Moreover, algorithmic trading even accounts for 50% of electronic message traffic [11]. While algorithmic trading, in terms of market share, has been virtually non-existent a decade ago, the electronification of trading systems has accelerated the double-digit growth since then. With the emergence of algorithmic trading, the interest in the topic by both the public and researchers has significantly grown. The media, however, has primarily satisfied the increased public interest with rather one-sided stories: While the Wall Street Journal concentrates on "*the dark side of algorithms*" (May 7th 2010), the New York Times exemplarily highlights reputed disparities by "*reward[ed] bad actors*" (August 3rd 2009). Beyond that the Financial Times asserts that "*algorithmic trades heighten volatility*" (December 4th 2008) or, to be more precise, "*algorithmic trades produce snowball effects on volatility*" (December 5th 2008). Researchers though were – until recently – neither able to confirm nor to defeat the propositions made by the media. Being largely hampered by limited data availability, academics have not been able to conduct rigorous research. In particular the relation between algorithmic trading activity and volatility has not yet been sufficiently investigated. On financial markets, stock prices should ideally reflect the underlying *real* (fundamental) value of listed companies. Thus, high levels of stock price volatility indicate large uncertainty about the value of respective companies. While uncertainty may open up trading opportunities for speculators, most market participants perceive volatility as rather iniquitous. This is particularly true whenever volatility levels do not reflect uncertainty about company values, i.e. expected future cash flows, because a certain group of investors / traders systematically impairs volatility levels.

Rigorous insights into the behavior of algorithmic traders, i.e. computers, on financial markets are essential for – at least – the following reasons:

First, the constant evolution to fully automated securities trading [15] calls for respective adjustments to electronic market design to secure both market integrity and fairness. In order to do so, a thorough analysis of market participants, i.e. agents, is necessary [20]. In this context, algorithmic traders can be classified as autonomous agents that (proactively) interact with other agents [8]. In other words, the rising share of algorithmic trading activity may entail necessary changes to electronic market design from both a "mechanism designer" and a "system designer" viewpoint.

Second, gained insights on the influence of information technology (IT) on market quality are most useful in the current discussion on regulation of algorithmic or high frequency traders. In a similar context, [19] also see the necessity to investigate "how the use of intelligent agents influences the market structure, the market process, the interaction of market participants, the role of intermediaries, and the efficiency of electronic markets".

To sum up, in order to fill inherent research gaps our overall research question is as follows: does algorithmic trading increase volatility?

Regarding algorithmic trading, there is no single accepted definition yet. Traditionally, algorithmic trading is often limited to "the automated, computer-based execution of equity orders […],

usually with the goal of meeting a particular benchmark" [7]. In other words, algorithmic traders are often limited to emulate a broker's core competence: as illustrated in Figure 1 (*Definition I*), investors interpret relevant information and make an investment decision. Brokers are then asked to implement the decision on the market by achieving best possible prices. Brokers (or alternatively also investors with direct market access) may, however, alternatively also use algorithmic trading engines to execute the orders, i.e. for example employ an algorithm that automatically slices a large order into smaller pieces to reduce market impact.

Nonetheless, algorithmic traders may also follow their own active trading strategies derived from available information (*Definition II*). The strategies are usually designed to close with a flat position at the end of the trading day. Therefore, from our point of view a broader definition of algorithmic trading seems more viable, i.e. the "use of computer algorithms to manage the trading process" [12].
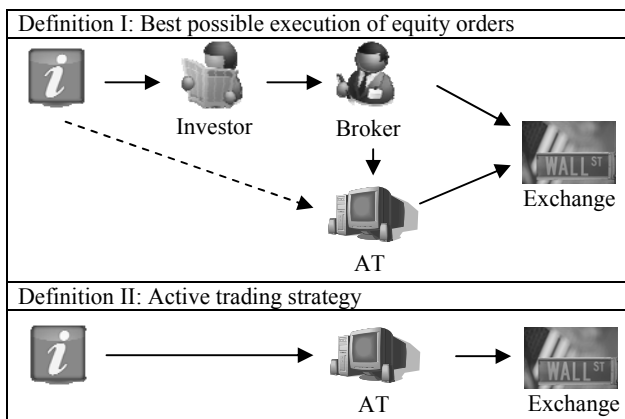

Figure 1: Simplified illustration of the trading value chain

We approach the overall research question as follows: we gather pieces of evidence that will allow us to answer whether or not algorithmic traders are – mainly – responsible for high volatility levels. Hereby, we pick up different arguments already brought forward by both researchers and the media. In particular, we analyze the heterogeneity of algorithmic trading strategies (section 4), the impact of algorithmic trading on volatility levels (section 5), the algorithmic trading liquidity supply during periods of high volatility (section 6), and the algorithmic trading order cancellation behavior during periods of high volatility (section 7). Consequently, we will finally be able to offer rigorous support in the relevant discussion on the role of technology and computers in today's financial markets. This knowledge is for example highly relevant in the course of the currently held discussion on algorithmic trading regulation (section 8).

2. RELATED WORK

Besides above depicted (German) information systems literature, i.e. "market engineering" and the "impact of IT", our research also contributes to finance literature. With the emergence of algorithmic trading concepts both researchers and practitioners were interested in whether or not algorithmic trading, compared to traditional brokerage with human intermediaries, actually creates additional value [3]. For example, [7] found the execution quality of the analyzed early-stage algorithms to be inferior; but as employing human brokers is considerably more costly, the use of algorithmic trading was assessed as basically creating value.

An additional, primarily practitioner-oriented, stream of research aims to answer which kind of algorithm to choose for which kind of task and how to actually evaluate its` success [14][22]. The continuously rising market shares of algorithmic trading during the last decade, however, also called for research on the influence of algorithmic trading on the market as a whole. Despite the eligible interest in this topic, there is only little empirical research in this area primarily because of the lack of appropriate data. The stream of research that concentrates on the influence of algorithmic trading on the market as a whole can be divided into two sub-categories in line with the analyzed market quality indicators, i.e. liquidity and volatility. In this context, the research questions are usually whether or not algorithmic trading decreases liquidity levels and / or whether or not algorithmic trading increases volatility levels.

For the U.S. market, [12] find that algorithmic trading likely improves liquidity rather than impairs liquidity levels. They are, however, not able to precisely identify algorithmic trading activity. Instead, [12] use the normalized measure of New York Stock Exchange (NYSE) electronic message traffic as a proxy for algorithmic trading activity. Given that both [9] and [11] observe that algorithmic traders, for example, tend to continuously adjust their existing orders on a (milli-) second basis, the assumption that there is a connection between electronic message traffic and algorithmic trading activity seems viable. Nonetheless, their proxy still remains very unspecific and may not appropriately pick up variations in algorithmic liquidity supply. In this context, [9] points out that algorithmic traders "blur traditional definitions" on how liquidity is supplied to the market. According to [9], the liquidity provided by algorithmic traders tends to be rather transient and therefore liquidity measures that are based on committed liquidity need to be questioned.

Being equipped with an intraday high-frequency dataset similar to ours that allows for the precise identification of algorithmic traders, [13] provide evidence that algorithmic traders should improve both price efficiency and market liquidity. Nonetheless, [13] were merely provided with data on orders submitted by algorithmic traders, i.e. not all order book events. Therefore, they were not able to for instance identify traded volumes between humans or between humans and algorithmic traders (see Section 4). Finally, [16] provide further evidence that algorithmic traders increase liquidity and the informativeness of prices. Their insights were gained through a natural experiment, i.e. a system upgrade (release 8.0) of the fully-electronic trading platform Xetra that reduced round trip system latency from 50ms to 10ms.

Regarding volatility, however, there are only very few rigorous research results available. The most important work by [2] has been conducted in the foreign exchange market where algorithmic trading is a far more recent phenomenon than in the equity market. Despite of less diverse trading strategies among algorithmic traders, [2] conclude that, if anything, the presence of algorithmic trading is associated with lower volatility. Similarly, [10] finds that algorithmic trading has the potential to lower market volatility. The study is, however, merely based on artificial data generated within a controlled simulation environment.

Overall, we may conclude that existing research suggests that algorithmic trading does not increase volatility. Nonetheless, we have also seen that this stream of research still lacks rigorous empirical research. Therefore, we aim to close this gap. While other authors make use of questionable proxies for algorithmic

trading activity [12], merely analyze an incomplete dataset [13], or work in an idealized and simplified simulation-based environment [10] we contribute to existing literature by analyzing a complete real-world equity market high-frequency dataset that contains the best currently available proxy for algorithmic trading activity.

3. DATA DESCRIPTION

Below empirical analyses are based on a unique high-frequency dataset directly provided by Deutsche Börse AG, i.e. the operator of the German Frankfurt Stock Exchange. The Frankfurt Stock Exchange offers both floor-trading and fully-electronic trading via Xetra. In 2007, 98.30% of order book turnover in German blue-chip DAX30 equities took place on Xetra [6].

The provided dataset contains all Xetra order book events during the period under investigation, i.e. between October 8th 2007 and October 12th 2007. Each order, which is assigned a unique order number by the trading system, should at least trigger two events (Table 1): first, a submission event and second either a full execution or a cancellation / deletion event. Each order can be partially executed and / or modified more than once. In Xetra, a modification event merely refers to a reduction of order volume. An increase in order volume would negatively affect the priority or execution probability of other orders. In this case, the system automatically generates a deletion event for the modified order and a new order entry event with increased volume. Analogue, technical deletion and insertion events occur due to changing trade restrictions that do not affect the price-time priority of other orders.

Table 1: Summary of order book events

Event	Event frequency		% of all events
	AT	H	
Submission	2,171,613	1,936,909	46.47 %
Cancellation	1,558,511	1,518,736	34.81 %
Full execution	583,638	374,028	10.83 %
Partial execution	230,430	300,730	6.01 %
Others	28,138	56,255	0.95%
Modification	51,722	30,260	0.93 %

For each event the following additional information is provided: timestamp, international security identification number (ISIN), order number, auction trade flag, order type, buy/sell indicator, (hidden) size, price / limit, event code, trade restriction, and ATflag.

The auction trade flag indicates the trading phase, e.g. continuous trading, during which the specific event occurred. One order may reveal different auction trade flags as for example order submission and order execution can take place during different trading phases. Order type indicates whether an order is a limit order, market order, iceberg order or market-to-limit order. Orders may also be restricted to be exclusively executed during a certain trading phase (trade restriction), e.g. auctions.

The ATflag indicates whether (ATflag = 1) or not (ATflag = 0) a certain event has been triggered by an algorithm. It does not allow the identification and exploitation of activities of single market participants though. The identification of algorithms is made possible because Deutsche Börse AG offers its clients a special pricing model for computer generated trades called Automated

Trading Program (ATP). Participants of the Automated Trading Program oblige themselves to exclusively make use of the rebate-relevant Automated Trading User-ID whenever transactions have been generated by an electronic system. The definition of an electronic system is as follows:

The electronic system has to determine two out of the three following order parameters: price (order type and / or order limit where applicable), timing (time of order entry) and quantity (quantity of the order in number of securities). [...] The electronic system must generate buy or sell orders independently, i.e. without frequent manual intervention, using a specified program or data. [5]

Considering both above "electronic system" definition and granted financial incentives (fee rebates), the algorithmic trading flag can be appreciated as the best proxy for algorithmic trading activity currently available. In other words, we are able to differentiate between orders submitted by humans and orders submitted by algorithms, i.e. by one of the two groups. It shall, however, be noted that despite of the strong financial incentives not all algorithmic traders may take part in the program. As exemplarily shown by [13] the fee rebates "for high-frequency trading firms, whose turnover is much higher than the amount of capital invested, the savings [associated with the automated trading program] are significant". Therefore, the identification of algorithmic traders via the automated trading program is seen as the best currently available proxy for algorithmic trading activity. During the following we will therefore assume that ATP members are equivalent to algorithmic traders (AT) and that the remaining non-ATP members are humans (H).

The dataset allows for an order book reconstruction of covered DAX30 securities at any time during the period under investigation, including all trading phases. Basically, all orders submitted prior to the time of interest, i.e. order book reconstruction, that are not fully executed, cancelled or deleted (including "deleted" invalid day orders) remain in the order book. The actual order limits are determined by further incorporating partial executions and modifications. The order entry timestamp allows for the consideration of time priority.

4. DIVERSITY OF ALGORITHMIC TRADING STRATEGIES
4.1 Motivation

In this section we do not attempt to approach the overall research question by directly linking algorithmic trading activity and observed volatility levels. Instead, we first want to gather rather indirect evidence for or against the volatility-increasing proposition of algorithmic traders by assessing the diversity of algorithmic trading strategies.

If algorithmic traders tended to follow similar active trading strategies, these should crowd on the same side of the market. [17] also state that "volatility increases under one-sided conditions". For example, let us assume that all algorithmic traders were following a strategy that capitalizes on an expected relationship between the movement of stock A and the time-delayed movement of stock B. Let us further assume that the developers of this algorithmic trader employ similar models because of identical available input, i.e. historical price time series. If the price of stock A increases (by for instance more than 2%), the algorithmic traders expect the price of stock B to increase, too. Therefore, all

algorithmic traders should try to build up a long position in stock B, i.e. buy shares in stock B. Depending on their level of aggressiveness, these would then either try to instantaneously buy shares via submitting buy market orders or try to get hit by another traders' aggressive sell orders by submitting passive buy limit orders. In both cases algorithmic traders were crowding on the same side of the market, i.e. the bid side. The combination of many aggressive buy market orders and no passive liquidity supplying sell orders (provided by algorithms) would result in large swings of prices, i.e. high volatility. It is against this background that the evaluation of algorithmic trading strategies should enable us to gain valuable insights to answer our research question.

We evaluate whether or not algorithmic traders – in aggregate – follow less diverse trading strategies by means of two different research approaches. If both approaches, i.e. methodologies, yield equivalent results, then our inferred implications are seen as robust.

4.2 Methodology I: Benchmark Model

In order to investigate the correlation of algorithmic strategies we adopt the approach proposed by [2] who argue that algorithmic traders are expected to trade less among themselves and more with humans, if the algorithmic traders follow homogeneous trading strategies. "At the extreme, if all computers used the very same algorithms and had the exact same speed of execution, we would observe no trading volume among computers. Therefore, the fraction of trades conducted between computers contains information on how correlated their strategies are" [2].

On the basis of a simple benchmark model that assumes random and independent matching of trades we are able to determine the theoretical probabilities of trades conducted between and among the two groups of traders, i.e. humans and algorithmic traders. Additionally differentiating between the aggressive counterparty who triggered a trade and the passive counterparty who has been hit by an aggressive order, four possible trade combinations are possible (*passive/aggressive*): *human/human*, *algorithmic trader/human*, *human/algorithmic trader*, and *algorithmic trader/algorithmic trader*.

After minor transformations (see Appendix I), the relation between the four trade combination probabilities can be re-written as follows [2]:

$$\underbrace{Prob(H/H) \ / \ Prob(AT/H)}_{R\_H} = \underbrace{Prob(H/AT) \ / \ Prob(AT/AT)}_{R\_AT}$$

From above equation two ratios can be extracted: "R_H" (i.e. aggressive human) and "R_AT" (i.e. aggressive algorithmic trader). Both ratios might be above one if the number of human traders is larger than the number of algorithmic traders. Given above assumption of a random matching process the ratio of ratios R = R_AT / R_H, however, will be equal to one. This is the case if humans and algorithmic traders, both being the aggressive trading counterparty, take about the same proportion of liquidity from (other) passive humans.

Next, ex-post proxies for R_H, R_AT and R are calculated on a daily basis for each security in above introduced dataset in the following manner:

R_H$_{proxy}$ = $Vol(H/H)$ / $Vol(AT/H)$

R_AT$_{proxy}$ = $Vol(H/AT)$ / $Vol(AT/AT)$

R$_{proxy}$ = R_H$_{proxy}$ / R_AT$_{proxy}$

Thereby *Vol(passive/aggressive)* marks the daily trading volume between the respective passive and aggressive trading counterparties. Finally, the mean / median of the cross-sectional daily ratio of ratios R$_{proxy}$ is compared to the theoretically derived ratio of ratios R. We consequently formulate the following null- and alternative hypotheses:

$$H_0 : \mu\left(R_{proxy}\right) = 1 \quad vs. \quad H_A : \mu\left(R_{proxy}\right) \neq 1$$

The rejection of the null hypothesis would provide evidence that algorithmic trading strategies are more homogeneous than the trading strategies applied by human traders.

4.3 Empirical Results I: Benchmark Model

Descriptive statistics for the ratio of ratios R$_{proxy}$ can be found in Table 2. Given the descriptive statistics it can be observed that both the mean and the median values are close to one, i.e. close to the theoretically derived benchmark ratio or ratios R.

Table 2: Descriptive statistics for the ratio of ratios

	Mean	Median	Standard deviation
R$_{proxy}$	1.0211	0.9874	0.2541
N	150		

Test results for above defined hypotheses are summarized in Table 3. Test results for both the mean (t-test) and the median (Wilcoxon signed rank test) provide further evidence that above defined null hypotheses may not be rejected.

Table 3: Test results for the ratio of ratios

Hypothesis	μ	Significance
$H_0 : \mu\left(R_{proxy}\right) = 1$	mean	0.310
$H_0 : \mu\left(R_{proxy}\right) = 1$	median	0.892

The results provide evidence that algorithmic traders trade with each other as much as random matching would predict. Therefore, we conclude that algorithmic strategies are just as diverse as human strategies. It follows that algorithmic traders as a whole should not particularly increase volatility because of homogeneous trading strategies, i.e. herding.

This is, however, only true for the equity market (Xetra). In the foreign exchange market, for example, [2] find the opposite to be true. One possible explanation might be found with the fact that algorithmic traders in the equity market follow active profit-generating trading strategies, compared to mere execution strategies, less exclusively than algorithmic traders in the foreign exchange market. Such a concentration on active profit-generating strategies would result in a less diverse trading strategy universe.

4.4 Methodology II: Market Sidedness

In order to further investigate the role both humans and algorithmic traders play in volatile market phases we additionally apply the market sidedness measure proposed by [17]. Similarly to above introduced ratio of ratios, the market sidedness measure also reveals whether or not market participants follow similar strategies.

4.4.1 Market Sidedness

If algorithmic traders followed homogeneous trading strategies, we would expect them to trade on the same side of the market, i.e. either primarily buy or sell. The market sidedness measure provides us with insights into whether trading has been more one-sided or more two-sided during a certain period of time. If algorithmic traders trade more one-sided than human traders during periods of high volatility, we could assume them to follow less diverse trading strategies than humans.

Market sidedness (MS) is estimated by the correlation between ZBUY and ZSELL [17]:

ZBUY = (BUY – *Mean(*BUY*)) / SD(*BUY*)*

ZSELL = (SELL – *Mean(*SELL*)) / SD(*SELL*)*

BUY (SELL) is the number of buyer- (seller-) initiated trades in an interval, i.e. on which side of the market has the pressure to execute been larger. The identification of relevant intervals is explained in more detail in section 4.4.2. Mean and SD are the sample mean and standard deviation [17].

4.4.2 Identification of High Volatility Intervals

As we are mostly interested in how algorithmic traders and humans behave during periods of high volatility, we need to identify those. Being provided with a high-frequency intraday dataset, we divide the trading day into 5-minute intervals. For reasons of better comparison we concentrate on continuous trading phases only. Those 5-minute intervals than contain any kind of auction (e.g. closing auction) have been excluded from the analysis.

For the calculation of volatility levels within each of the 5-minute intervals we apply a risk model from [4] that is based on short-term price volatility. Realized volatility, $\sigma_{[t1,t2]}$, is calculated using transaction returns in the following manner:

$$\sigma_{[t1,t2]} = \sum_{i=1}^{N} r_{i,[t1,t2]}^2$$

Here, $r_{i,[t1,t2]}$ is defined as the return of the ith transaction during time interval [t1, t2], i.e. each 5-minute interval. In financial risk research, the importance of realized volatility has been emphasized by for example [1].

In order to account for microstructure effects such as negative autocorrelation that may negatively affect the validity of our volatility measure, we adjust the $\sigma_{[t1,t2]}$ measure with a *Bias* factor [4]:

$$Bias = \frac{\sqrt{q} \; \sigma_{\Delta t}}{\sigma_{\Delta t \; ref}} \qquad \text{with } \Delta t_{ref} = q \cdot \Delta t$$

The bias factor is calculated by observing a bias-free reference case (with a large enough time interval Δt_{ref}) to judge the bias of smaller intervals Δt. As proposed in [4], one working day has been used as Δt_{ref} and *Bias* has been calculated on the basis of a one year price history for each DAX30 security. On the basis of the calculated Bias that is measured in terms of how much *Bias* deviates from 1, a corrected realized volatility measure $RV_{[t1,t2],corr}$ is calculated by:

$$RV_{[t1,t2],corr} = \sigma_{[t1,t2]} / Bias$$

Given the realized (corrected) volatility value for each 5-minute interval during the period under investigation for each security,

the high (low) volatility intervals were identified as those being in the top (bottom) 5% (percentile). The empirical distribution of realized volatility for all 30 securities during the period of investigation is presented in Figure 2.
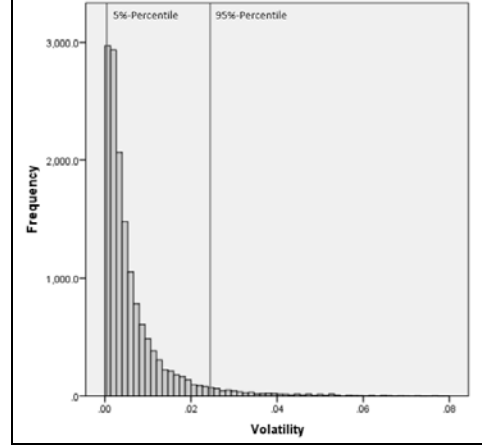

Figure 2: Empirical distribution of realized volatility

As it can be observed in Figure 3 the distribution of high (low) volatility intervals is in line with what literature suggests about intraday volatility shapes, i.e. high volatility levels at the beginning of the trading day and rather low volatility levels at the middle of the trading day [21].
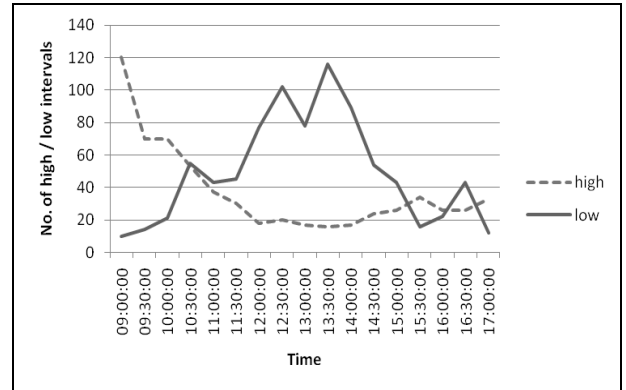

Figure 3: Distribution of high / low volatility intervals

4.4.3 Evaluation Setup

ZBUY and ZSELL are calculated for each 5-minute high-volatility interval. The market sidedness measure MS, i.e. the correlation between ZBUY and ZSELL, is calculated for each security resulting in 30 MS values.

If algorithmic traders followed less diverse trading strategies than humans, we would expect them to trade more one-sided. If algorithmic traders consequently followed less diverse trading strategies during periods of high volatility, we could – at least – argue that there is an interrelation between algorithmic trading activity and volatility because of the use of homogeneous trading strategies by algorithmic traders. We consequently formulate the following null- and alternative hypotheses:

$$H_0 : \mu\!\left(MS_{high}^{H}\right) \leq \mu\!\left(MS_{high}^{AT}\right) \quad vs. \quad H_A : \mu\!\left(MS_{high}^{H}\right) > \mu\!\left(MS_{high}^{AT}\right)$$

Hereby, MS$_{high}$ depicts market sidedness for periods of high volatility only. MS^H (MS^{AT}) depicts market sidedness for humans (algorithmic traders) only. Due to the fact that we were provided with detailed high-frequency order book information, we are able to determine ZBUY and ZSELL for the trader groups human and algorithmic traders separately.

4.5 Empirical Results II: Market Sidedness

Descriptive statistics for the market sidedness measure MS can be found in Table 4. Mean correlation values were calculated based on Z_{fisher}-transformed correlation coefficients. The descriptive results provide first evidence that both groups of traders exhibit a similar degree of market sidedness, i.e. during the N = 752 periods (5-minute intervals) of high-volatility.

Table 4: Descriptive statistics for market sidedness

	Mean	Median
MS^{AT}_{high}	0.3866	0.3577
MS^{H}_{high}	0.3802	0.4022

Test results for above defined hypotheses are summarized in Table 5. The median correlations are compared using the Wilcoxon test. Test results provide further evidence that above defined null hypothesis cannot be rejected.

Table 5: Wilcoxon test results for market sidedness

Hypothesis	Significance
$H_0 : \mu\left(MS^{H}_{high}\right) \leq \mu\left(MS^{AT}_{high}\right)$	0.926

Given above results we may conclude that both groups of traders, i.e. both algorithmic traders and humans, exhibit similar degrees of market sidedness during periods of high volatility. In other words, both groups exhibit a similar diversity of trading strategies. In line with the evidence found in section 4, we therefore do not expect algorithmic traders to increase market volatility because of rather homogeneous trading strategies; at least not more than human traders do.

5. IMPACT OF ALGORITHMIC TRADING ACTIVITY ON VOLATILITY

5.1 Motivation

After having gathered rather indirect evidence on the role algorithmic trading plays regarding its' influence of volatility, this section's approach is more straight forward. If algorithmic trading had a worsening influence on volatility, we should observe a causal positive relationship between the two figures. In other words, an increase in algorithmic trading activity should result in an increase in volatility.

5.2 Methodology

In order to derive a causal relationship between algorithmic trading activity and volatility, we estimate a regression equation by means of OLS:

$$RV_{t,corr} = \alpha + \beta \cdot AT_t + \gamma \cdot DT_t + \sum_{k=1}^{5} \delta \cdot RV_{t-k,corr} + \varepsilon_t$$

Hereby, $RV_{t,corr}$ is equivalent to above defined realized volatility measure during interval t = 1, …, T. AT_t stands for two different measures that indicate the degree of algorithmic trading activity. First, the fraction of executed volume with any algorithmic

trading participation, i.e. both passive and aggressive (AT_{any}). Second, the fraction of executed volume with algorithmic traders being aggressive, i.e. taking liquidity ($AT_{aggressive}$).

$$Vol(all) = Vol(H/H) + Vol(H/AT) + Vol(AT/AT) + Vol(AT/H)$$

$$AT_{any} = (Vol(H/AT) + Vol(AT/AT) + Vol(AT/H)) / Vol(all)$$

$$AT_{aggressive} = (Vol(H/AT) + Vol(AT/AT)) / Vol(all)$$

DT_t constitutes a set of time dummies to control for intraday variations of volatility. Moreover, k lags of the realized volatility measure $RV_{t,corr}$ are included to control for the strong serial correlation in volatility [1]. Finally, ε_t is the error term. While the analysis in section 5 has been restricted to a certain selection of volatility intervals, the respective inputs into the OLS regression are calculated for each 5-minute interval (but still without those that contain auctions).

As highlighted by [2], the error term ε_t may not be uncorrelated with AT_t. This potential endogeneity issue arises because algorithmic trading activity may not only have an influence on volatility, but volatility levels may also influence the behavior of algorithmic traders. As it is not clear in which direction a possible bias will go, we additionally adopt an instrumental variable (IV) approach to deal with the endogeneity issue.

Hereby, we need to find an instrumental variable that is uncorrelated with the error term ε_t, but correlated with AT_t. As it is very difficult to find an instrumental variable that meets these criteria, we adopt an alternative method. Thereby, the instrumental variable is created by lagging the troublesome variable, i.e. AT_t. Operationally, the problem is solved by means of Two-Stage Least-Squares Regression (2SLS).

5.3 Empirical Results

Results for both the OLS and the 2SLS regression can be found in Table 6. While the model is not capable to explain variations in volatility too well – given the low R^2 value – the coefficients for the algorithmic trading activity measure AT_t are significant at very high levels of significance. Moreover, the coefficients of both the OLS and the 2SLS regression point into the same direction. The negative coefficients imply that the participation of algorithmic traders is not associated with higher levels of volatility, but – if at all – with lower levels of volatility.

Table 6: Regression results of algorithmic trading impact

	OLS estimation	2SLS estimation
Coeff. on AT_{any}	- 0.005 ***	- 0.007 ***
R^2	0.176	0.174
Coeff. on $AT_{aggressive}$	- 0.004 ***	- 0.006 ***
R^2	0.177	0.174
*** indicates significance at the 1%-level.		

To conclude, the results of the regressions provide further evidence that algorithmic traders should not be made responsible for high(er) levels of volatility.

6. VOLATILITY & THE ROLE OF LIQUIDITY SUPPLY

6.1 Motivation

So far, we have primarily concentrated on the aggressive / liquidity-taking behavior of algorithmic traders. [13], however,

note that "AT could also exacerbate volatility by not supplying liquidity". In other words, volatility may not only be increased by submitting aggressive market (or limit) orders, but additionally by not supplying liquidity during periods of high volatility. Moreover, [9] argues that the liquidity contribution of algorithmic traders is more transient anyway. For example, think of the following – rather extreme – limit order book situation:

Table 7: Exemplary order book

Bid		Ask	
Size	Limit (€)	Limit (€)	Size
1,000	50.00	50.10 *	500 *
5,000	49.50	50.50 *	1,000 *
5,000	49.10	55.00	1,000
1,000	47.70	80.30	1,000

While the absolute spread, i.e. the difference between best bid (€ 50.00) and best ask (€ 50.10) is comparatively small, the committed liquidity on the ask side is already comparatively low. The mid-point, i.e. the middle between best bid and best ask, is at € 50.05. If those orders that are marked with a star (*) are now deleted, without being executed, the new mid-point would already be € 52.25. If additionally a market buy order is submitted, it will also be matched at € 55.00 (price).

If we observed algorithmic traders to actually withdraw much liquidity from the order book during periods of high volatility, we could assume that there exists a connection between their actions and the observed volatility levels.

6.2 Methodology
Aiming to analyze whether algorithmic traders are responsible for higher levels of volatility because of lower liquidity contributions, we need to define appropriate liquidity proxys. In this case, we believe that liquidity contribution should not be scrutinized in isolation. Instead, it should also be taken into account how much liquidity has been taken from the order book. This allows us a better view on the trend in overall liquidity levels.

The first proposed liquidity-balance measure is based on committed liquidity, i.e. the liquidity provided by means of limit orders in the limit order book. Submitted orders increase the volume of committed liquidity and cancelled orders decrease the volume of committed liquidity. Therefore, the liquidity-balance during a certain interval is given by the net-submissions (*NSub*).

NSub = Vol(Submission) – Vol(Cancellation)

In order to compare the net-submission values among trader groups, a net-submission ratio (*R_NSub*) is additionally calculated for each trader group and each interval.

R_NSub = Vol(Submission) / Vol(Cancellation)

If *R_NSub* is larger than one, then more volume has been provided than withdrawn during the relevant period. If algorithmic traders were responsible for high volatility levels, we would expect them to have a significantly lower R_NSub^{AT} than humans during the relevant intervals (and an R_NSub^{AT} below one). We consequently formulate the following null- and alternative hypotheses:

$$H_0 : \mu\left(R\_NSub_{high}^{AT}\right) \le \mu\left(R\_NSub_{high}^{H}\right)$$
vs.
$$H_A : \mu\left(R\_NSub_{high}^{AT}\right) > \mu\left(R\_NSub_{high}^{H}\right)$$

In order to find evidence whether or not *R_NSub* actually has a worsening influence on volatility, the following regression equation is estimated:

$$RV_{t,corr} = \alpha + \beta \cdot R\_NSub_t + \gamma \cdot DT_t + \sum_{k=1}^{5} \delta \cdot RV_{t-k,corr} + \varepsilon_t$$

Compared to the regression conducted in section 6, the only difference can be found with R_NSub_t that replaces AT_t. Within 2SLS, of course, R_NSub_t is lagged. The second proposed liquidity-balance measure is based on the liquidity provided during executions. The net-provided trading volume figure is calculated as follows:

$R\_Nlq^{AT} = (Vol(AT/H) + Vol(AT/AT)) / (Vol(H/AT) + Vol(AT/AT))$
$R\_Nlq^{H} = (Vol(H/AT) + Vol(H/H)) / (Vol(AT/H) + Vol(H/H))$

Analogue to above net-submission ratio we formulate the following null- and alternative hypotheses for the net-provided trading volume ratio:

$$H_0 : \mu\left(R\_Nlq_{high}^{AT}\right) \le \mu\left(R\_Nlq_{high}^{H}\right)$$
vs.
$$H_A : \mu\left(R\_Nlq_{high}^{AT}\right) > \mu\left(R\_Nlq_{high}^{H}\right)$$

Both *R_NSub* and *R_Nlq* are calculated for both trader groups algorithmic traders and humans.

6.3 Empirical Results
First descriptive statistics on associated volumes of liquidity-related events during high / low volatility intervals can be found in Table 8. Submitted volume, if associated with non-aggressive limit orders, adds liquidity to the order book. In contrast, cancelled volume withdraws committed liquidity from the order book. Therefore, the net-submission volume provides an indication of overall liquidity levels.

It can be observed that algorithmic traders, compared to human traders, seem to be more active during periods of low volatility. Moreover, high volatility intervals exhibit a negative net-submission volume while low volatility intervals exhibit a positive net-submission volume. In other words, during periods of high volatility more volume is actively withdrawn from the order book than added back to the order book (not even taking into account executed volume).

Table 8: Descriptive statistics on mean volumes associated with events

Events	Intervals: High		Intervals: Low	
	ALL	% AT	ALL	% AT
Submission	705,709	0.352	102,620	0.461
Cancellation	1,324,147	0.169	73,316	0.385
Net-Submission (*NSub*)	- 618,438		29,304	

This insight is in line with our argument that volatility may not only be caused because of aggressive market behavior, but also because of extremely passive / cautious liquidity-withdrawing behavior. The descriptive statistics, however, do not tell us whether the net-submission volume is negative because of high volatility levels or whether the volatility levels are high because of negative net-submissions. The regression results that were expected to shed more light on this are not unambiguous (Table 9). Given that the 2SLS estimation is more accurate, we may

conclude though that the net-submission ratio has no significant influence on volatility levels.

Table 9: Regression results for net-submission ratio

	OLS estimation	2SLS estimation
Coeff. on $(R\_NSub^{AT\&H} - 1)$	- 0.002 ***	0.001
R^2	0.179	0.171
*** indicates significance at the 1%-level.		

Nevertheless, aiming to answer whether or not algorithmic trading increases volatility, we analyze the net-submission behavior of algorithmic traders and humans during periods of both high and low volatility (Table 10). Given the descriptive statistics and in particular R_NSub, it can be observed that both groups of traders exhibit lower net-submission volumes during periods of high volatility compared to periods of low volatility. During periods of high volatility, however, algorithmic traders still provide more liquidity than these withdraw liquidity.

Table 10: Descriptive statistics on net-submission volume

	Intervals: High		Intervals: Low	
	Mean	Median	Mean	Median
$NSub^{AT}$	76,164	22,146	25,023	2,188
$NSub^{H}$	- 694,602	- 354,604	4,281	1,201
R_NSub^{AT}	1.349	1.221	1.508	1.231
R_NSub^{H}	0.687	0.436	1.322	1.075

The test results for above defined hypotheses on the mean (t-test) and median (Wilcoxon test) differences in net-submission ratios are depicted in Table 11. For high volatility intervals, both the mean and the median test reveal the same results: the null hypotheses can be rejected at high levels of significance. In other words, humans provided significantly less committed liquidity during high volatility periods. For low volatility intervals, the mean and the median test do not reveal the same results: the null hypothesis for the median, however, can be rejected at a high level of significance.

Table 11: Test results for net-submission ratios

Hypothesis	μ	Significance
$H_0 : \mu\left(R\_NSub_{high}^{AT}\right) \leq \mu\left(R\_NSub_{high}^{H}\right)$	mean	0.001
$H_0 : \mu\left(R\_NSub_{low}^{AT}\right) \leq \mu\left(R\_NSub_{low}^{H}\right)$	mean	0.109
$H_0 : \mu\left(R\_NSub_{high}^{AT}\right) \leq \mu\left(R\_NSub_{high}^{H}\right)$	median	0.000
$H_0 : \mu\left(R\_NSub_{low}^{AT}\right) \leq \mu\left(R\_NSub_{low}^{H}\right)$	median	0.000

Independent of whether not providing committed liquidity leads to higher volatility levels – or at least facilitates larger swings in prices – we may already conclude that algorithmic traders are not to be made responsible for a potential liquidity dry-up effect. Our results indicate that algorithmic traders provide more liquidity to the market than they withdraw from the market during periods of high volatility. It shall, however, also be noted that our analysis does not take into account where – at which level – the liquidity has been added, i.e. has the liquidity contribution been useful to the market. Against this background, we additionally evaluate the net-provided liquidity in actual transactions (Table 12).

Table 12: Descriptive statistics on net-provided traded volume

	Intervals: High		Intervals: Low	
	Mean	Median	Mean	Median
R_Nlq^{AT}	1.174	1.009	1.407	0.836
R_Nlq^{H}	1.098	0.992	2.365	1.035

Descriptive statistics on the net-provided traded volume reveal that the difference in R_Nlq_{high} ratios between algorithmic traders and humans is not as obvious as it was in the earlier net-submission volume (R_NSub) case. T-test and Wilcoxon test results for above defined hypotheses give an additional indication on the relationship of ratios (Table 13).

Table 13: Test results for net-provided trading volume

Hypothesis	μ	Significance
$H_0 : \mu\left(R\_Nlq_{high}^{AT}\right) \leq \mu\left(R\_Nlq_{high}^{H}\right)$	mean	0.107
$H_0 : \mu\left(R\_Nlq_{low}^{AT}\right) \leq \mu\left(R\_Nlq_{low}^{H}\right)$	mean	0.006
$H_0 : \mu\left(R\_Nlq_{high}^{AT}\right) \leq \mu\left(R\_Nlq_{high}^{H}\right)$	median	0.725
$H_0 : \mu\left(R\_Nlq_{low}^{AT}\right) \leq \mu\left(R\_Nlq_{low}^{H}\right)$	median	0.000

The test results show that the difference in net-provided trading volume is not significantly different between algorithmic traders and humans in periods of high volatility. To sum up: While we have seen a significant different in R_NSub_{high}, there is no significant different in R_Nlq_{high}. In other words, algorithmic traders do seem to provide more committed liquidity ($NSub$) during times of high volatility. The provided liquidity is, however, not used by the market in transactions, i.e. it does not translate into higher degrees of supplied liquidity in executions. Consequently, the negative net-submission ratios of humans during periods of high volatility might simply be the reaction to unintentional liquidity-supply during in executions. Furthermore, for periods of low volatility the relative strength of R_NSub and R_Nlq is even twisted. For example humans exhibit a smaller R_NSub than algorithmic traders, but simultaneously show a larger R_Nlq.

Nonetheless, overall we do not find evidence that algorithmic traders withdraw more liquidity from the market than humans do. Therefore, we may conclude that algorithmic traders do not increase volatility by not supplying liquidity; at least not more than humans do.

7. CANCELLATION BEHAVIOR OF ALGORITHMIC TRADERS
7.1 Motivation
Closely related to above evaluated provision of liquidity, we analyze the order adjustment behavior of algorithmic traders in this section. [11], for instance, suggest that algorithmic traders constantly observe the market and adapt their placed orders accordingly. Consequently, the lifetimes of orders submitted by algorithmic traders are significantly shorter than the lifetimes of orders submitted by humans [9].

If algorithmic traders were somehow responsible for higher volatility levels, then we should – at least – observe a significantly different order adjustment behavior of algorithmic traders during periods of high and low volatility respectively. As order adjustment usually takes place by cancelling an existing order and

submitting another order (Table 1), we analyze the order cancellation behavior.

7.2 Methodology

The evaluation of cancellation behavior is straight forward. For each high / low volatility 5-minute interval we examine the orders that were actively cancelled during the interval. For each of these orders we calculate the time to cancellation Tc, i.e. the time difference between submission and cancellation. As we are able to differentiate between orders that were submitted by humans and orders that were submitted by algorithmic traders, we calculate the time to cancellation Tc for both groups of traders separately.

If algorithmic traders were responsible for high volatility levels, we would expect them to exhibit a significantly different cancellation behavior during periods of high volatility compared to periods of low volatility. Moreover, we would even expect Tc^{AT} to be smaller during periods of high volatility because algorithmic traders need to adjust their orders more frequently. In periods of frequent trading activity, humans also need to adjust their orders more frequently, but these are not necessarily able to do so (technically / operationally). We consequently formulate the following null- and alternative hypotheses:

$$H_0 : \mu\!\left(Tc_{high}^{trader}\right) \ge \mu\!\left(Tc_{low}^{trader}\right) \quad vs. \quad H_A : \mu\!\left(Tc_{high}^{trader}\right) < \mu\!\left(Tc_{low}^{trader}\right)$$

Hereby, Tc^{AT} (Tc^{H}) depicts the time to cancellation for algorithmic traders (humans). Tc_{high} (Tc_{low}) depicts time to cancellation for high (low) interval subsamples.

7.3 Empirical Results

Descriptive statistics for the lifetimes of cancelled orders can be found in Table 14. First, it can be observed that independent of the intervals, i.e. high or low volatility, the time to cancellation for algorithmic traders is shorter than the time to cancellation for humans. With regard to algorithmic traders only, it can be seen that the mean time to cancellation is very similar during both high and low periods of volatility. In other words, algorithmic traders do not seem to particularly adjust their cancellation behavior to existing volatility levels. Contrary, humans cancel their orders earlier during periods of high volatility.

Table 14: Descriptive statistics for orders cancelled

	Mean (in sec)	Standard deviation
Tc_{high}^{AT}	777	6,565
Tc_{low}^{AT}	752	5,971
Tc_{high}^{H}	4,448	13,102
Tc_{low}^{H}	7,294	44,322

T-test results for above defined hypotheses are presented in Table 15.

Table 15: Test results for orders cancelled

Hypothesis	Significance
$H_0 : \mu\!\left(Tc_{high}^{AT}\right) \ge \mu\!\left(Tc_{low}^{AT}\right)$	0.469
$H_0 : \mu\!\left(Tc_{high}^{H}\right) \ge \mu\!\left(Tc_{low}^{H}\right)$	0.061

The test results affirm our descriptive analysis with regard to the fact that the cancellation behavior of algorithmic traders in high

and low periods of volatility is not significantly different from each other and that the cancellation behavior of humans in high and low periods of volatility is significantly different from each other.

To conclude, we have seen that algorithmic traders cancel their orders after shorter periods of time than humans do. Nonetheless, in terms of a potential influence on volatility we cannot find a significantly different cancellation behavior of algorithms during periods of high volatility. Consequently, their cancellation behavior is most likely not responsible for the increased volatility levels.

8. CONCLUSION & DISCUSSION

Having conducted a literature review, we identified the impact of algorithmic trading on volatility (in the equity market) as an area of research that still lacks sufficient insights. Against this background, we empirically evaluated a unique high-frequency Xetra dataset that allowed us to precisely differentiate between algorithmic trading activity and human activity. Hereby, we picked up different arguments on how algorithmic traders could potentially increase volatility.

Overall, our results provide sufficient evidence that algorithmic traders do not increase volatility more than humans do. In particular, we found that algorithmic traders in aggregate follow trading strategies that are as diverse as human strategies. Moreover, algorithmic trading participation does not significantly increase volatility levels; actually the opposite seems to be true. With regard to liquidity supply, algorithmic traders do not withdraw liquidity during periods of high volatility. Finally, algorithmic traders do not seem to adjust their order cancellation behavior to the respective volatility levels.

It shall, however, be noted that our research results are limited with regard to the period of investigation. In the course of time, algorithmic traders may for instance have adapted similar trading strategies. Given that applied methodologies and models have become increasingly sophisticated, we do not believe that algorithmic trading strategies will turn less diverse though.

Moreover, our results may not hold true in periods of extreme market movements. Even though we already investigated the top 95%-percentile volatility intervals (Figure 2), more extreme market situations such as the "flash crash" observed on May 6th 2010 are not represented by our data subset. On May 6th the Dow Jones Industrial Average fell nearly 1,000 points and single stocks such as Accenture fell to one-cent, i.e. almost lost all its' value, in a 10-second period. While the market already recovered the same day, the Securities and Exchange Commission (SEC) is still looking for explanations and potential solutions. Against the background of our results, we would like to contribute to the discussion of possible causes and recall that algorithmic traders – as often blamed – do not seem to systematically withdraw liquidity from the market during periods of high volatility. In cases of extremely high volatility, however, these may naturally bail out of the market to reduce their own risk. This behavior should also be observed with human traders. The only difference is that algorithmic traders are able to make these decisions within milliseconds. Consequently, markets should also be "designed" [20] to handle actors that react within milliseconds to market movements. In other words, as algorithmic traders do not increase volatility during "normal" trading weeks, whatsoever constraint on high-frequency trading cannot be the solution to the observed

problem. Instead market safeguards should be adapted to European standards, i.e. volatility interruptions on a single stock basis (Xetra) instead of a market circuit-breaker. Appropriately employed market interruptions will enable for example market makers to adequately adjust their quotes, so that their stub quotes are not hit [18].

Future research should therefore also concentrate on the role of algorithmic (or high-frequency) trading in periods of extremely high volatility.

9. APPENDIX I: BENCHMARK MODEL

As stated above, the benchmark model is taken from [2]. In the model there are $H_{passive}$ potential human liquidity providers, $H_{aggressive}$ potential human liquidity takers, $AT_{passive}$ potential algorithmic trader liquidity providers, and $AT_{aggressive}$ potential algorithmic trader liquidity takers. For a given period, the probability of an algorithmic trader providing liquidity is equal to (passive/aggressive):

$Prob(AT/ALL) = AT_{passive} / (H_{passive} + AT_{passive}) = \alpha_{passive}$

Analogue, the probability of an algorithmic trader taking liquidity is equal to:

$Prob(ALL/AT) = AT_{aggressive} / (H_{aggressive} + AT_{aggressive}) = \alpha_{aggressive}$

Assuming that these events are independent, the following probabilities result for the four possible passive/aggressive combinations:

$Prob(H/H) = (1 - \alpha_{passive}) (1 - \alpha_{aggressive})$

$Prob(H/AT) = (1 - \alpha_{passive}) \alpha_{aggressive}$

$Prob(AT/H) = \alpha_{passive} (1 - \alpha_{aggressive})$

$Prob(AT/AT) = \alpha_{passive} \alpha_{aggressive}$

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10. REFERENCES

[1] Andersen, T., Bollerslev, T., Diebold, F., and Labys, P. 2003. *Modeling and Forecasting Realized Volatility*. Econometrica, 71, 2.

[2] Chaboud, A., Chiquoine, B., Hjalmarsson, E., and Vega, C. 2009. *Rise of the Machines: Algorithmic Trading in the Foreign Exchange Market*. International Finance Discussion Papers – Board of Governors of the Federal Reserve System. No. 980 (Oct. 2009).

[3] Chakravarty, S., Kalev, P., and Pham, L.T. 2005. *Stealth Trading in Volatile Markets*. Working Paper, Monash University, Australia.

[4] Dacorogna, M. M., Gençay, R., Müller, U. A., Olsen, R. B., and Pictet, O. V. 2001. *An Introduction to High-Frequency Finance*. Academic Press, San Diego, CA.

[5] Deutsche Börse AG 2007. *ATP Agreement for Participation in the "Automated Trading Program" (ATP) via the Electronic Trading System Xetra*. DOI=http://deutsche-boerse.com.

[6] Deutsche Börse AG 2007. *Factbook 2007*. DOI= http://deutsche-boerse.com.

[7] Domowitz, I., and Yegerman, H. 2005. *The Cost of Algorithmic Trading - A First Look at Comparative Performance*. Bruce, B. (eds.) *Algorithmic Trading: Precision, Control, Execution*. Institutional Investors Inc. 30-40.

[8] Gomber, P., Budimir, M., Kosciankowski, K., Urtheil, R., Lohmann, M., Nopper, N., and Henning, P. 1999. *Agentenbasierter Rentenhandel*. Wirtschaftsinformatik, 41, 2.

[9] Groth, S. S. 2009. *Algorithmic Trading Engines & Liquidity Contribution – The Blurring of "Traditional" Definitions*. Software Services for e-Business and e-Society, 9th IFIP WG 6.1 Conference, I3E 2009; Springer, Boston, 210-224.

[10] Gsell, M. 2008. *Assessing the Impact of Algorithmic Trading on Markets: A Simulation Approach*. Proceedings of the 16th European Conference on Information Systems (ECIS), 587-598.

[11] Gsell, M., and Gomber, P. 2009. *Algorithmic Trading Engines versus Human Traders – Do They Behave Different in Securities Markets?* Proceedings of the 17th European Conference on Information Systems (ECIS), Verona, Italy.

[12] Hendershott, T., Jones, C., and Menkveld, A (forthcoming). *Does Algorithmic Trading Improve Liquidity?* The Journal of Finance.

[13] Hendershott, T., Riordan, R. (2009). *Algorithmic Trading and Information*. NET Institute Working Paper #09-08.

[14] Kissel, R., and Malamut, R. 2006. *Algorithmic Decision-Making Framework*. Journal of Trading, 1, 12-21.

[15] Picot, A., Bortenlaenger, C., and Roehrl, H. 1995. *The Automation of Capital Markets*. Journal of Computer-Mediated Communication, 1, 3.

[16] Riordan, R., and Storkenmaier, A. 2008. *Exchange System Innovation and Algorithmic Liquidity Supply*. Australasian Finance and Banking Conference.

[17] Sarkar, A., and Schwartz, R. A. 2009. *Market Sidedness: Insights into Motives for Trade Initiation*. The Journal of Finance. 64, 1, 375-423.

[18] Schapiro, M. L. 2010. *Testimony Concerning the Severe Market Disruption on May 6, 2010*. U.S. Securities and Exchange Commission. DOI= http://www.sec.gov/.

[19] Weinhardt, C., and Gomber, P. 1999. *Elektronisierung des Außerbörslichen Wertpapierhandels*. Wirtschaftsinformatik, 41, 6.

[20] Weinhardt, C., Holtmann, C., and Neumann, D. 2003. *Market Engineering*. Wirtschaftsinformatik, 45, 6.

[21] Wood, R.A., McInish, T. H., and Ord, J. K. 1985. *An Investigation of Transactions Data for NYSE Stocks*. The Journal of Finance, 40, 3.

[22] Yang, J., and Jiu, B. 2006. *Algorithm Selection: A Quantitative Approach*. Algorithmic Trading II. Institutional Investor, New York, 26-34.